Antibunching in an optomechanical oscillator

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Mechanical effects of light

Kepler's observation

Maxwell's equations

$$\nabla \cdot \vec{E} = \rho / \epsilon_0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

Lebedev's demonstration

Light mill configuration

Optical forces were extremely feeble and seemed to be of academic vale.

Invention of the laser

PHYSICAL REVIEW VOLUME 112, NUMBER 6 DECEMBER 15, 1958

Infrared and Optical Masers

A. L. SCHAWLOW AND C. H. TOWNES*
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received August 26, 1958)

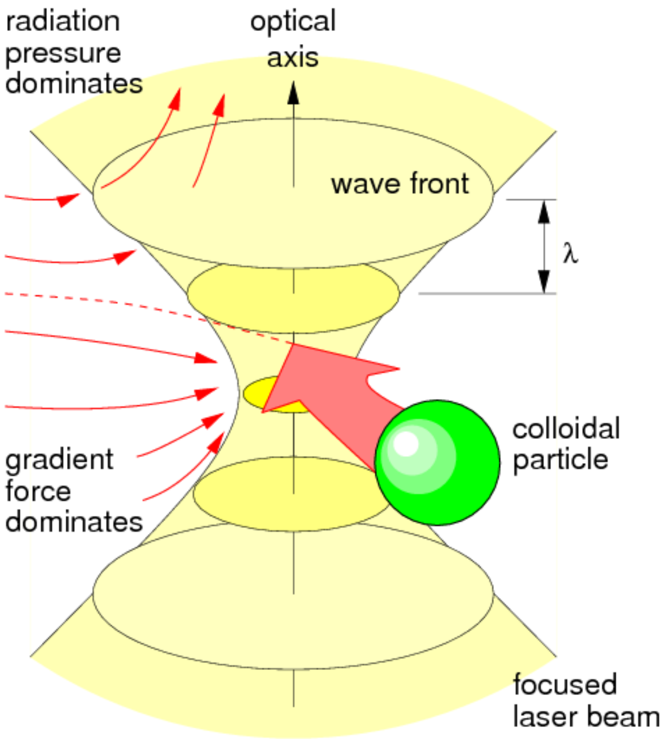
The extension of maser techniques to the infrared and optical region is considered. It is shown that by using a resonant cavity of centimeter dimensions, having many resonant modes, maser oscillation at these wavelengths can be achieved by pumping with reasonable amounts of incoherent light. For wavelengths much shorter than those of the ultraviolet region, maser-type amplification appears to be quite impractical. ...y making only the end ...ely monochromatic and ...system using potassium vapor.

The invention of the LASER became a game changer!

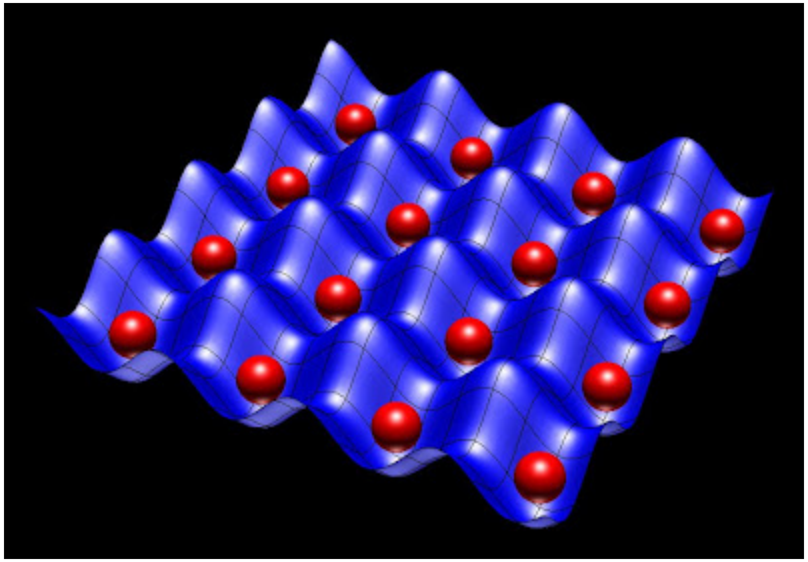
Theodore Maiman made the first laser operate on 16 May 1960 at the Hughes Research Laboratory in California, by shining a high-power flash lamp on a ruby rod with silver-coated surfaces. He promptly submitted a short report of the work to the journal *Physical Review Letters*, but the editors turned it down. Some have thought this was because the *Physical Review* had announced that it was receiving too many papers on masers—the longer-wavelength predecessors of the laser—and had announced that any further papers would be turned down. But Simon Pasternack, who was an editor of *Physical Review Letters* at the time, has said that he turned down this historic paper because Maiman had just published, in June 1960, an article on the excitation of ruby with light, with an examination of the relaxation times between quantum states, and that the new work seemed to be simply more of the same. Pasternack's reaction perhaps reflects the limited understanding at the time of the nature of lasers and their significance. Eager to get his work quickly into publication, Maiman then turned to *Nature*, usually even more selective than *Physical Review Letters*, where the paper was better received and published on 6 August.

Optical potential

Optical tweezers



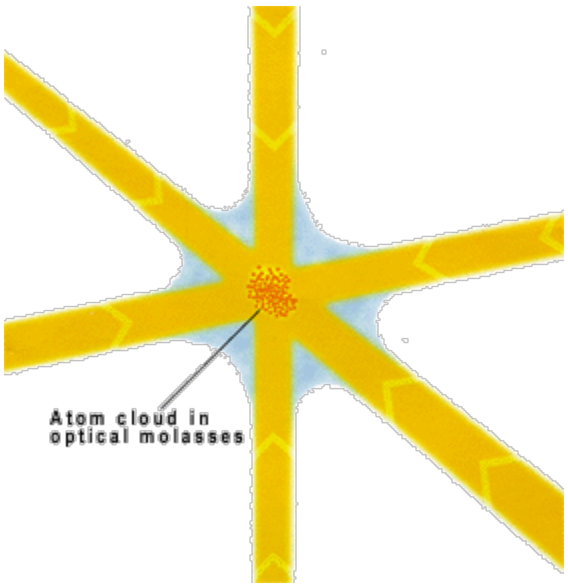
Optical lattices



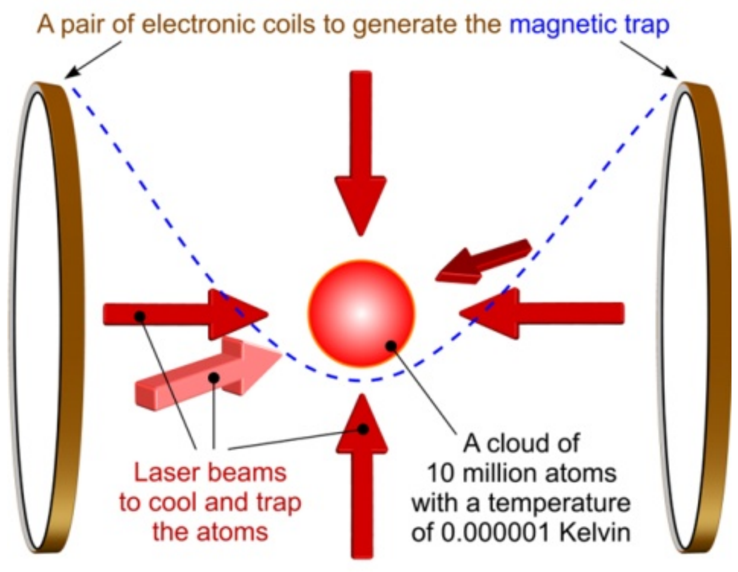
A. Ashkin, Phys. Rev. Lett. 24, 156 (1970)

Laser cooling

Optical Molasses



Magneto-Optical trap



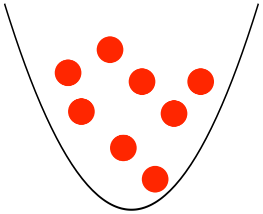
Ultracold atoms, Bose-Einstein condensates, etc.

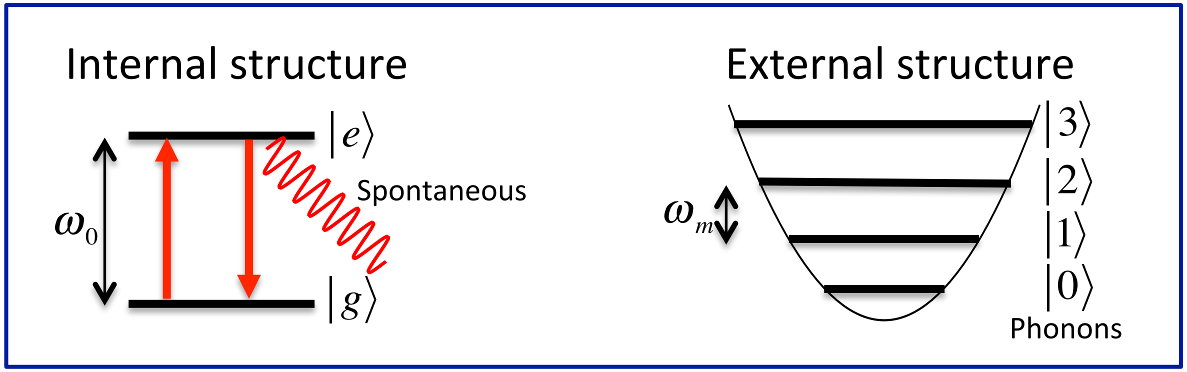
Would it be possible to use light to control and manipulate MACROSCOPIC objects?

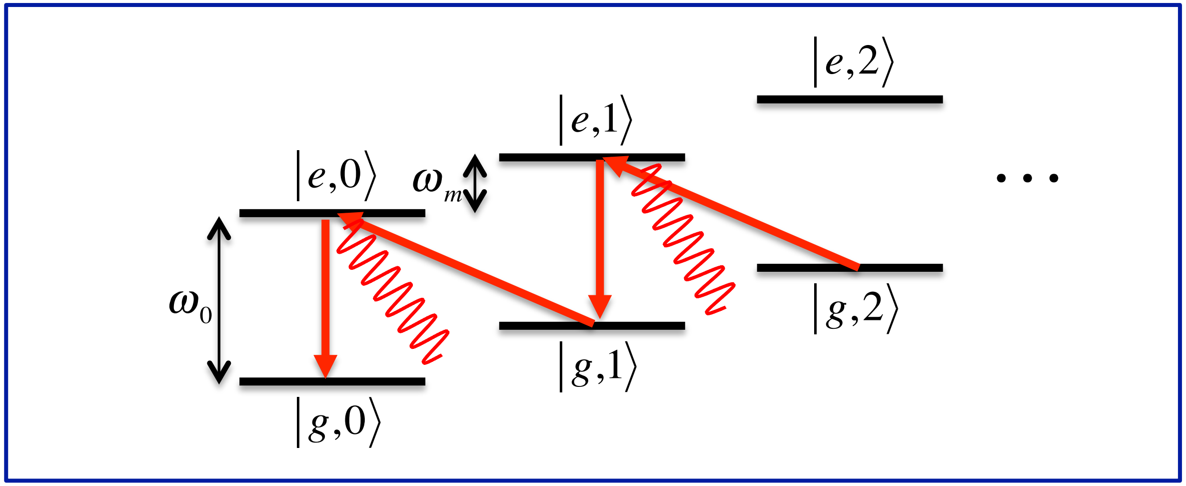
T. W. Hansch and A. L. Schawlow, Opt. Comm. 13, 68 (1975)

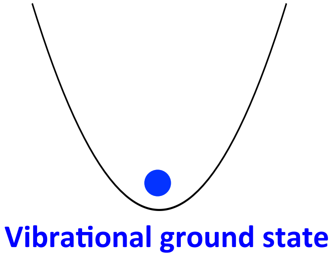
Laser cooling mechanism

Harmonically trapped two-level atoms

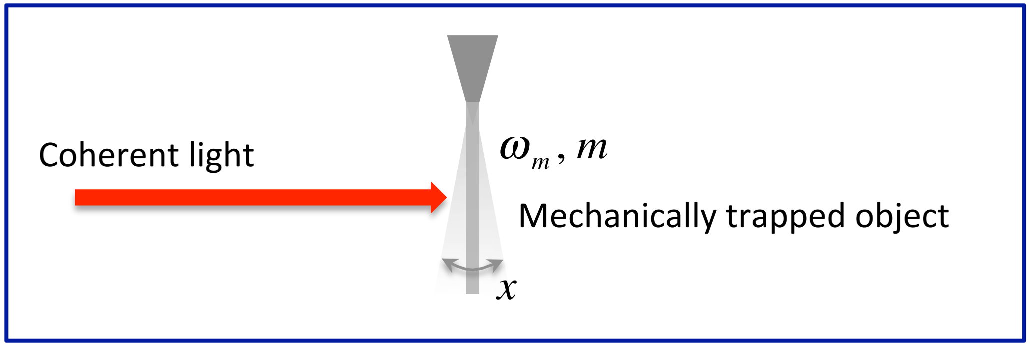






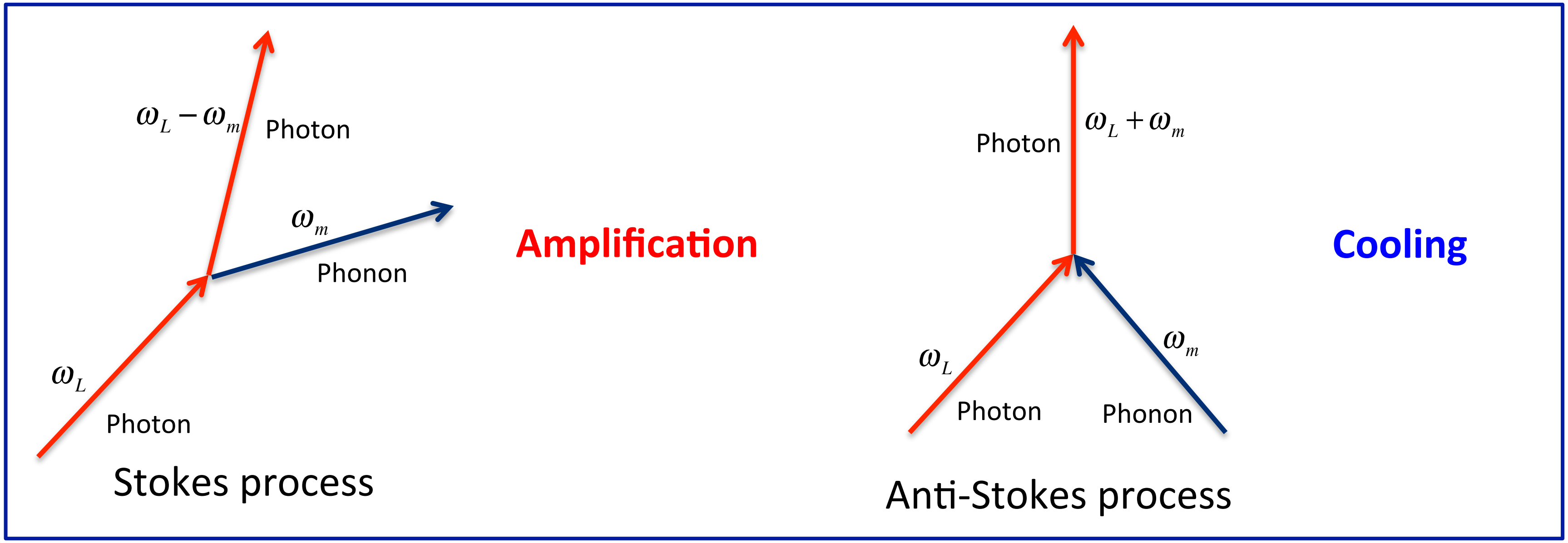


Macroscopic trapped objects

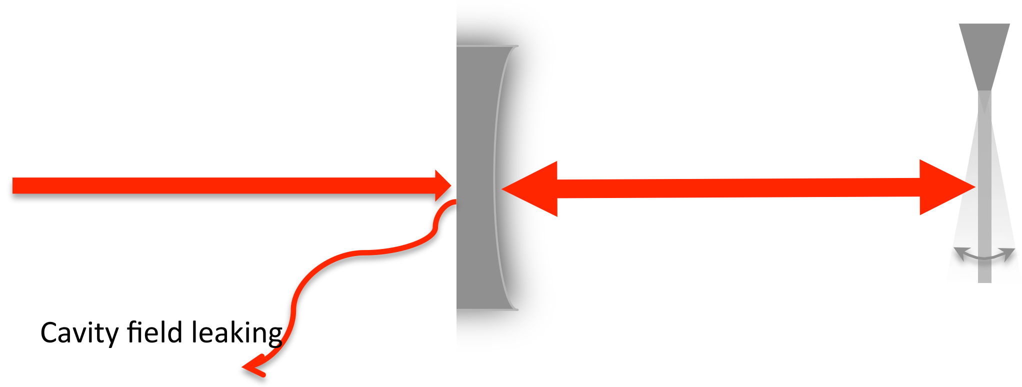


Radiation pressure force is too weak to change the dynamics of the mechanics.
Macroscopic objects do not have internal level structures and thus no spontaneous emission.

Raman scattering

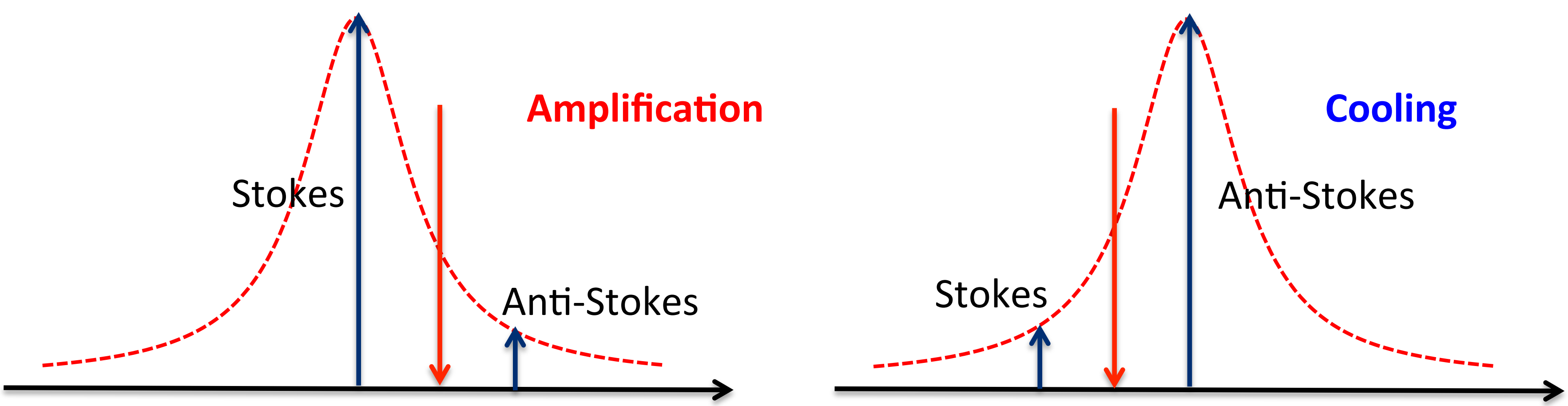


Cavity assisted configuration



Coherent many photons

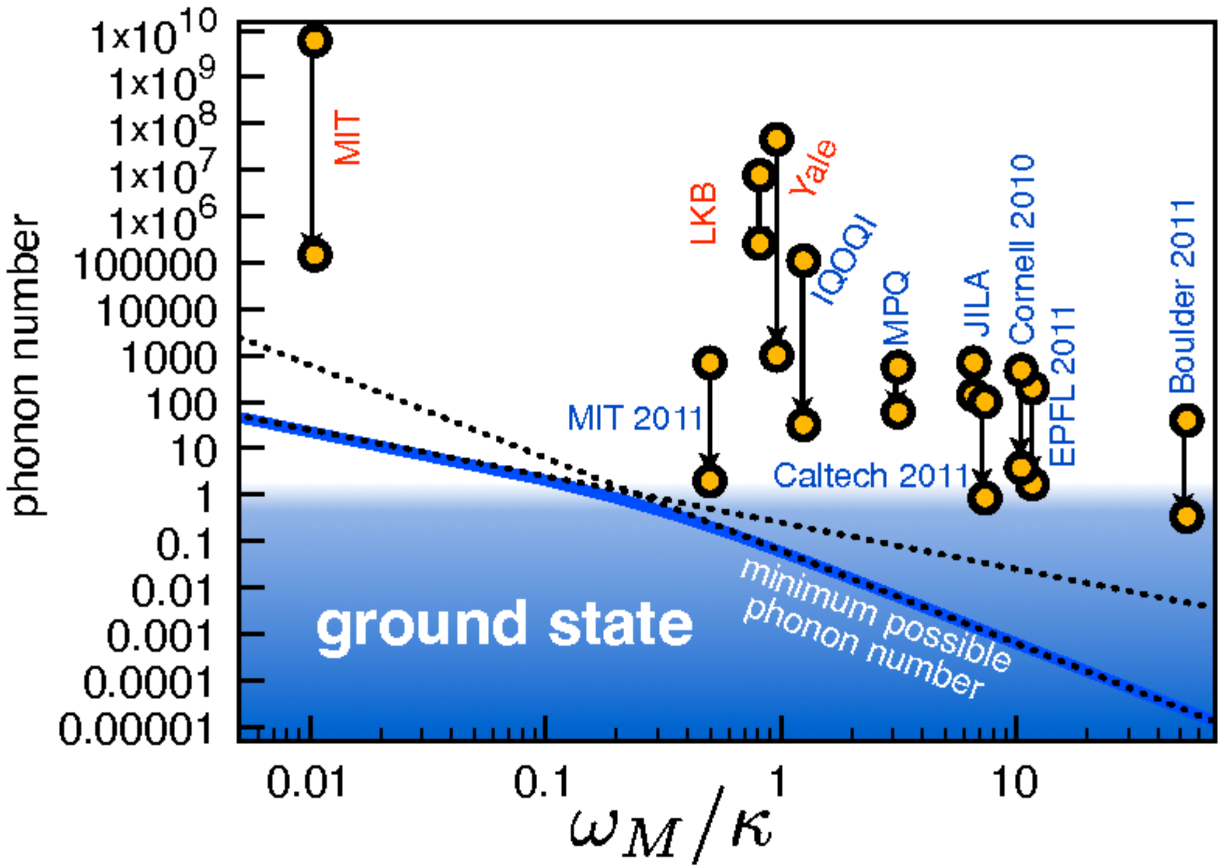
New leaking channel from light fields, cavity leaking



Blue-detuned cavity

Red-detuned cavity

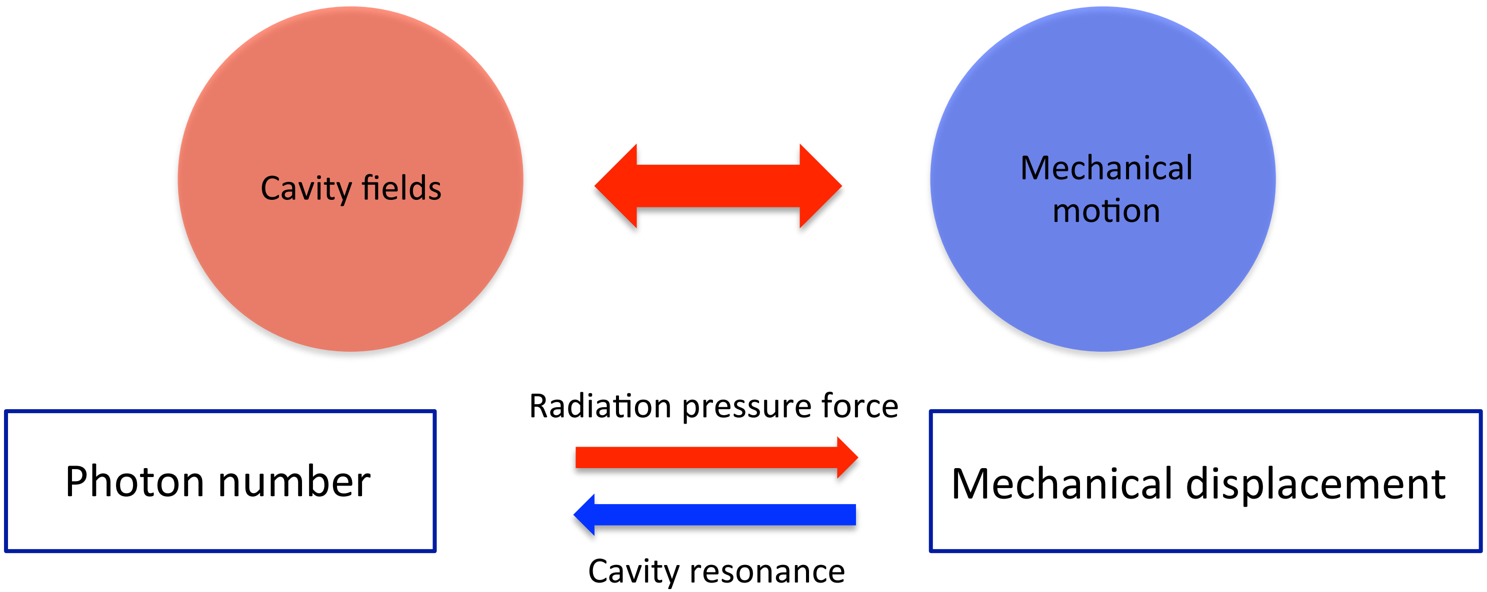
Macroscopic quantum objects



M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. **86**, 1391 (2014)

Cavity optomechanics

In high precision physical experiments with test bodies, it is important to know how strongly the test body is influenced by the optical system used to register the small displacement.



Precision measurements of feeble forces and fields
: gravitational wave detection

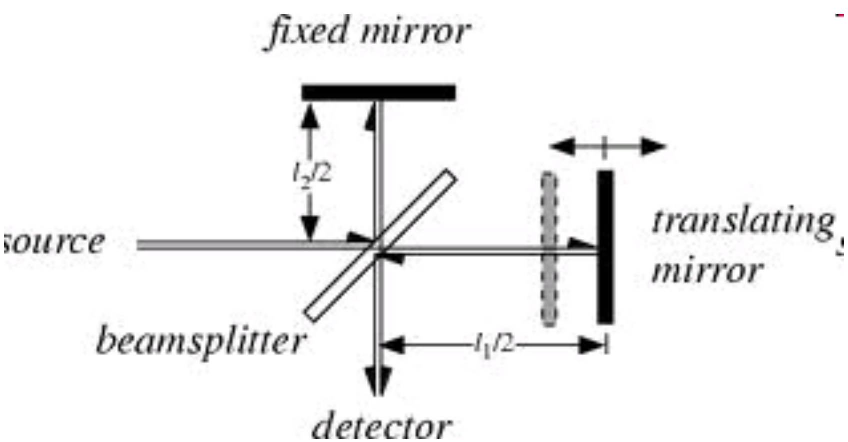
Develop fabrication techniques for high finesse optical resonators and mechanical oscillators with high quality factors

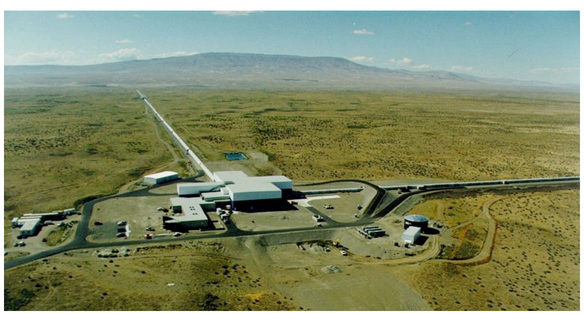
Braginsky and A. B. Manukin, JETP 25, 653 (1967)
Braginsky et. al., JETP 31, 829 (1970)

Measuring weak forces and displacements

Mechanical oscillators + lasers make great sensors

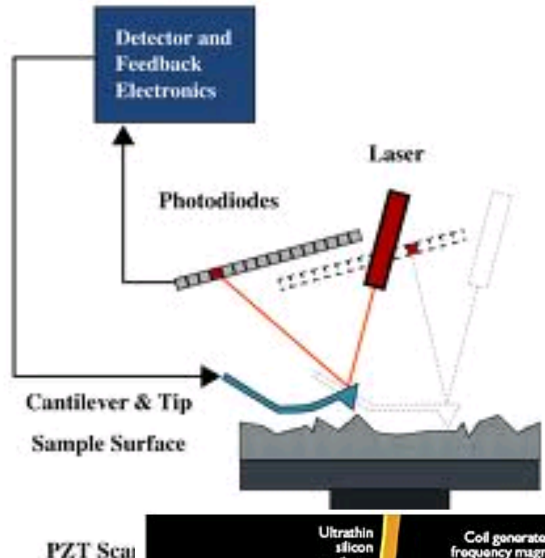
Michelson interferometer

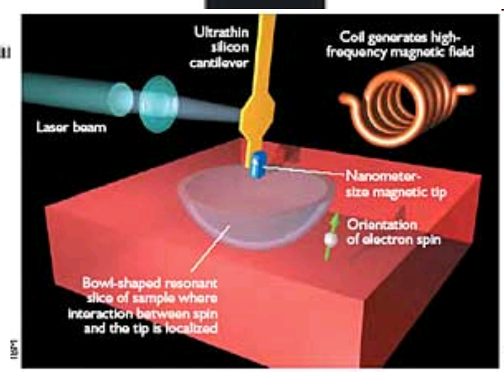




LIGO

Atomic Force Microscope

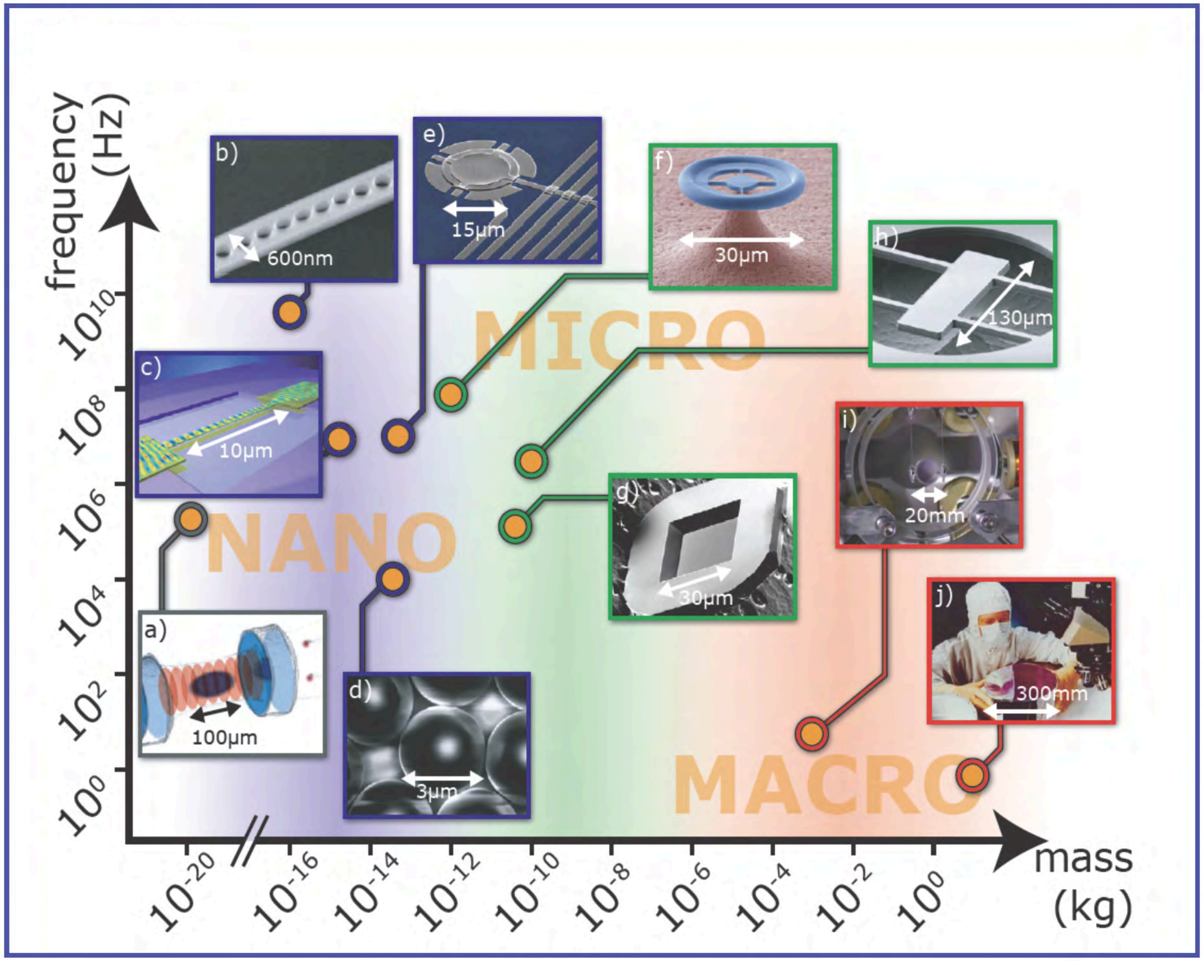




Single spin detection setup

D. Rugar et. al., Nature 430, 329 (2004)

Optomechanical systems



Foundational interests:

Exhibits superposition of macroscopic degrees of freedom.

Test of quantum mechanics over different sizes.

Technological interests:
Can couple to qubits and photons- quantum communication protocols

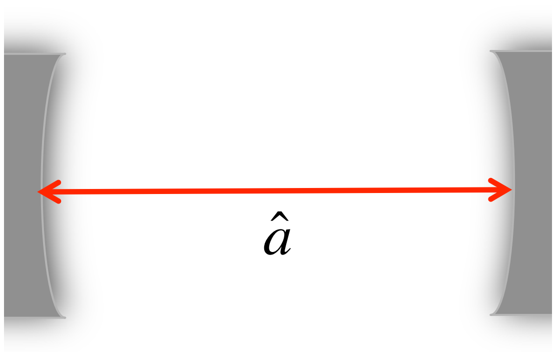
Make unprecedented sensors for small displacements and forces.

Image courtesy of P. Meystre

Basic theory

Basic setup

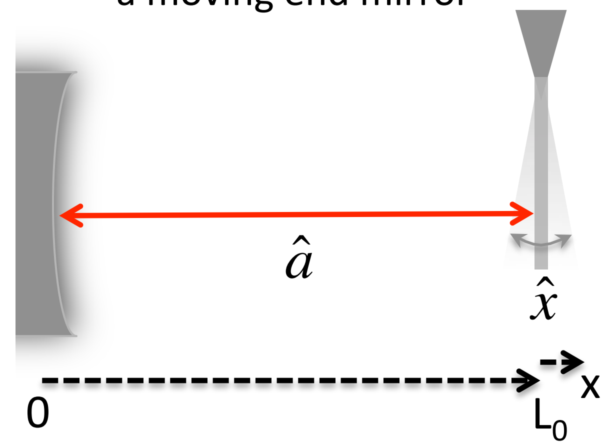
Fabry-Perot optical resonator



$$\omega_c = \frac{\pi c}{L} n$$

$$H_{\text{opt}} = \hbar \omega_c \hat{a}^\dagger \hat{a}$$

Fabry-Perot optical resonator with a moving end mirror



$$\omega_c(x) = \frac{\pi c}{L_0 + x} n \approx \omega_c - Gx$$

$$H'_{\text{opt}} = \hbar \omega_c \hat{a}^\dagger \hat{a} - \hbar g_0 \hat{a}^\dagger \hat{a} \hat{x}$$

$$g_0 = G\sqrt{\frac{\hbar}{m\omega_m}} \approx \frac{\omega_c}{L_0}\sqrt{\frac{\hbar}{m\omega_m}}$$

$$H_{\text{total}} = \hbar \omega_c \hat{a}^\dagger \hat{a} + \frac{\hbar \omega_m}{2}(\hat{p}^2 + \hat{x}^2) - \hbar g_0 \hat{a}^\dagger \hat{a} \hat{x} + H_{\text{pump}} + H_{\text{loss}}$$

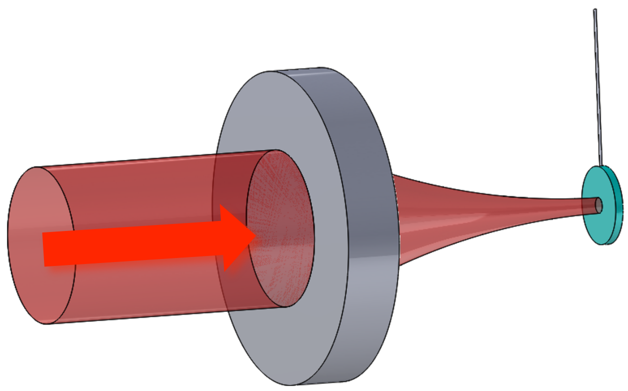
Linearized radiation pressure force

$$\dot{\hat{b}} = -i\omega_m \hat{b} + iF_{RP} - \gamma \hat{b} + \hat{\xi}$$

EOMs of the mechanics

$$F_{RP} = g_0 \hat{a}^\dagger \hat{a}$$

Radiation pressure force



If the cavity field is driven by a classical field,

$$\hat{a} \to \alpha + \hat{a} \qquad F_{RP} = g_0 |\alpha|^2 + g_0 \alpha(\hat{a}^\dagger + \hat{a}) + g_0 \hat{a}^\dagger \hat{a}$$

$$|\alpha|^2 \gg \langle \hat{a}^\dagger \hat{a} \rangle$$

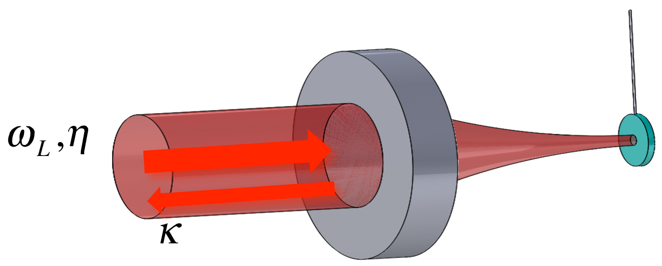
$$F_{RP} \approx g_0 |\alpha|^2 + g(\hat{a}^\dagger + \hat{a})$$

Linearized radiation pressure force

Mean radiation pressure force

$$\dot{\beta} = -i\omega_m \beta + i\langle F_{RP} \rangle - \gamma\beta$$

Mean field $\beta = \langle \hat{b} \rangle$



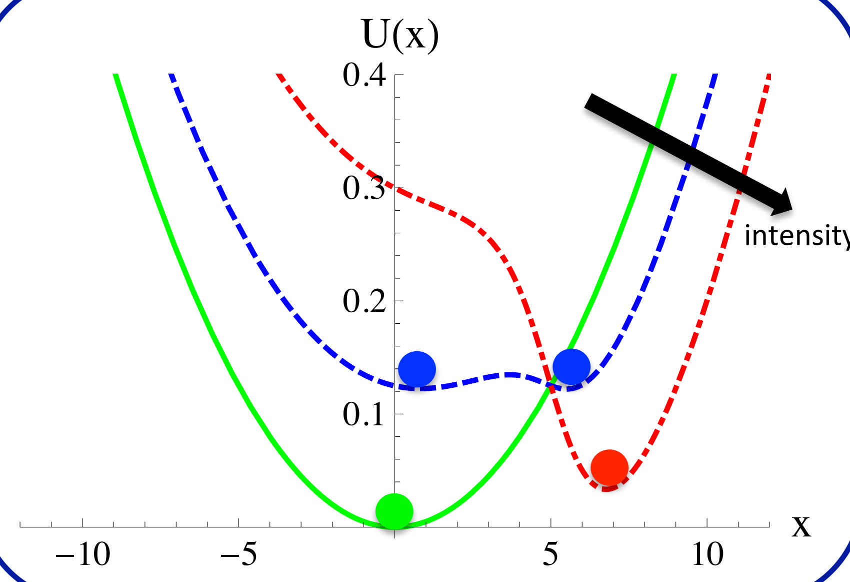
Mean radiation pressure force

$$\langle F_{RP} \rangle = g_0 |\alpha|^2$$

$$\rightarrow \frac{g_0 |\eta|^2}{(\Delta_c + g_0(\beta + \beta^*))^2 + \kappa^2/4}$$

$$\Delta_c = \omega_L - \omega_c$$

Mechanical potential

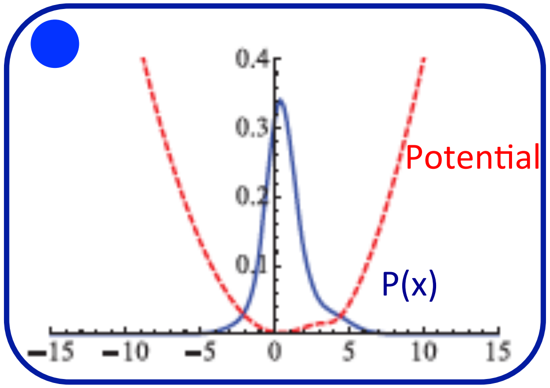


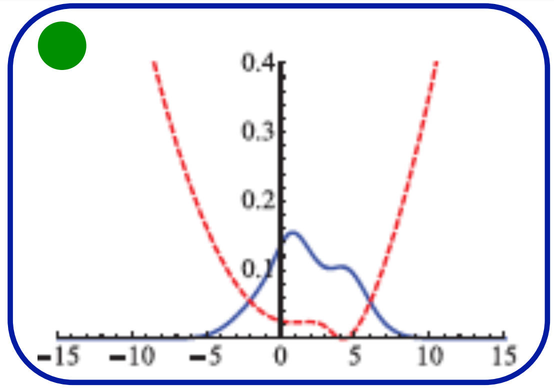
Two stable equilibrium positions

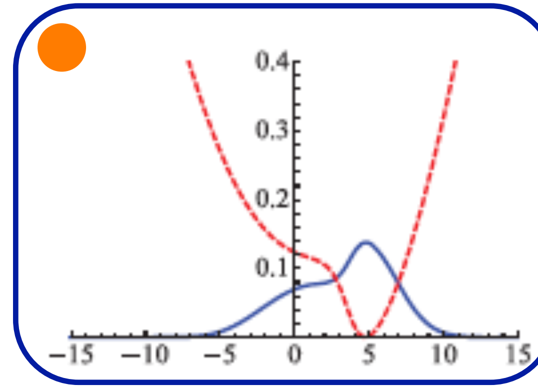
Optical bistability

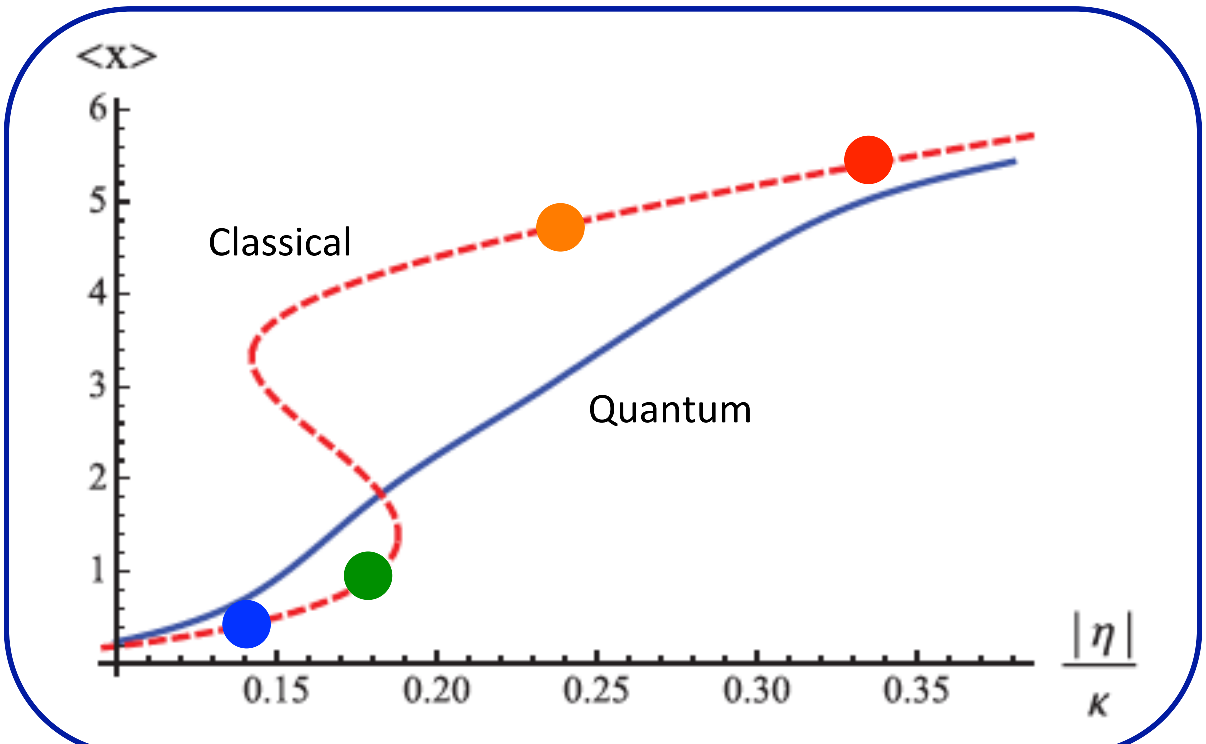
"Static optical spring effect"

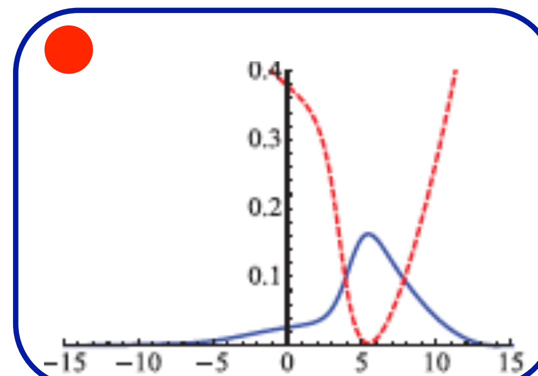
Optical bistability











Fluctuating radiation pressure force

$$\dot{\hat{b}} = -i\omega_m \hat{b} + \delta F_{RP} - \gamma \hat{b} + \hat{\xi}$$

$\kappa \gg g_0 |\alpha|$

Radiation pressure force fluctuations

$$\delta F_{RP} = g(\hat{a}^\dagger + \hat{a}) \rightarrow -i\Omega\hat{b} - \Gamma\hat{b} + ig_0\hat{N}$$

Fluctuations $\hat{b} = \langle \hat{B} \rangle - \beta_{ss}$

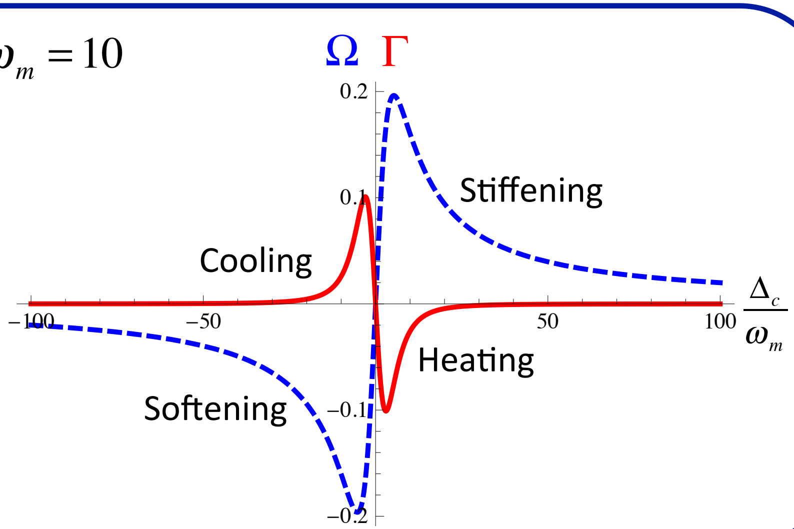
$$\Omega = g_0^2 \alpha_{ss}^2 \left[\frac{\Delta_c + \omega_m}{(\Delta_c + \omega_m)^2 + \kappa^2/4} + \frac{\Delta_c - \omega_m}{(\Delta_c - \omega_m)^2 + \kappa^2/4} \right]$$

Dynamic optical spring effect

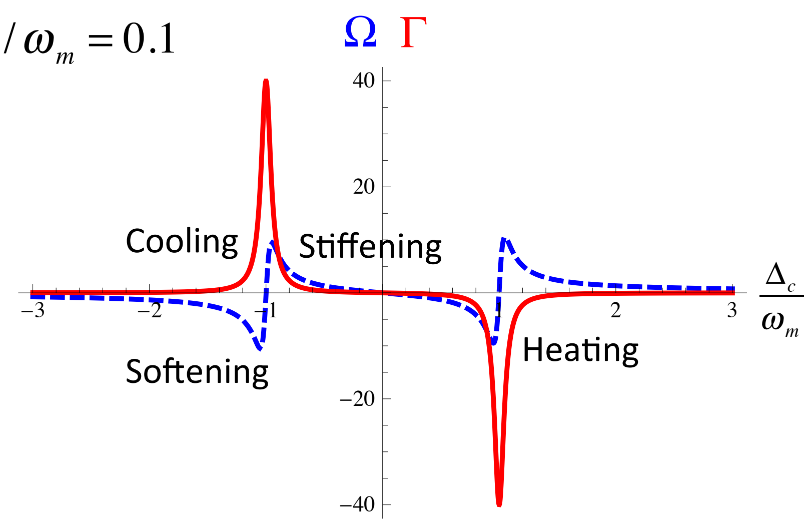
$$\Gamma = g_0^2 \alpha_{ss}^2 \left[\frac{\kappa/2}{(\Delta_c + \omega_m)^2 + \kappa^2/4} - \frac{\kappa/2}{(\Delta_c - \omega_m)^2 + \kappa^2/4} \right]$$

Radiation pressure cooling
or amplification

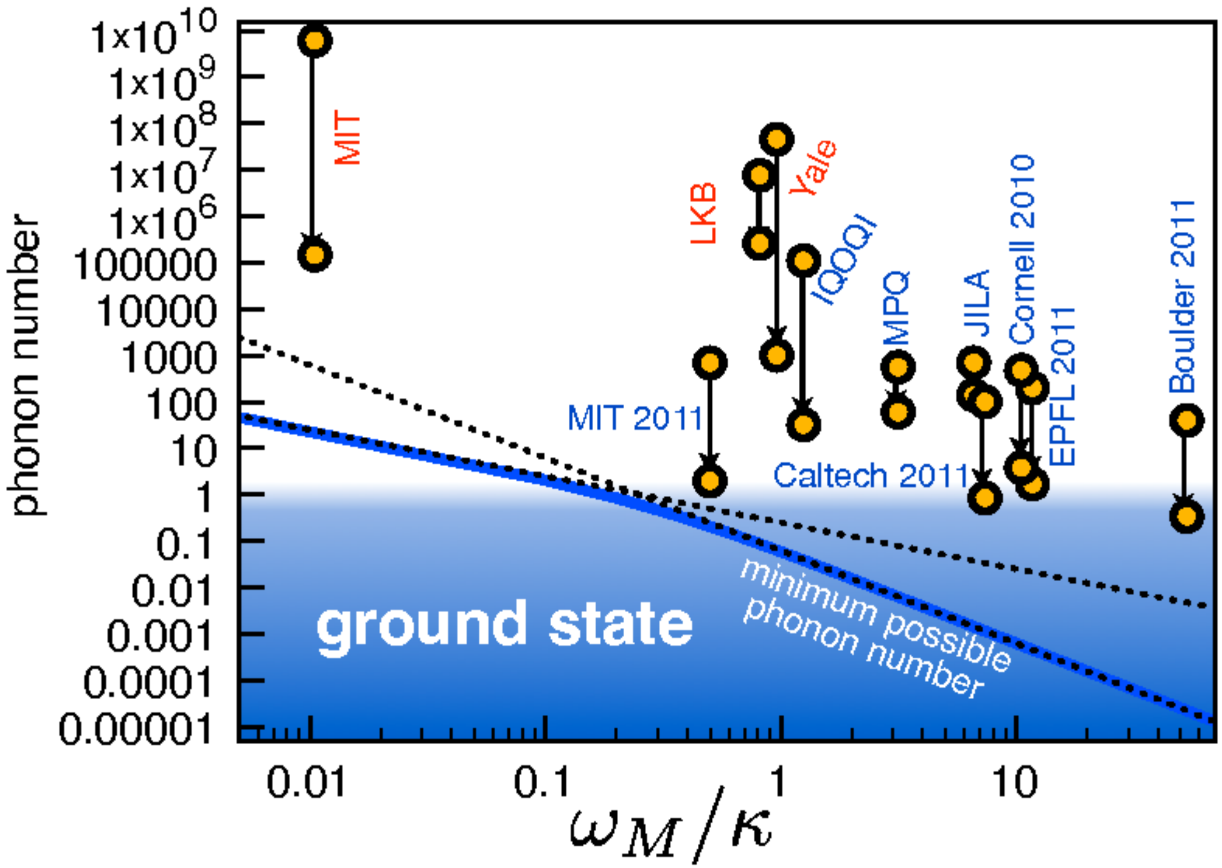
$\kappa / \omega_m = 10$



$\kappa / \omega_m = 0.1$



Macroscopic quantum objects

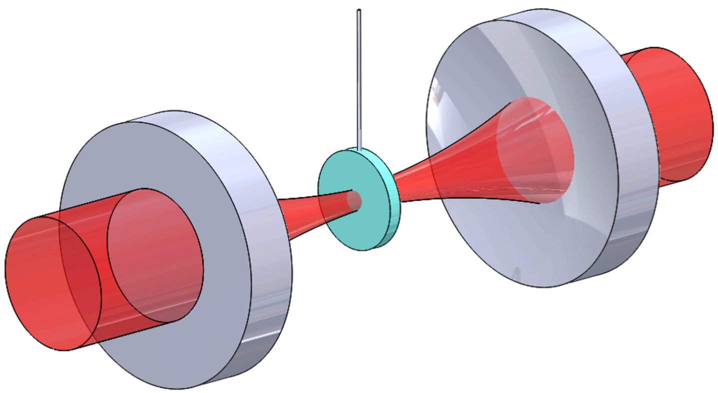


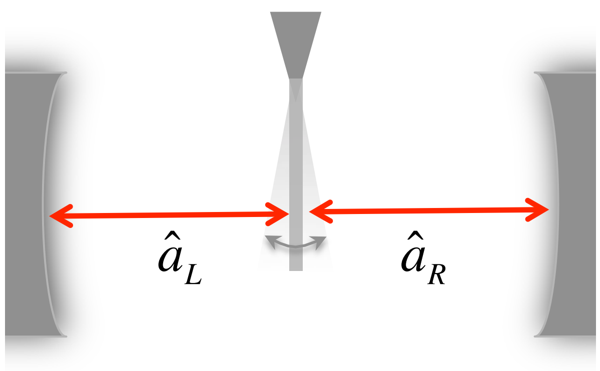
M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. **86**, 1391 (2014)

Antibunching in an optomechanical oscillator

Quadratic optomechanical interaction

Membrane-in-the-middle geometry





$$\hat{a}_{\pm} = \frac{1}{\sqrt{2}}(\hat{a}_L \pm \hat{a}_R)$$

Radiation pressure forces in opposite directions

Tunneling through the membrane

Optical eigenmode splitting

Quadratic optomechanical interactions

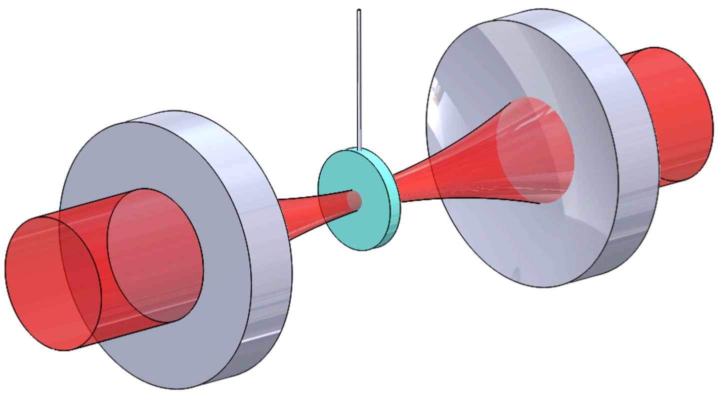
$$H = \sum_{k=+,-} \left[\hbar\omega_k \hat{a}_k^{\dagger}\hat{a}_k + \hbar g_0^{(2)} \hat{a}_k^{\dagger}\hat{a}_k \hat{x}^2 \right] + H_{\mathrm{mech}}$$

$$\omega_c(x) = \frac{\pi c}{L_0 + x} n \approx \omega_c + G^{(2)} x^2$$

$$H'_{\mathrm{opt}} = \hbar\omega_c \hat{a}^{\dagger}\hat{a} + \hbar g_0^{(2)} \hat{a}^{\dagger}\hat{a}\,\hat{x}^2$$

Quadratic optomechanical interaction

Membrane-in-the-middle geometry



Equations of motion

$$\dot{\hat{x}} = \omega_m \hat{p}$$

$$\dot{\hat{p}} = -\omega_m \hat{x} + F_{RP} - \frac{\gamma}{2}\hat{p} + \hat{\xi}$$

$$\dot{\hat{a}} = i(\Delta_c + g_0^{(2)}\hat{x}^2)\hat{a} - \frac{\kappa}{2}\hat{a} + \eta + \sqrt{\kappa}\hat{a}_{\text{in}}$$

$$F_{RP} = 2g_0^{(2)}\hat{a}^\dagger\hat{a}\hat{x}$$

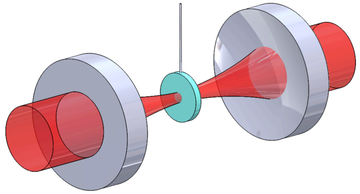
$$H_{\text{total}} = \hbar\omega_c\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_m}{2}(\hat{p}^2 + \hat{x}^2) + \hbar g_0^{(2)}\hat{a}^\dagger\hat{a}\,\hat{x}^2 + H_{\text{pump}} + H_{\text{loss}}$$

Master equation

The equations of motion can be highly nonlinear and open.

The master equation describing the evolution of the total density operator is

$$\boxed{\dot{\tilde{\rho}} = -\frac{i}{\hbar}[\hat{H}_{\rm opt} + \hat{H}_{\rm mech} + \hat{H}_{\rm om}, \tilde{\rho}] + \frac{\kappa}{2}D[\hat{a}]\tilde{\rho} + \frac{\gamma}{2}\bar{n}_{\rm th}D[\hat{b}^\dagger]\tilde{\rho} + \frac{\gamma}{2}(\bar{n}_{\rm th}+1)D[\hat{b}]\tilde{\rho}}$$



$\tilde{\rho}$: Density operator for the optomechanical system

$\hat{b}$: Annihilation operator for the mechanics

$\mathcal{D}[\hat{o}]\hat{\rho} = 2\hat{o}\hat{\rho}\hat{o}^\dagger - \hat{o}^\dagger\hat{o}\hat{\rho} - \hat{\rho}\hat{o}^\dagger\hat{o}$: Lindblad superoperator describing dissipation

κ : Decay rate of the cavity field

γ : Damping rate of the mechanics

$\bar{n}_{\rm th}$: Thermal occupation number of the mechanical heat bath

Unitary transformations

Introducing the unitary operator that transforms to a frame rotating at the driving frequency

$$\hat{U}_1 = e^{-i\omega_L \hat{a}^\dagger \hat{a} t},$$

and the unitary operator capturing the steady-state mean amplitude of the cavity field resulting from the external pump

$$\hat{U}_2 = e^{(\alpha \hat{a}^\dagger - \alpha^* \hat{a})},$$

the master equation for the transformed density operator $\bar{\rho} = \hat{U}_2^\dagger \hat{U}_1^\dagger \tilde{\rho} \hat{U}_1 \hat{U}_2$ then becomes

$$\dot{\bar{\rho}} = i\Delta_c[\hat{a}^\dagger \hat{a}, \bar{\rho}] - i\omega_m^{'}[\hat{b}^\dagger \hat{b}, \bar{\rho}] - ig_0 n_c[\hat{b}^{\dagger 2} + \hat{b}^2, \bar{\rho}] - ig[(\hat{a} + \hat{a}^\dagger)(\hat{b}^\dagger + \hat{b})^2, \bar{\rho}] - ig_0[\hat{a}^\dagger \hat{a}(\hat{b}^\dagger + \hat{b})^2, \bar{\rho}]$$
$$+ \frac{\kappa}{2} D[\hat{a}]\bar{\rho} + \frac{\gamma}{2} \bar{n}_{\text{th}} D[\hat{b}^\dagger]\bar{\rho} + \frac{\gamma}{2}(\bar{n}_{\text{th}} + 1) D[\hat{b}]\bar{\rho},$$

Δ_c : Detuning of the pump laser from the cavity resonance

$\omega_m^{'}$:Shifted frequency of the mechanical oscillator

n_c : Intracavity photon number

g : Enhanced optomechanical coupling strength

Interaction picture

If the external pump is red-detuned by twice the effective mechanical frequency, invoking the rotating wave approximation in the interaction picture implemented by the unitary transformation

$$\hat{U}_3 = e^{i(\Delta_c \hat{a}^\dagger \hat{a} - \omega_m' \hat{b}^\dagger \hat{b})t}$$

simplifies the master equation as

$$\dot{\rho} = -ig[\hat{a}^\dagger \hat{b}^2 + \hat{b}^{\dagger 2}\hat{a}, \rho] + \frac{\kappa}{2} D[\hat{a}]\rho + \frac{\gamma}{2}\bar{n}_{\text{th}} D[\hat{b}^\dagger]\rho + \frac{\gamma}{2}(\bar{n}_{\text{th}} + 1) D[\hat{b}]\rho.$$

Physically the Hamiltonian representing the Schroedinger evolution in the master equation reads

$$\boxed{\hat{H} = \hbar g(\hat{a}^\dagger \hat{b}^2 + \hat{b}^{\dagger 2}\hat{a})}$$

Notice that it is identical to the interaction picture Hamiltonian describing a parametric amplifier in quantum optics and is well-known to generate two photons in the subharmonic mode destroying a photon in the pump mode. It is thus expected that two phonons of the mechanics can be destroyed by creating a single photon which is eventually leaked out the optical resonator by the cavity field dissipation.

Adiabatic elimination of the cavity field

In the regime where cavity dissipation is the dominant source of damping, the state of the cavity field tends to approach to a coherent state in a timescale of $1/\kappa$ and thus the density operator describing the optomechanical system can be approximated as a product state

$$\boxed{\rho(t) \approx \rho_o(t) \otimes \rho_m(t).}$$

Adiabatic elimination of the reduced density operator for the cavity field gives rise to the effective master equation for the mechanics only,

$$\boxed{\frac{d\rho_m}{dt} = \frac{\Gamma_{\text{opt}}}{2} D[\hat{b}^2]\rho_m + \frac{\gamma}{2}\bar{n}_{\text{th}} D[\hat{b}^\dagger]\rho_m + \frac{\gamma}{2}(\bar{n}_{\text{th}} + 1) D[\hat{b}]\rho_m,}$$

Γ_{opt} : Nonlinear optomechanical damping rate $\Gamma_{\text{opt}} = \dfrac{8g^2}{\kappa}$

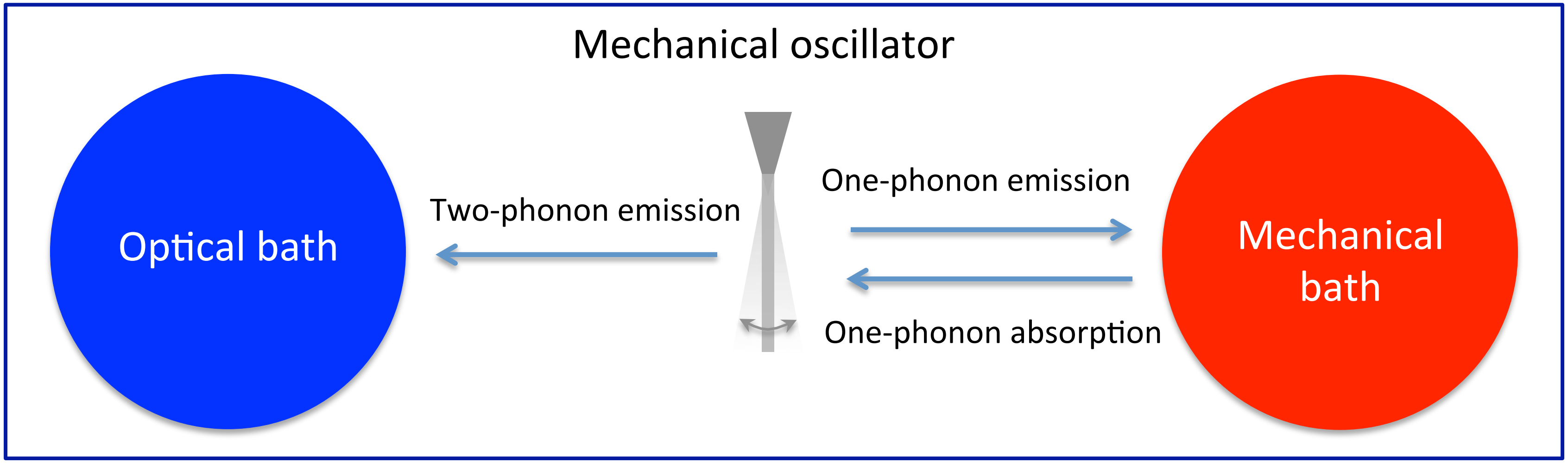
Scaling time to the inverse of the mechanical decay rate, $\tau = \gamma t$, we have

$$\frac{d\rho_m}{d\tau} = \frac{C}{2} D[\hat{b}^2]\rho_m + \frac{1}{2}\bar{n}_{\text{th}} D[\hat{b}^\dagger]\rho_m + \frac{1}{2}(\bar{n}_{\text{th}} + 1) D[\hat{b}]\rho_m$$

C : Multiphoton optomechanical cooperativity $C = \dfrac{8g^2}{\kappa\gamma}$

Competition between one- and two-phonon processes

$$\frac{d\rho_m}{d\tau} = \frac{C}{2} D[\hat{b}^2]\rho_m + \frac{1}{2}\bar{n}_{\rm th} D[\hat{b}^\dagger]\rho_m + \frac{1}{2}(\bar{n}_{\rm th}+1) D[\hat{b}]\rho_m$$



Phase-space methods

As is well-known, a nonlinear quantum mechanical problem can be mapped into a classical stochastic process by an appropriate phase space representation.

Expanding the density operator for the mechanics as

$$\rho_m = \int \frac{|\mu\rangle\langle\nu^*|}{\langle\nu^*|\mu\rangle} P(\mu,\nu)\mathrm{d}\mu\mathrm{d}\nu,$$

the master equation for the mechanics takes the form of the Fokker-Planck equation

$$\boxed{\frac{dP(\chi)}{d\tau} = -\sum_i \frac{\partial}{\partial\chi_i}[A(\chi)]_i P(\chi) + \frac{1}{2}\sum_{i,j}\frac{\partial}{\partial\chi_i}\frac{\partial}{\partial\chi_j}[D(\chi)]_{i,j} P(\chi)}$$

Drift vector

$$A(\chi) = \begin{pmatrix} -\frac{1}{2}\mu - C\nu\mu^2 \\ -\frac{1}{2}\nu - C\mu\nu^2 \end{pmatrix}$$

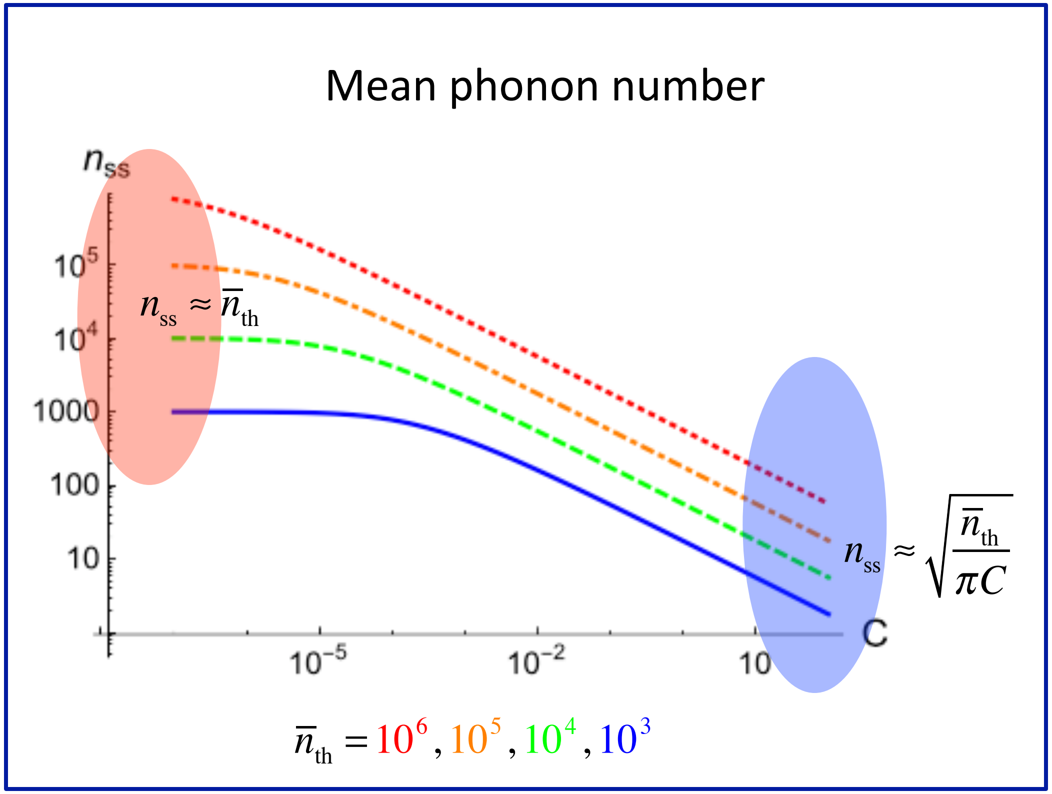
Diffusion matrix

$$D(\chi) = \begin{pmatrix} -C\mu^2 & \bar{n}_{\text{th}} \\ \bar{n}_{\text{th}} & -C\nu^2 \end{pmatrix}$$

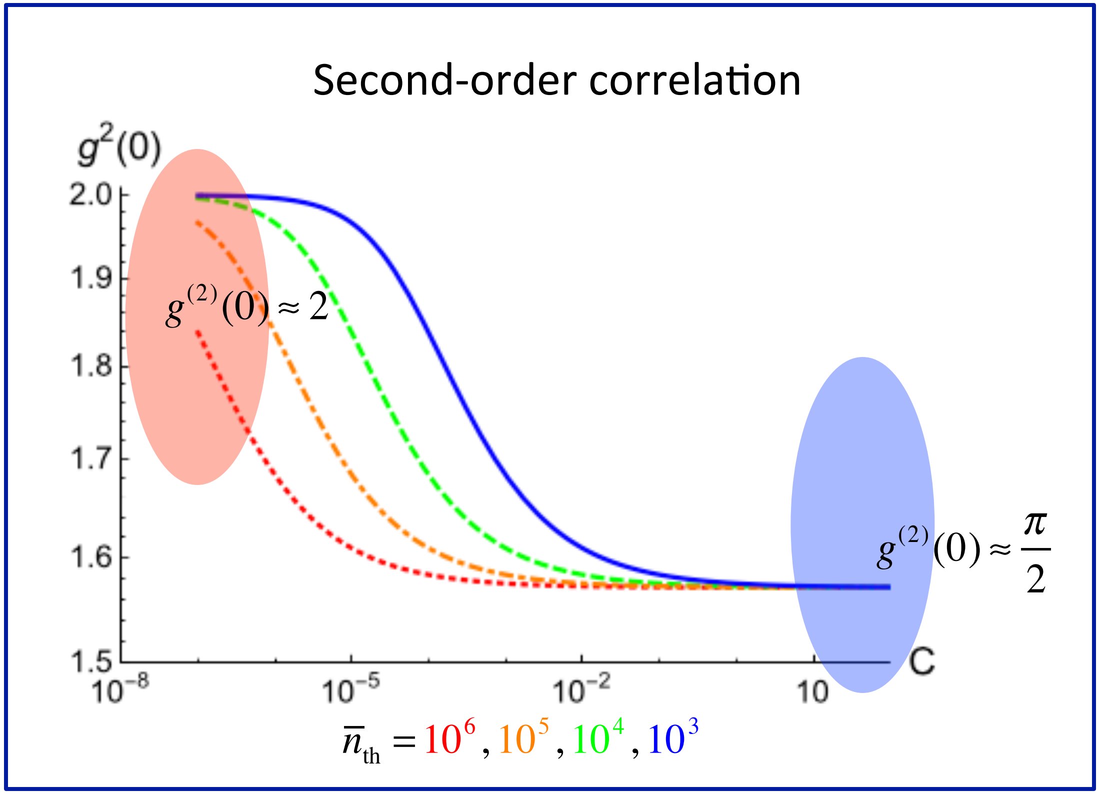
High temperature regime

The steady-state complex P distribution in the high temperature regime can be obtained as

$$P_s(\mu,\nu) = N \exp\left(-\frac{1}{\bar{n}_{\text{th}}}\mu\nu\right)\exp\left(-\frac{C}{\bar{n}_{\text{th}}}\mu^2\nu^2\right)$$



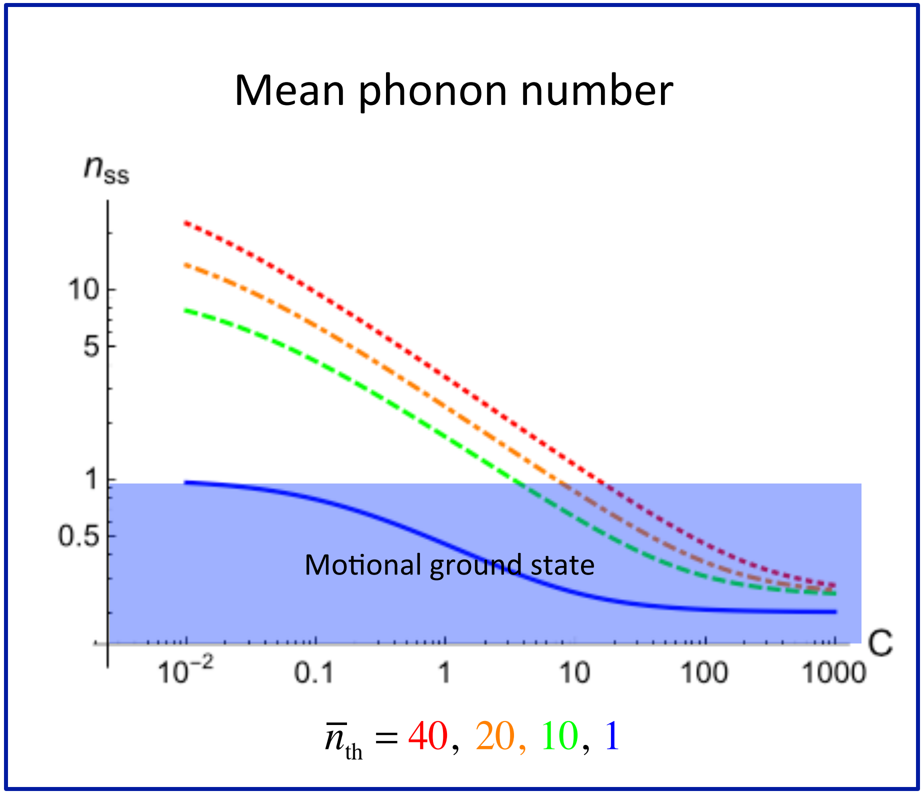
High temperature regime



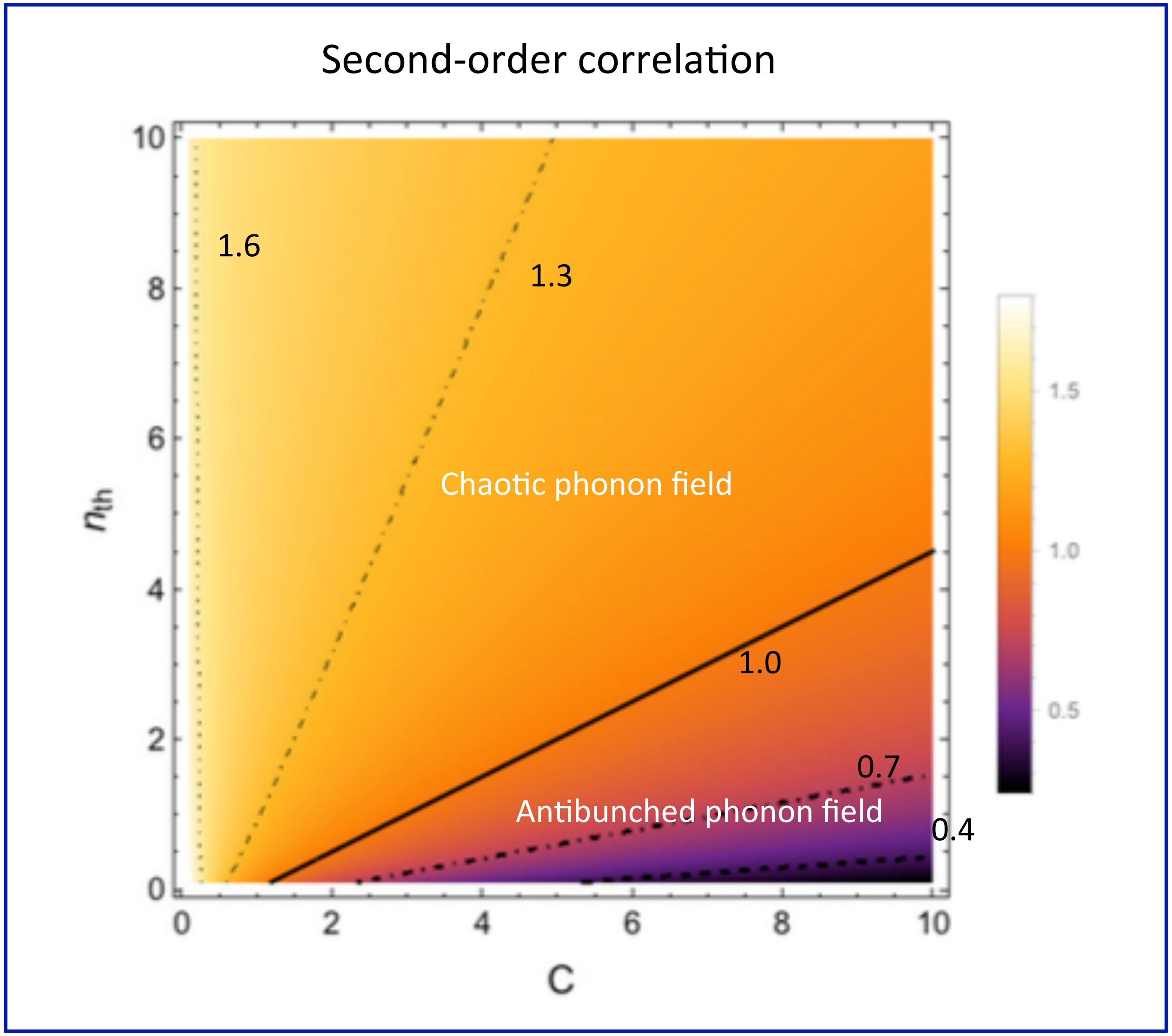
Low temperature regime

The steady-state complex P distribution in the low temperature regime can be obtained as

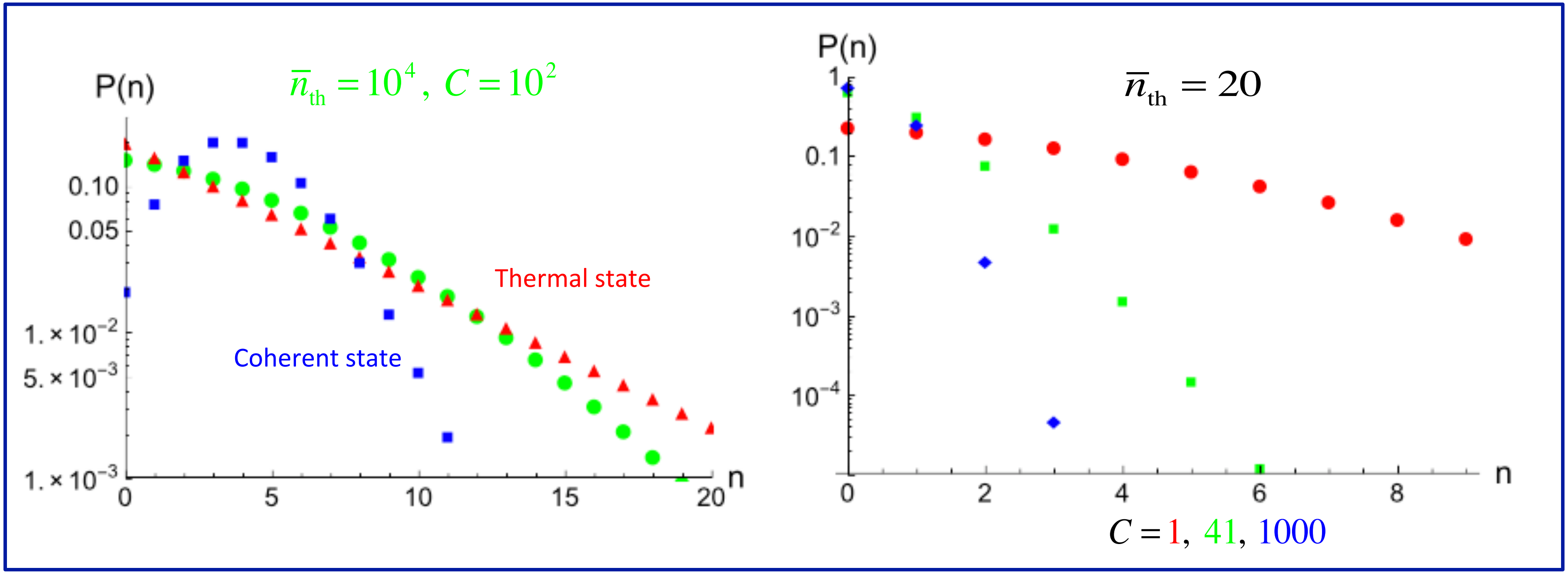
$$P_s(\mu,\nu)=\frac{2Ae^{2\mu\nu}}{(1+2\bar{n}_{\rm th}-C)\mu\nu}\,{}_2F_1\left(1,1;\tfrac{1+2\bar{n}_{\rm th}}{C};\tfrac{\bar{n}_{\rm th}}{C\mu\nu}\right)+\frac{2Ae^{2\mu\nu}}{\bar{n}_{\rm th}}\sum_{r=1}^{\infty}\frac{(-2\mu\nu)^r}{rr!}\times{}_2F_1\left(1,2+r-\tfrac{1+2\bar{n}_{\rm th}}{C};1+r;\tfrac{C\mu\nu}{\bar{n}_{\rm th}}\right)$$



Low temperature regime



Phonon number distribution



Conclusions

1. Our key result is that the steady-state phonon field is chaotic if the multiphoton cooperativity obeys $C<2n_{th}+1$ whereas it antibunched if $C>2n_{th}+1$.

2. This calculation opens the door to control of the second-order correlation of the mechanical oscillator in the weak coupling regime, and the observation of phonon antibunching.