Macroscopic Quantumness in Many-body systems

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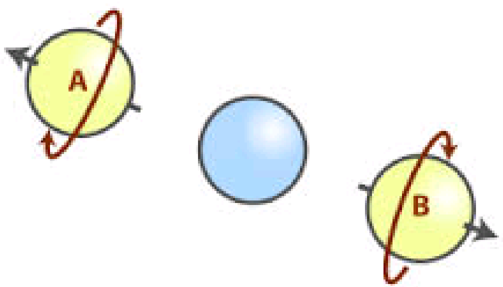
Two Quantum Paradoxes

EPR



$$|01\rangle + |10\rangle$$

This state yields the violation of local realism

Schrödinger's Cat

$$|\alpha\rangle + |-\alpha\rangle$$

Contradiction to macroscopic realism?

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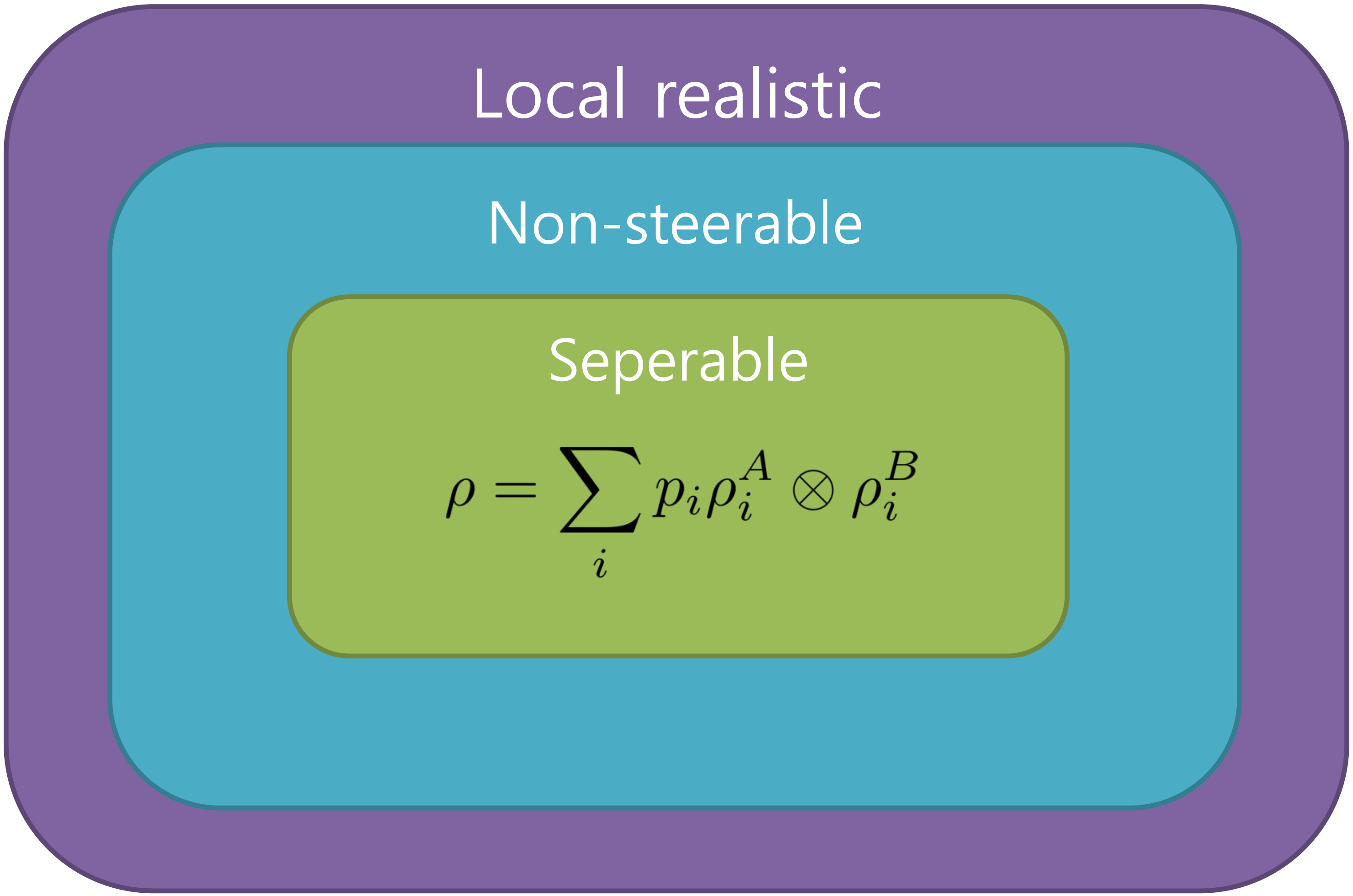
I. Introduction to Entanglement

Local Realism

- Reality
  - All objects must objectively have a pre-existing value for any possible measurement before the measurement is made.
- Locality
  - If the objects A and B are space-like separated, any external influence on A cannot directly influence the object B.
- Introduces local hidden-variable theory, which is rejected by the Bell inequality test.

Locality hierarchy



Why entanglement is important?

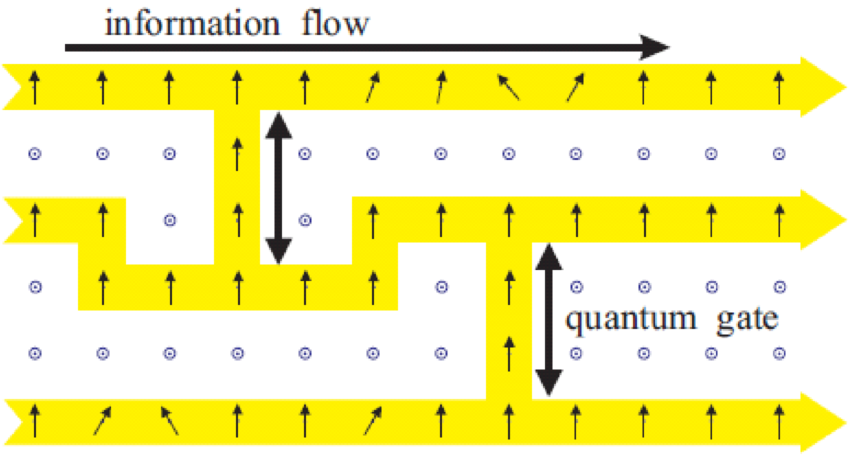
- Resources for quantum information processing
  - Quantum computation, quantum teleportation, quantum cryptography, …
- Explains why the classical models fail in many-body physics.
  - Mean field theory, spin frustration, …
- Can be used to detect special behaviors of many-body systems.
  - Quantum phase transitions, non-equilibrium phases, topological order, …

Cluster state quantum computation*

- Cluster State

$$|\phi_N\rangle = \frac{1}{2^{N/2}} \bigotimes_{a=1}^{N} (|0\rangle_a \sigma_z^{(a+1)} + |1\rangle_a)$$

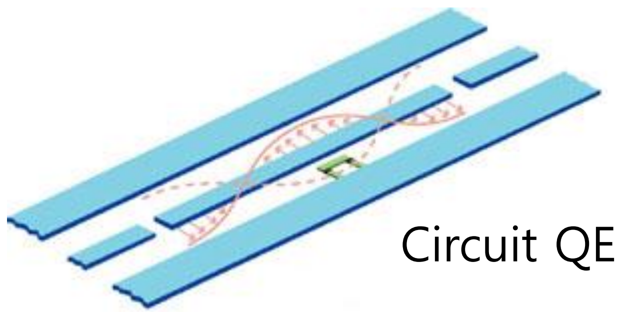
- Projection measure on one qubit is equivalent to applying unitary gate to the next qubit.



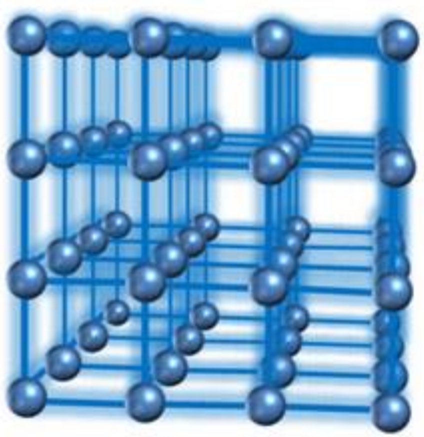
*: H. J. Breigel and R. Raussendorf, Phys. Rev. Lett. 56, 910 (2001); M. A. Neilson, Rep. Math. Phys. 57, 147 (2006)

Entanglement and many-body systems

Make entangled states using many-body systems



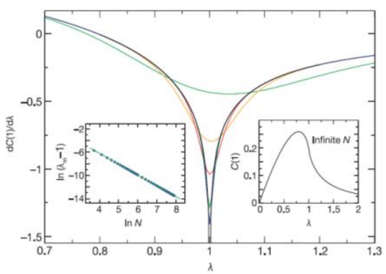
Circuit QED



Optical lattice

Using entanglement to study many-body physics

Quantum Phase Transition



Matrix Product states

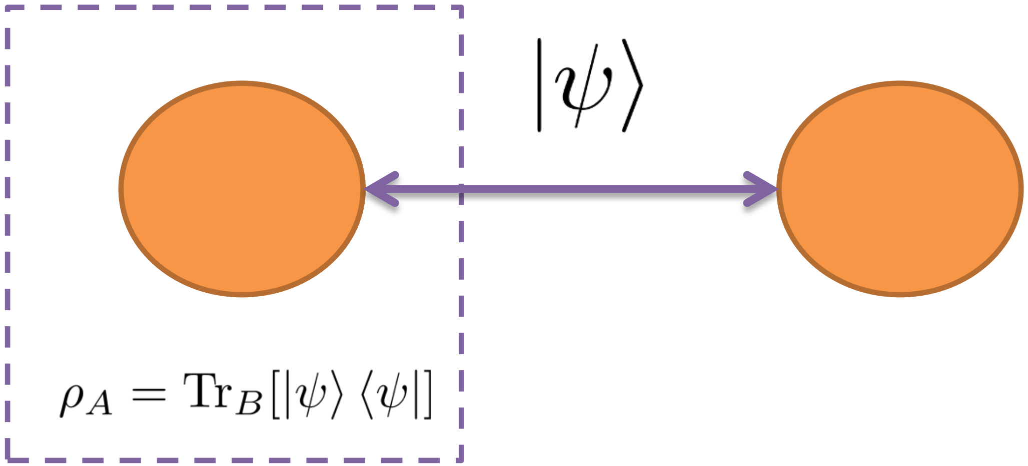
$$|\Psi\rangle = \sum_{\{s\}} \mathrm{Tr}[A_1^{(s_1)} A_2^{(s_2)} \cdots A_N^{(s_N)}] |s_1 s_2 \ldots s_N\rangle,$$

Topological Entanglement Entropy

$$S(\rho) = \frac{\partial}{\partial T}(T \log Z) = \alpha L - \log(\mathcal{D}/d_a).$$

Bipartite Entanglement

- How to measure the size of entanglement?



$$S(\rho_A) = \mathrm{Tr}[\rho_A \log \rho_A]$$

- It is difficult to make appropriate measure for general mixed states.
- And even more unclear for multipartite case.

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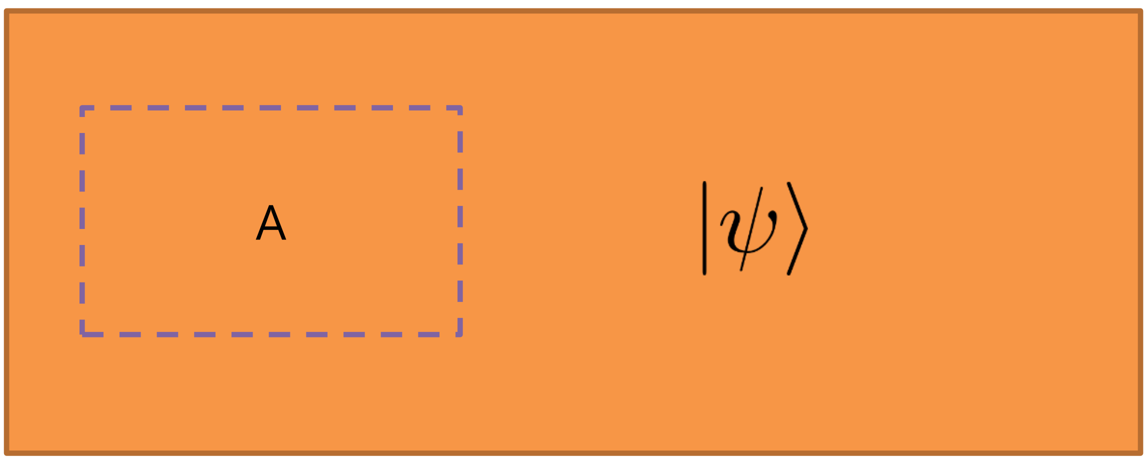
Entanglement Area Law

- Consider the infinite system with local interacting Hamiltonian H.
  - Ex. transverse Ising mode: $H = \sum \sigma_x^i \sigma_x^{i+1} + J \sum \sigma_z^i$
- For the ground state $|\psi\rangle$, the entanglement entropy of the subsystem of block of area A:



$$\rho_A = \mathrm{Tr}_B[|\psi\rangle\langle\psi|] \qquad S(\rho_A) = \mathrm{Tr}[\rho_A \log \rho_A] \sim \partial A$$

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Matrix Product States

- From the entanglement area law, entanglement entropy of any block in 1D is bounded by a constant (non-critical).
- Efficient description of quantum states can be possible.

$$|\psi\rangle = \sum_{\sigma_1,\sigma_2,\ldots,\sigma_N} \psi_{\sigma_1,\sigma_2,\ldots,\sigma_N} |\sigma_1,\sigma_2,\ldots,\sigma_N\rangle \qquad 2^N \text{ parameters}$$

$$|\psi\rangle = \sum_{\sigma_1,\sigma_2,\ldots,\sigma_N} \mathbf{A}^{\sigma_1}\mathbf{A}^{\sigma_2}\cdots\mathbf{A}^{\sigma_N} |\sigma_1,\sigma_2,\ldots,\sigma_N\rangle \qquad DN \text{ parameters}$$

A is matrices smaller than $D \times D$

- The bond dimension D is related to the entanglement entropy.

Multipartite Entanglement

- A widely accepted measure does not exist.
- Geometric Entanglement

$$E_G(|\psi\rangle) = -\log_2 \sup_{|\phi_{sep}\rangle} |\langle \phi_{sep} | \psi \rangle|^2$$

- The properties are not much known.
- Large geometric entanglement means it is hard to use for measurement based quantum computation.*

*Phys. Rev. Lett. **102**, 190502 (2009).

II. Macroscopic Quantumness

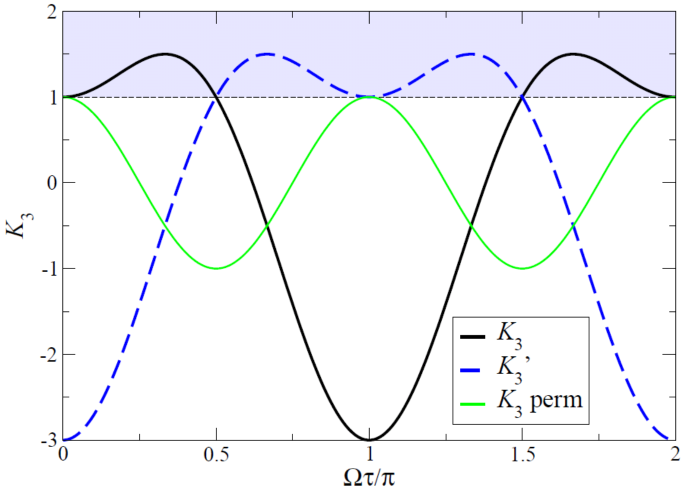
Macroscopic Realism*

- Two postulates
  - Macrorealism per se: "A macroscopic object, which has available to it two or more macroscopically distinct states, is at any given time in a definite one of those states.“
  - Noninvasive measurability: "It is possible in principle to determine which of these states the system is in without any effect on the state itself, or on the subsequent system dynamics."

*: A. J. Leggett and A. Garg, PRL **54** 857 (1985).

Leggett-Garg inequality

- For successive times $t_1 < t_2 < t_3$,
$$K = C_{12} + C_{23} - C_{13} \leq 1.$$
- C_{ij} is the temporal correlation function for t_i and t_j.
- The inequality is always violated in microscopic scale.
- Ex:
  - Qubit rotating with x axis: $H = \frac{1}{2}\Omega\sigma_x$.
  - We measure σ_z, the violation can be seen.
  - The initial state does not matter.



*: Rep. Prog. Phys. 77, 016001 (2014)

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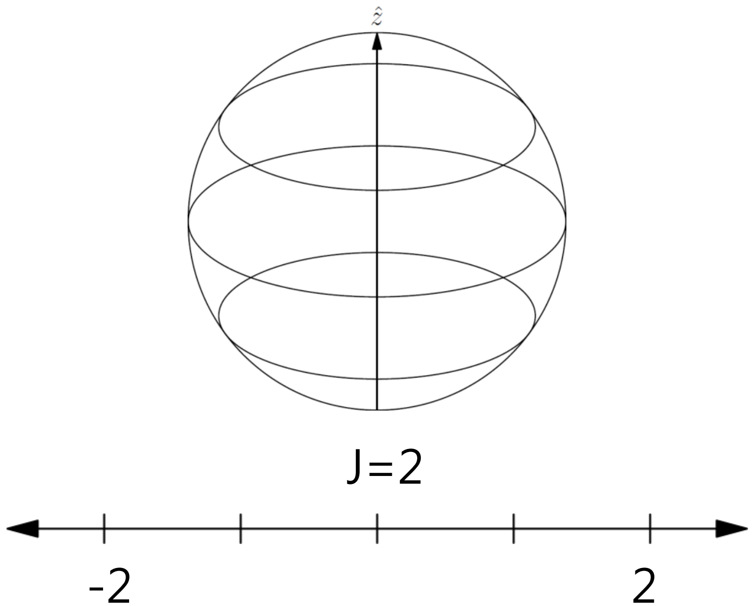
Coarse grained measurement*

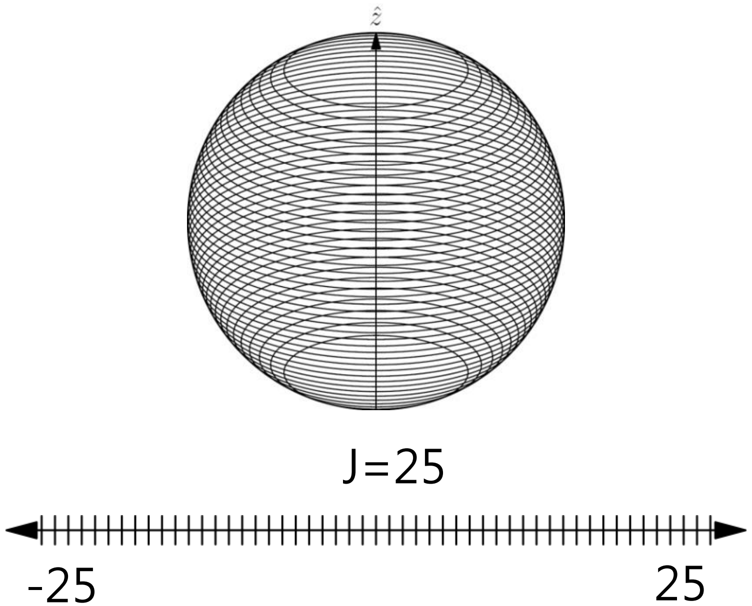
- If we can resolve the all eigenvalues of the projection measurement, the violation of Leggett-Garg inequality is general.
- Why we cannot see the violation of macroscopic realism in everyday life?
- Classical measurement
  - The eigenvalues of the measurement operator have direct physical meaning.
  - Cannot distinguish the neighboring eigenvalues.

*: J. Kofler and C. Brukner, PRL **99** 180403 (2007).

Coarse grained measurement (cont.)

- Suppose we measure J_z of spin-j system.





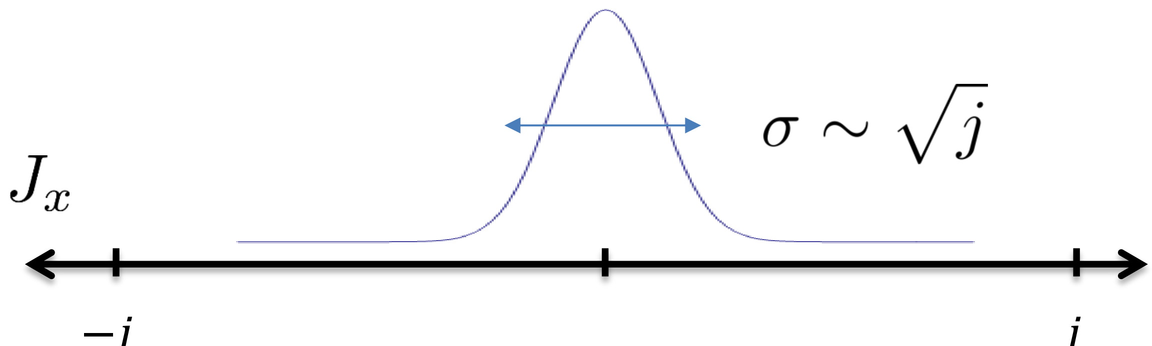
- For large j system, we cannot identify each eigenvalue.

Coarse grained measurement (cont.)

- The spin coherent state in z direction

$$J_z\left|j,j\right\rangle = j\left|j,j\right\rangle.$$

- If we measure the state in x direction, the distribution of measurement outcomes is a Gaussian shape in $j \gg 1$.



- The dispersion of the measurement outcomes is $\sigma{\sim}\sqrt{j}$.
- If the resolution of our measurement operator is $\Delta m \gg \sqrt{j}$, the outcomes can be treated as a single value.

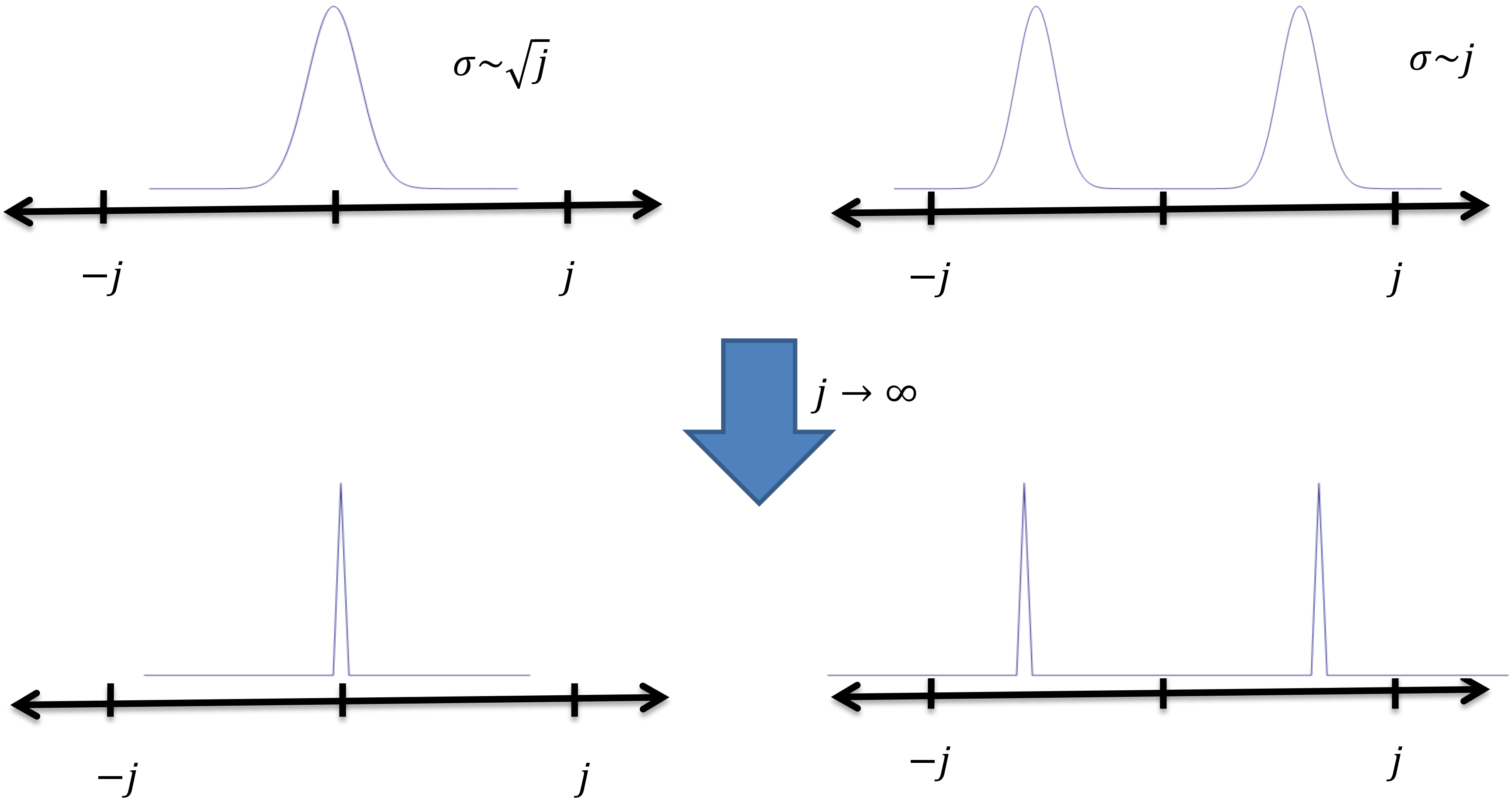
Coarse grained measurement (cont.)

- Any pure state $\sigma \sim \sqrt{j}$ give a single measurement outcome and is not disturbed by the coarse-grained measurement.

- If a pure quantum state give $\sigma \sim j$, the state can be considered as a superposition between macroscopically distinct states (in terms of eigenstate of J_z).

- Macroscopic realism is not violated unless the Hamiltonian generate a macroscopic quantum superposition from the classical-like state*.

*: J. Kofler and C. Brucker, PRL **101** 090403 (2008).



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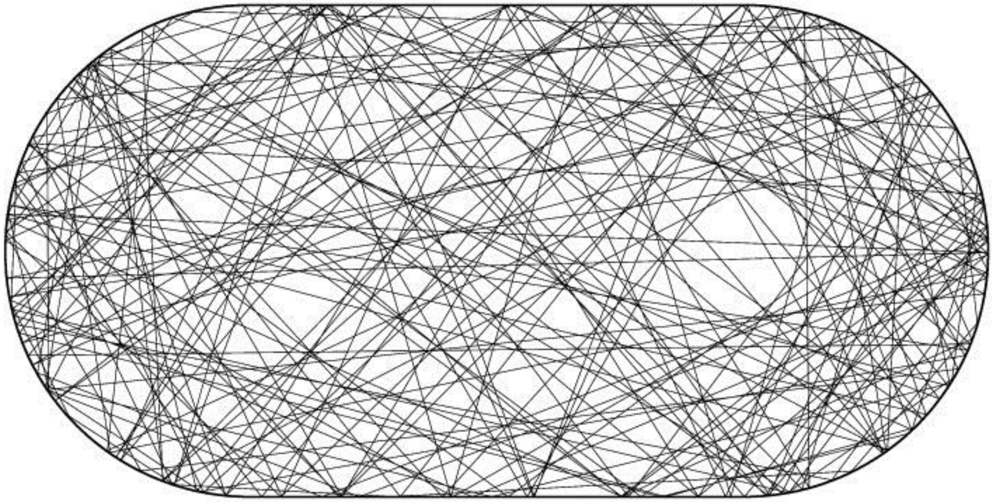
III. Application to the thermalization of a closed system*

Thermalization of a Closed System

- In classical mechanics, ergodicity makes the time average of the observable would be the same to the ensemble average.
- A non-equilibrium systems evolve into equilibrated thermal state.



Eigenstate Thermalization Hypothesis

- Is a closed quantum system equilibrate?
- When $\{|\psi_n\rangle\}$ are eigenstates of the Hamiltonian with energy E_n, the expectation value of the observable A for initial state $\sum_n c_n|\psi_n\rangle$ is

$$\langle A\rangle = \sum_n |c_n|^2 \langle\psi_n|A|\psi_n\rangle + \sum_{n\neq m} e^{-i(E_n-E_m)t} c_m^* c_n \langle\psi_m|A|\psi_n\rangle .$$

- Equal to the expectation value using the diagonal ensemble after dephasing when $\langle\psi_m|A|\psi_n\rangle$ is small.

$$\rho = \sum_n |c_n|^2 |\psi_n\rangle\langle\psi_n|$$

- This value is equal to microcanonical ensemble average if $\langle\psi_n|A|\psi_n\rangle$ does not vary much (as a function of n).

Many-body localization*

- Many closed systems thermalize in the sense that an expectation value of a local observable approaches to thermal equilibrium value.
- There are some systems which does not thermalize
  - Anderson localization (non-interacting, presence of disorder)
  - Integrable system (many conservative laws)
- It is usually believed that the interacting non-integrable system thermalize.
- However, it is recently known that there are some interacting systems do not thermalize.

→ Many-body localization

*: D. M. Basko et al., Ann. Phys. **321**, 1126 (2006);
R. Nandkishore and D. A. Huse, Annu. Rev. Condens. Matter Phys. **6** 15(2015).

Entanglement Entropy*

- If a system thermalize, the entropy of a subsystem follows volume law as it is extensive quantity.
- It is the same for the closed system case. The entanglement entropy of the subsystem of typical eigenstates of the Hamiltonian obeys
  - Volume law for thermalization case.
  - Area law for localization case.
- Large entanglement between the subsystem and the remaining system needed for thermalization.

*: B. Bauer and C. Nayak, J. Stat. Mech. (2013) P09005

Thermalization and macroscopic quantumness

- If macroscopically distinct states evolve into the similar thermalized states, the superposition between two may not be a macroscopic superposition.

- In thermal system, the relative fluctuation of macroscopic observables to the system size is suppressed.

- For N spin-$1/2$ system, $\langle A^2 \rangle - \langle A \rangle^2 \propto N$ for $A \in S$ in thermal case*.

*: A. Shimizu and T. Miyadera, PRL **89** 270403 (2002).

Disordered Heisenberg Model

- We use disordered Heisenberg model to test this conjecture.

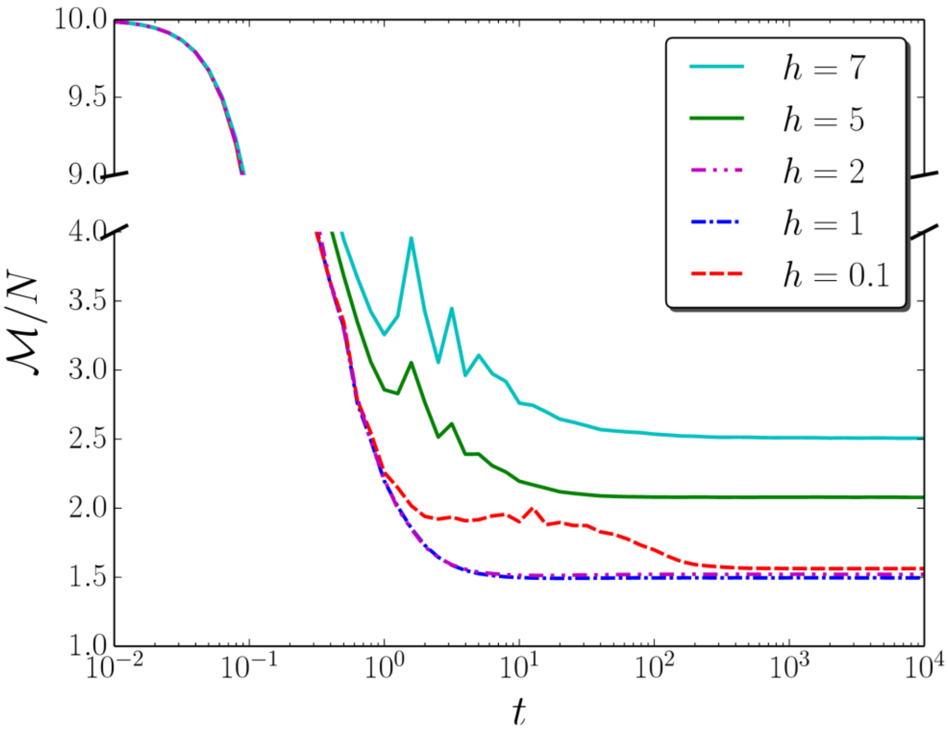
$$H = J\sum_{i=1}^{N}\left[\sigma_x^{(i)}\sigma_x^{(i+1)} + \sigma_y^{(i)}\sigma_y^{(i+1)} + \sigma_z^{(i)}\sigma_z^{(i+1)}\right] + \sum_{i=1}^{N} h_i\sigma_z^{(i)}$$

- h_i sampled randomly in $[-h, h]$. The Hamiltonian thermalize for small h but enters to many-body localization phase for $h \gtrsim 3.6$*.

*: A. Pal and D. Huse, PRB **82**, 174411 (2010).
D. Luitz, N. Laflorencie, and F. Alet, PRB **91** 081103(R) (2015).

Time evolution

- For $v = 0.6$, we averaged $\mathcal{M}/N$ for all realization of disorder and initial states.
- The dynamics of the averaged value of $\mathcal{M}/N$.



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