Phase phenomena in probabilistic theories

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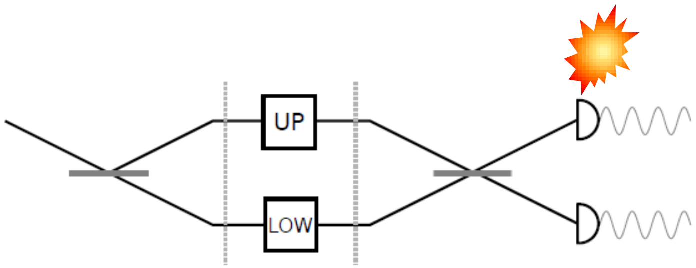
‘Phase phenomena in probabilistic theories’

- By *phase* I mean the $e^{i\theta}$ appearing in front of a ket in a quantum state, e.g.

$$|\psi\rangle = \frac{|0\rangle + e^{i\theta}|1\rangle}{\sqrt{2}}.$$

- By *phase phenomena* I mean things like interference.



- By *probabilistic theories* I mean something called the ‘convex framework for probabilistic theories’ (a.k.a. generalised probabilistic theories).
  - Quantum theory is a special case in this very wide framework.
  - Essentially any experiment yielding tables of data can be expressed in this way, it is an operational approach. *See e.g. Hardy: Quantum theory from five reasonable axioms, quant-ph/0101012*

Phase phenomena in probabilistic theories: motivation

The broad aim is to investigate phase phenomena in this wider framework for probabilistic theories.

Motivations:

1. To understand phase in quantum theory more deeply: more operationally and also which features of quantum theory are uniquely quantum, e.g. can other theories do Shor's algorithm?
2. To think about post-quantum scenarios: suppose quantum theory doesn't always hold, what is then possible? Phase seems to be at the heart of non-classical phenomena so is a natural focal point.

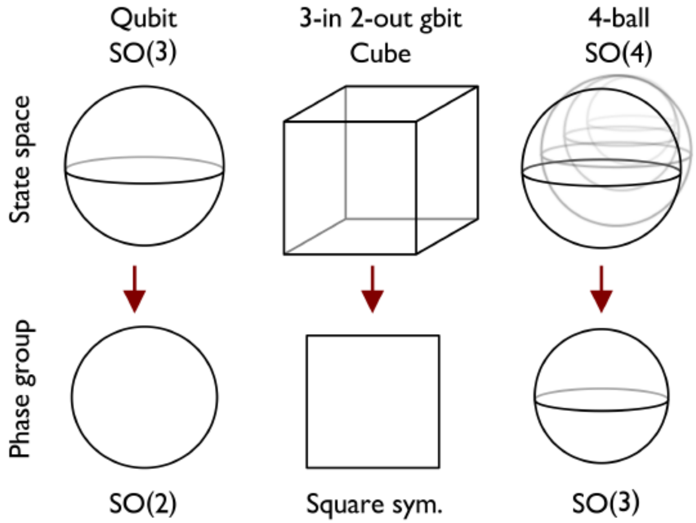
To get started with this we need to have a good definition of 'phase' in the wider framework.

For other treatments of phase beyond the quantum framework see also:
Sorkin ('94). Barnett, Dowker, Rideout ('07). Ududec, Barnum, Emerson ('11). Coecke, Duncan ('11) Coecke, Edwards, Spekkens ('11).

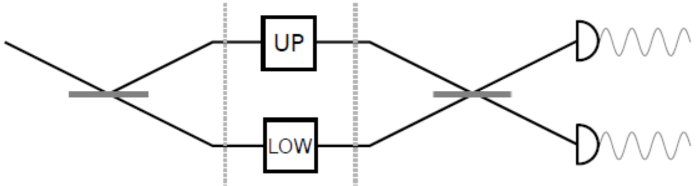
Overview

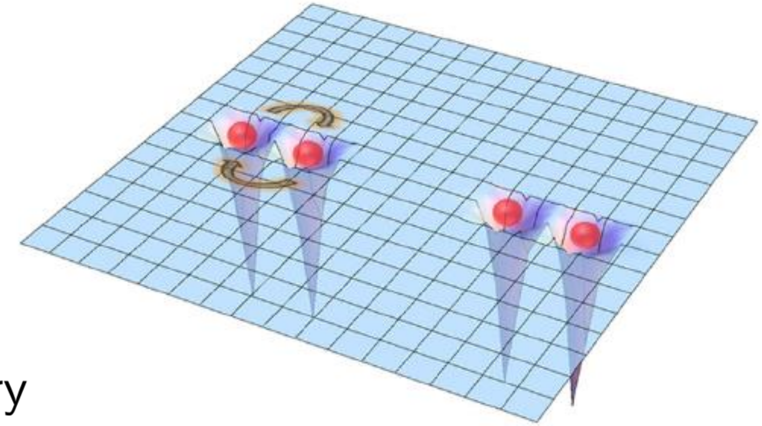
1. Probabilistic theories, how to go beyond quantum.

2. Defining phase in probabilistic theories



3. Phase in interferometers: how uncertainty enables phase dynamics





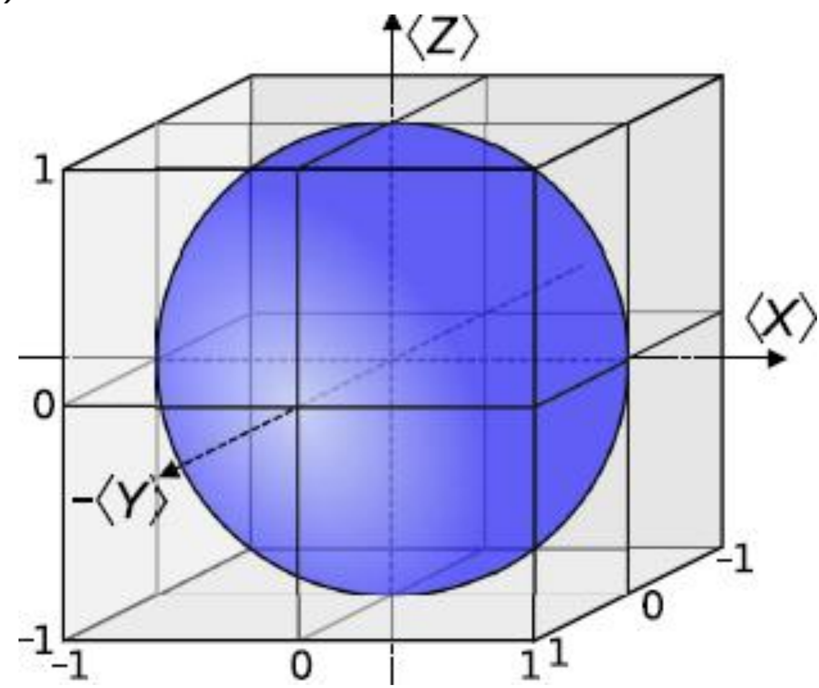
4. Particle exchange phase beyond quantum theory

1. Probabilistic theories

Probabilistic theories: 1/5 “Everything is real”

There are measurement outcomes (effects), states and transformations.

- States are real vectors
  (e.g. in quantum case may write $\rho = \sum_i \xi_i g_i$ where $\xi_i \in \mathcal{R}, g_i = g_i^\dagger$ and take the vector of the ξ_i's as the state.)
- Measurement outcomes are real vectors. (e.g. in quantum case one may represent the POVM elements in this way)
- Transformations are real matrices.
- The framework can be viewed as a generalisation of the Bloch sphere



Probabilistic theories 2/5: Understanding states

- Suppose there are some measurements in your experiment with some outcomes. Repeating the experiment many times with the same settings gives some relative frequencies for all outcomes. This list of real numbers can be taken to be the state vector. Examples:

Measurement	Outcome	Tossing type 1	Tossing type 2
Coin side	H	0.7	0.5
	T	0.3	0.5

Tossing type 1 column: State 1. Tossing type 2 column: State 2. Relative frequencies

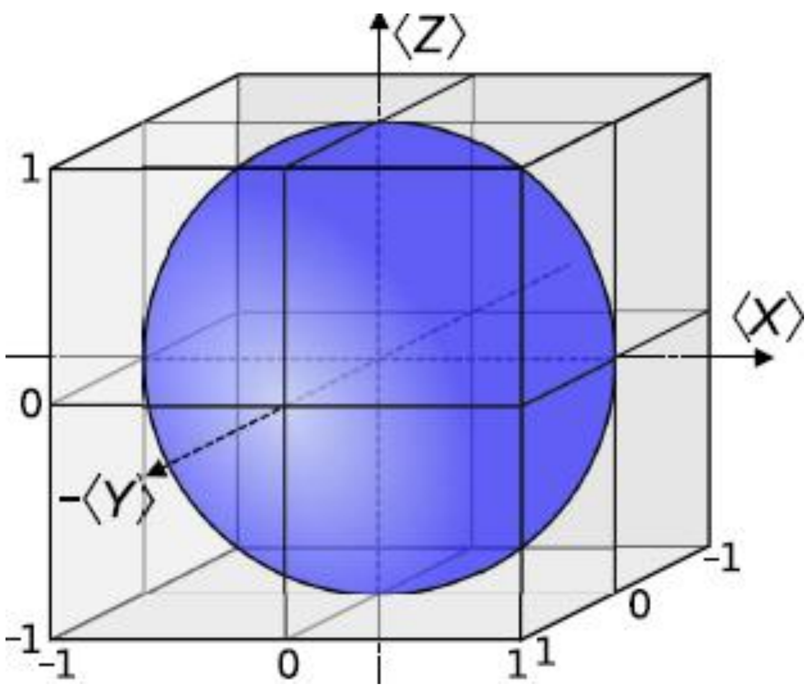
Another example

Measurement	Outcome	Preparation 1	Preparation 2
M1	+1	1	1
	-1	0	0
M2	+1	1	0
	-1	0	1

Preparation 1 column: State 1. Preparation 2 column: State 2

Probabilistic theories 3/5: State spaces

- If we take these state vectors from the data table and draw them as real vectors we get a bunch of vectors associated with a theory.
- We also allow for any probabilistic mixture (=convex combination) of the states (hence the name convex framework).
- This gives a convex body called the state space.

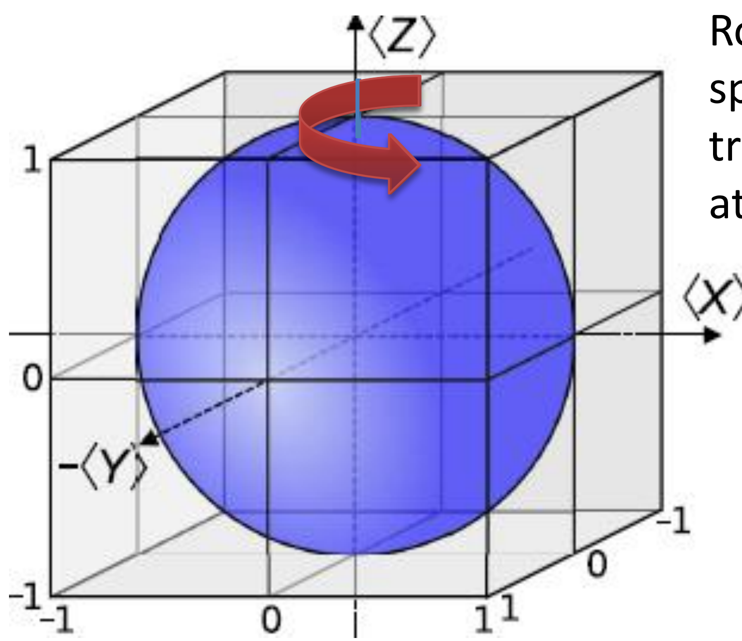


Bloch Sphere is one example of a state space

Probabilistic theories 4/5: Transformations of states

Transformations are
real matrices acting on the state vectors.

A *theory* is state space
plus a set of transformations.

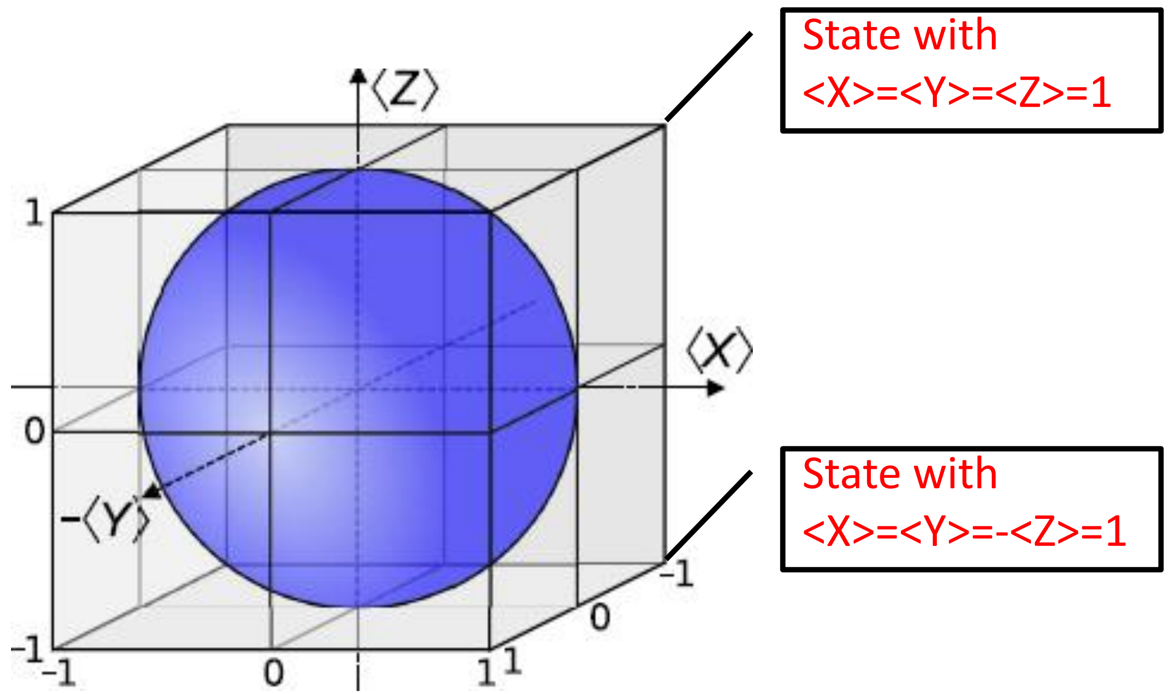


Rotation of sphere: a transform-ation

Probabilistic theories 5/5: Beyond quantum

How can things get more exotic than quantum?

- We can have incompatible measurements without an uncertainty relation, e.g. we could have states s.t. $\langle X \rangle = \langle Z \rangle = 1 \rightarrow$ the cube state space.



- Such a theory allows states ('PR Boxes') that maximally violate the CHSH Bell inequality, giving 4, whilst quantum respects Tsirelson's bound of $2\sqrt{2}$.

- There is in my opinion no killer reason to rule such systems out, though they have not been observed.

2. Phase in probabilistic theories

Phase in probabilistic theories (1/3)

How to define phase in probabilistic theories? Need operational definition.

- Recall quantum case:

Phase

$$|\psi\rangle = \sum_j r_j e^{i\theta_j} |j\rangle \text{ s.t. } r \in \mathbb{R}$$

  Phase arose from (i) choosing a basis, (ii) polar decomposing in that basis.

- Operationally speaking, changing the phases does not change the measurement statistics for the measurement in basis $|j\rangle$. So step (i) becomes more generally to pick a 'frozen' measurement.

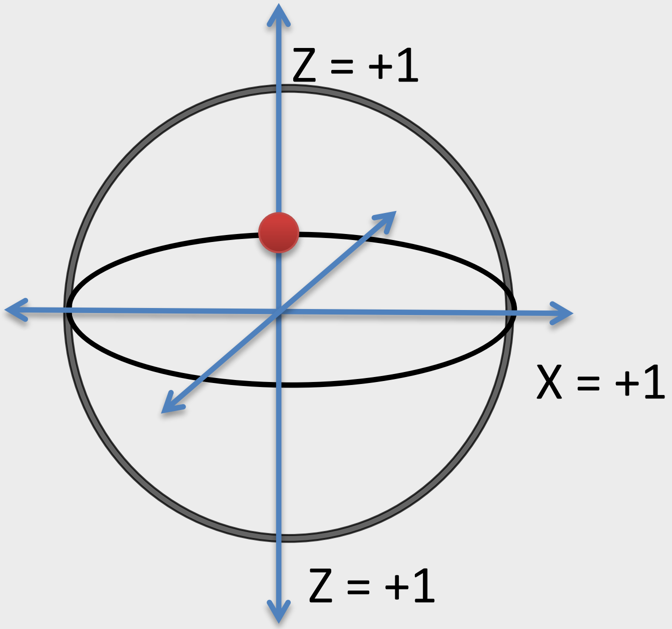
- Moreover operationally speaking only relative phases matter, so **we focus on phase transformations** relating states with different phases.

We define phase transformations associated with measurement M as those which leave the statistics of measurement M invariant for all states.

Phase in probabilistic theories (2/3)

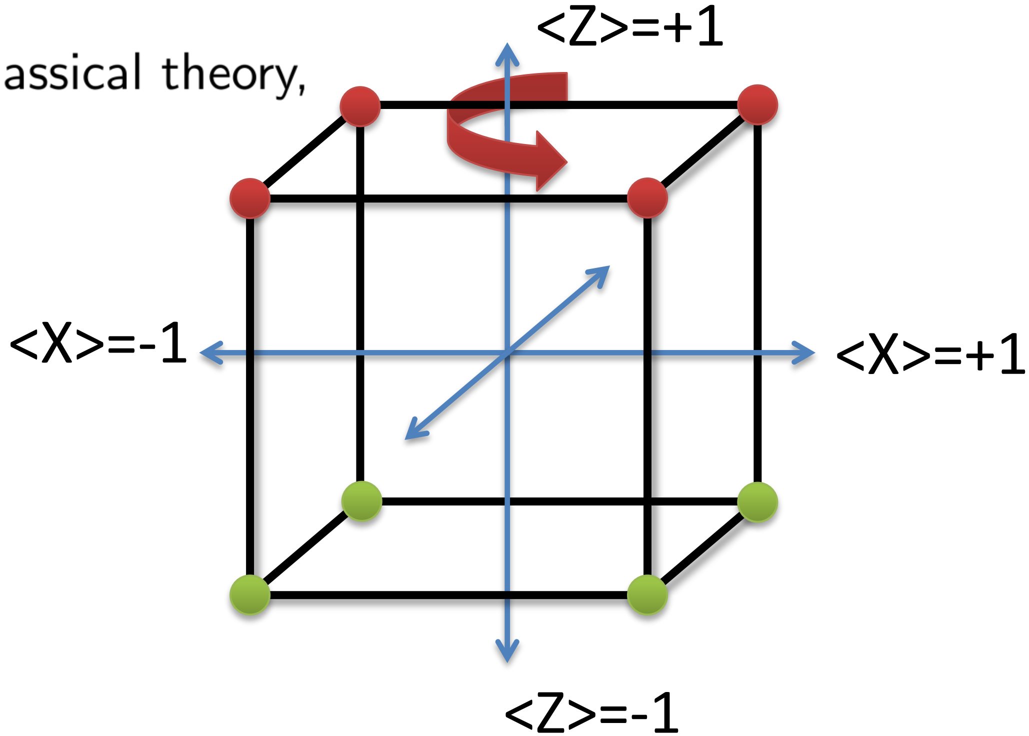
Again: We define phase transformations associated with measurement M as those which leave the statistics of measurement M invariant for all states.

Example: Bloch sphere visualisation of phase transformation associated with Z measurement: $U = e^{i\theta_0 t}|0\rangle\langle 0| + e^{i\theta_1 t}|1\rangle\langle 1|$, $|\psi\rangle = |0\rangle + e^{i\phi}|1\rangle$.



Phase in probabilistic theories (3/3)

Here is another example for a non-classical theory, again for phase dynamics associated with Z-measurement.

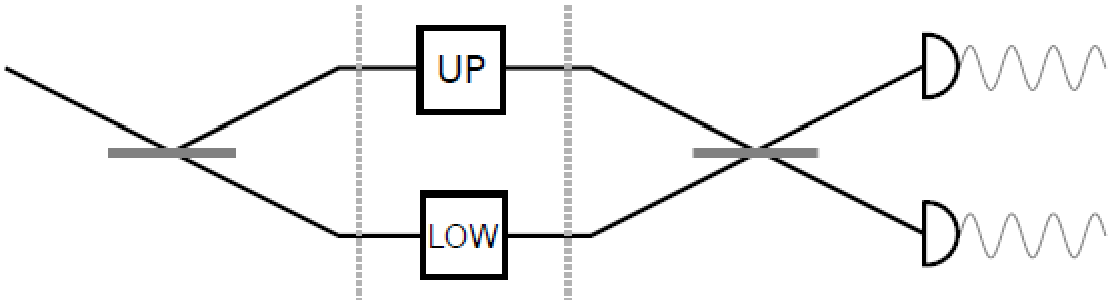


We show that **a theory has non-trivial phase dynamics associated with a maximal measurement if and only if the theory is non-classical**.
(maximal measurement=measurement that deterministically distinguishes maximal number of pure states in a single measurement; classical theory =theory where each state has a unique decomposition into mixture of extremal states).

3. Phase in interferometers: How uncertainty enables phase dynamics

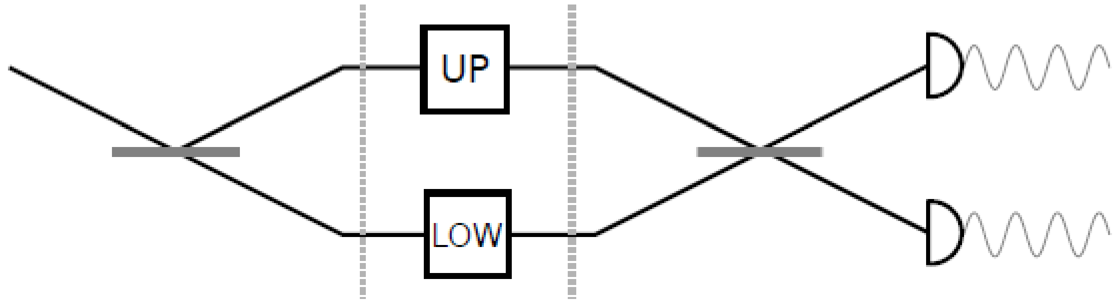
Photo courtesy of
English National Ballet

Why interferometers implement phase dynamics



The transformations on upper and lower branch respectively cannot make the system jump between the branches
$\rightarrow$ they must leave the position statistics invariant
$\rightarrow$ they are phase transformations associated with the position measurement.

A stronger restriction: branch locality



Physical actions on one region of space have no immediate effect on systems with no probability of being detected in that region.

If the system has no probability of being detected where a transformation is, then that transformation must leave the state of the system invariant. We call this *branch locality* (because we are thinking about branches of an interferometer).

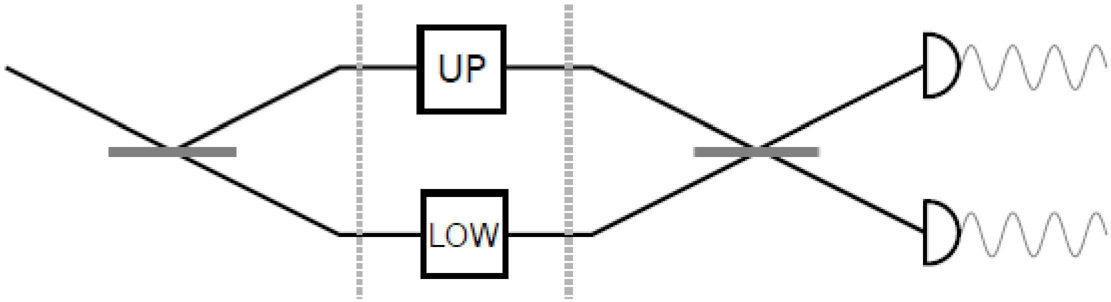
Mathematically, for $\vec{\nu}_{low}$ being a state where the particle has probability 1 of being in the lower branch, and T_{up} a transformation localised to the upper branch:

$$T_{up}\vec{\nu}_{low} = \vec{\nu}_{low},$$

similarly

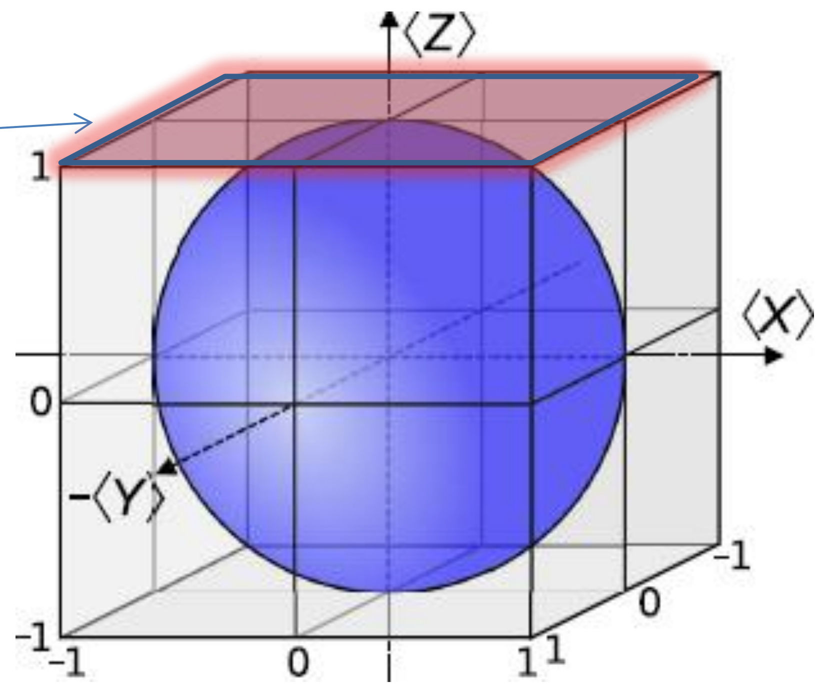
$$T_{low}\vec{\nu}_{up} = \vec{\nu}_{up}.$$

Branch locality, as visualised on Bloch sphere



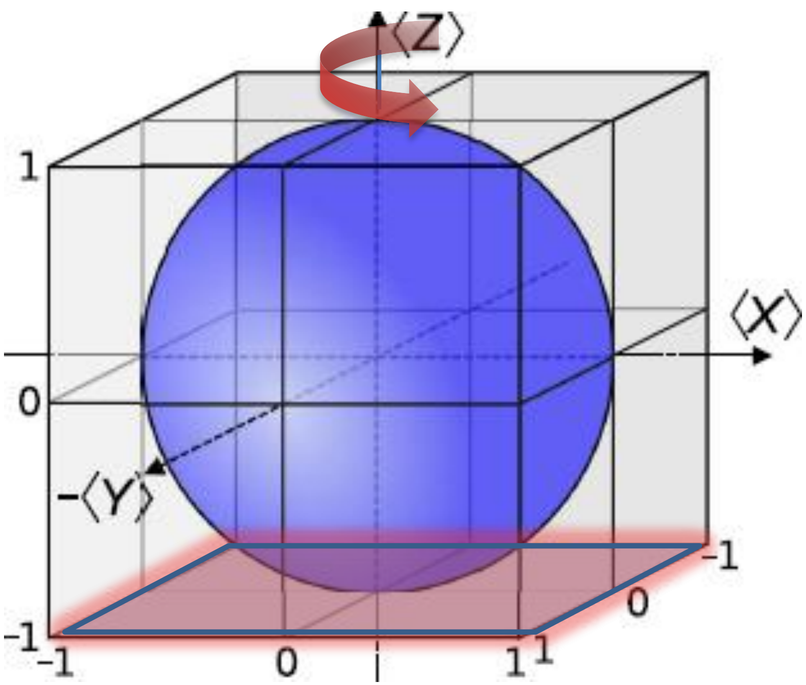
Recall $T_{low}\vec{\nu}_{up} = \vec{\nu}_{up}$.

- Let the Z measurement be the binary position measurement ($Z = \pm 1$ means upper/lower branch).
- Thus T_{low} must leave any state on upper plane ($\langle Z \rangle = +1$) invariant.
- We see this is a heavy restriction on T_{low} in the case of the cubic state space. We show it implies $T_{\text{low}} = I$, $\rightarrow$ no interferometer dynamics!.

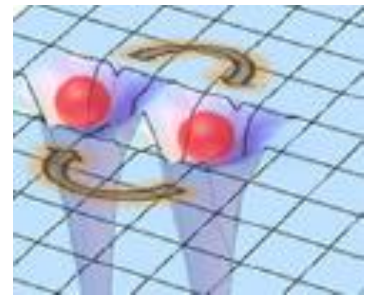


How uncertainty enables phase dynamics

- The qubit sphere however only touches the plane at one point. This allows the sphere to spin without violating Branch locality.
- The reason it only touches one point is essentially the uncertainty principle, which implies that when Z is known X and Y must be fully random.



4. Post-quantum particle exchange phase



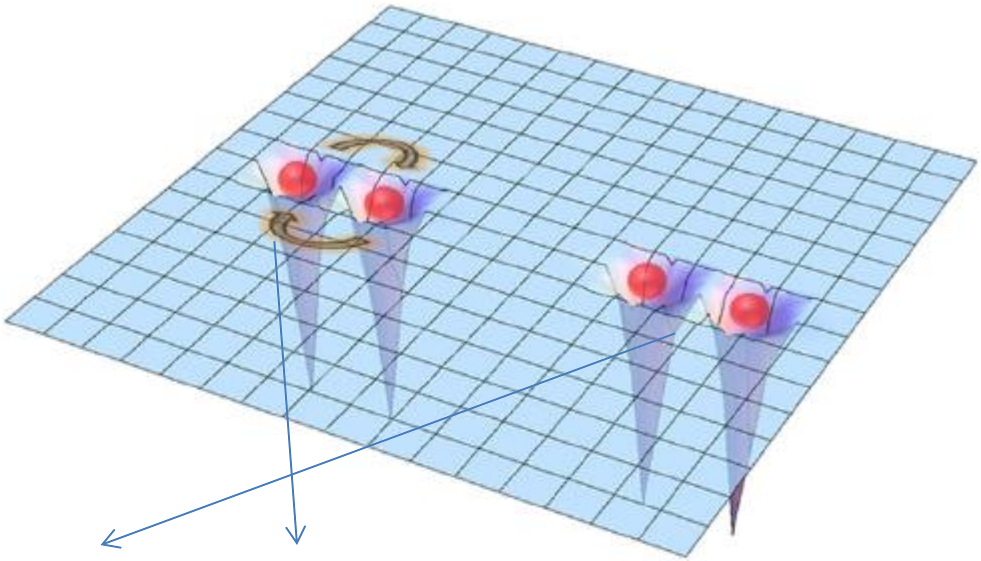
For particles indistinguishable under a swap,

$SWAP|\psi_{12}\rangle = e^{i\phi}|\psi_{12}\rangle$
$e^{i\phi} \in \{+1 \text{ bosons}, -1 \text{ fermions, other } e^{i\phi} \rightarrow \text{anyon}\}$

What about in theories other than quantum
-is there still an exchange phase?

Making particle exchange phase observable

Superposition of pair being on left ($|1\rangle$) or right ($|0\rangle$).



$$(|0\rangle|AB\rangle + |1\rangle|AB\rangle) \rightarrow |0\rangle|AB\rangle + e^{i\theta}|1\rangle|AB\rangle = (|0\rangle + e^{i\theta}|1\rangle)_C|AB\rangle.$$

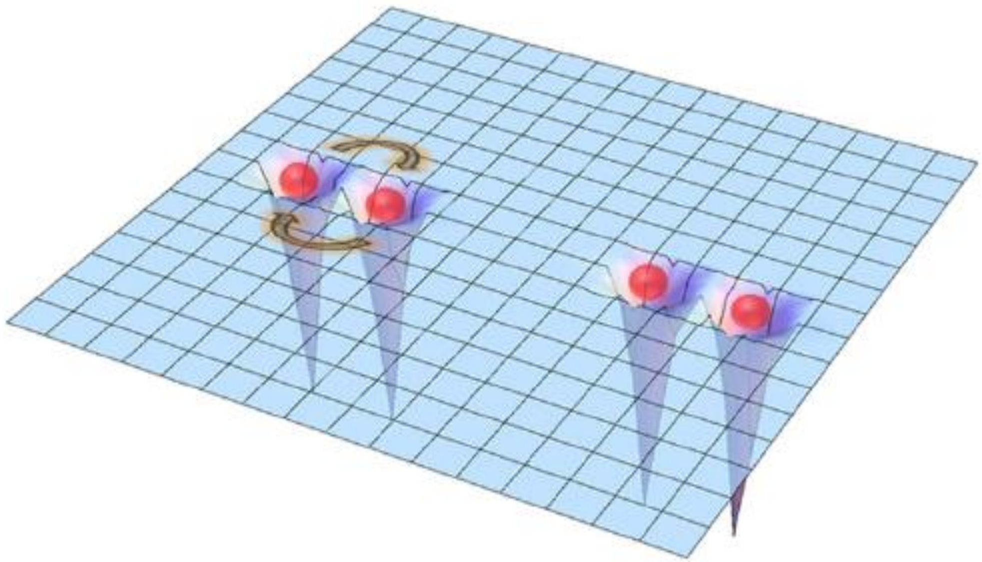
This results in a phase transformation on the reference degree of freedom C,

$$U(\theta) = |0\rangle\langle 0| + e^{i\theta}|1\rangle\langle 1|.$$

We may thus in quantum case associate a phase transformation of a qubit with each quantum particle type.

What if systems are not quantum?

Pair either on left or right, this is associated with some binary observable.

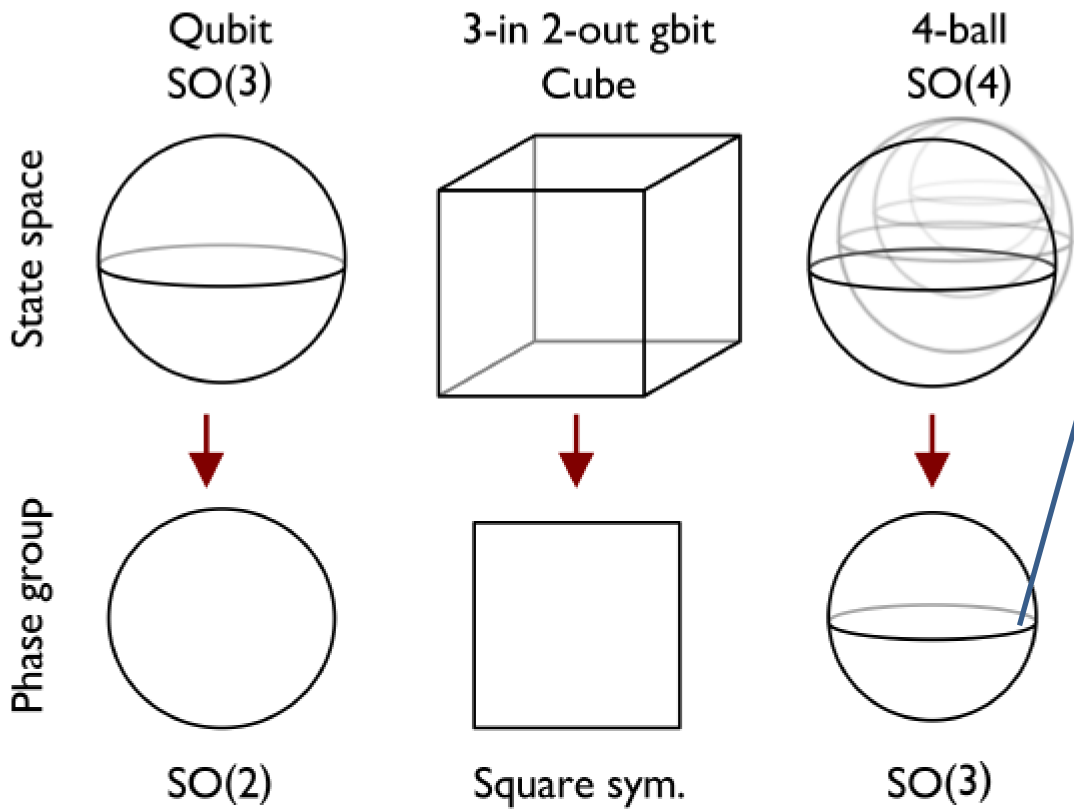


- As the pair gets swapped the statistical of the binary position observable cannot change
  $\rightarrow$ the swap is a phase transformation associated with that observable.

- We build an argument along those lines without assuming the particles are quantum, but assuming that they are indistinguishable, and conclude...

Particle exchange phase in post-quantum theories

We argue: The observable effect of exchanging two indistinguishable particles is restricted to the control system being transformed by an element of the phase group of a binary measurement. Hence the particle types perceivable in a theory correspond to elements of this phase group.



This example has phase elements that square to identity, yet do not commute-> non-Abelianicity in 3D space?

Summary and Outlook

Summary

1. We define phase transformations associated with measurement M as those which leave the statistics of measurement M invariant for all states.
2. Uncertainty principle enables non-trivial interferometer phase dynamics in quantum theory (enables escape from branch locality restriction).
3. The observable effect of exchanging two indistinguishable particles is restricted to the control system being transformed by an element of the phase group of a binary measurement.

Outlook

- Investigate post-quantum particles more.
- Role of phase in computing.

Thank you

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More general statements

Definition 1 (Fully conditionally restricted)**.** *A state space is* fully conditionally restricted *by a particular measurement if fixing any particular outcome of that measurement to occur with certainty is sufficient to completely specify the state.*

Definition 2 (Fully independent)**.** *A state space is fully independent of a given measurement if fixing the outcome of that measurement only reduces the number of degrees of freedom in the choice of state by the number of possible outcomes of the measurement.*

Theorem 1. *If the state space is fully restricted conditional on the position measurement, then* any *phase transformation can always be localised to a strict subset of branches without violating Branch Locality. If the state space is instead fully independent of the position measurement then* no *phase transformations (of form* $T \neq 1$*) can be localised to any strict subset of branches without violating Branch Locality.*

Particle exchange phase, extension to probabilistic theories

We take the input states to be products

$$\varphi^{in}_{ABC} = \varphi^{in}_{AB} \otimes \varphi^{in}_{C} \tag{1}$$

where φ_{AB} and φ_C are pure states.

For indistinguishable particles the post-exchange output state

$$\begin{aligned} \varphi^{out}_{ABC} &= \varphi^{out}_{AB} \otimes \varphi^{out}_{C}, \\ &= \varphi^{in}_{AB} \otimes \varphi^{out}_{C}. \end{aligned} \tag{2}$$

The first line follows from noting that by the assumption of reversibility φ^{out}_{AB} is pure and that Theorem 2 of [Hardy 2009] shows that such states are uncorrelatable. The second line follows from the definition of indistinguishability which implies that the marginal state on AB is invariant under the swap.

Particle exchange phase, extension to probabilistic theories

We thus have the exchange acting on the given input state type as

$$T(\varphi_{ABC}^{in}) = (1_{AB} \otimes T_C)\,(\varphi_{ABC}^{in}) \tag{1}$$

for some T_C acting on C exclusively.
Moreover as the two locations of the particle pairs are space-like separated, T_C is in the phase group of that binary position measurement(!)

> *The observable effect of exchanging two indistinguishable particles is restricted to the control system being transformed by an element of the phase group of a binary measurement. Hence the particle types perceivable* in a theory correspond to elements of this phase group.*

*) There could be further restrictions.