Hanbury Brown and Twiss correlations of Cooper pairs in helical liquids

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We propose a Hanbury Brown and Twiss (HBT) experiment of Cooper pairs on the edge channels of quantum
spin Hall insulators. The helical edge channels provide a well-defined beam of Cooper pairs and perfect Andreev
reflections from superconductors. This allows our setup to be identical in spirit to the original HBT experiment.
Interestingly, the cross correlation is *always* negative and provides no hint of the bosonic nature of Cooper pairs.
This counterintuitive result is attributed to the perfect Andreev reflection and the true beam splitter in the setup.

I. INTRODUCTION

The Bose-Einstein condensation of constituent particles
results in superfluidity in bosonic systems [1]. Superconductivity in electronic systems is attributed to the pairing of
electrons into so-called Cooper pairs [2]. It is thus plausible
to expect Cooper pairs to bear some bosonic nature, and
to regard superconductivity as a condensation. Nevertheless,
mathematically, Cooper pairs are not pure bosons because the
pair creation and annihilation operators do not obey strictly
the boson commutation relations [2], and the bosonic nature
of Cooper pairs still remains controversial. Therefore, it will be
invaluable to examine the issue directly in experiments. Here
we propose a Hanbury Brown and Twiss (HBT) experiment
of Cooper pairs on the edge channels of quantum spin Hall
insulators [3,4], a recently discovered new state of matter.
Surprisingly [5], the cross correlation is always negative and
shows no hint of the bosonic nature of Cooper pairs once
they are emitted from the superconductor. Previously, theoretical [6–9] and experimental [10,11] works showed positive
or negative correlation depending on the system parameters.
Interestingly, the cross correlation has been predicted to be
always negative in a *diffusive* multiterminal superconductor-
normal-metal contacts [12].

A HBT effect is an *intensity interference* between two partial beams. It was originally introduced in order to overcome
the technical difficulties in measuring the size of stars with
Michelson interferometers. After the pioneering experiment
by HBT in 1956 [13,14], it was soon realized that the effect
can determine quantum-statistical properties of a stream of
particles [15,16]: The intensity correlation is positive for
bosons (obeying Bose-Einstein statistics) while negative for
fermions (obeying Fermi-Dirac statistics) [17].

An ideal HBT experiment requires a well-defined beam
of particles and a tunable beam splitter (BS). For normal
electrons, an electron beam is achieved on the chiral edge
states of an integer quantum Hall insulator (IQHI), for which a
quantum point contact (QPC) serves as a beam splitter [18,19].
It does not work for Cooper pairs because the high magnetic
field required for IQHI breaks them.

A quantum spin Hall insulator (QSHI) is a prototype
example of topological insulators. Topological insulators are

characterized by bulk excitation energy gap and gapless edge
modes, the latter being intimately connected to the topological
nature [3,20]. In QSHIs, the spin-orbit coupling gives the
edge modes a helical feature, spin-up (↑) electrons moving
to the right and spin-down (↓) electrons moving to the left,
and the time-reversal invariance prevents back scattering even
in the presence of disorder [3,20]. The helical edge states are
thus a Kramers pair and duplicate copies of a chiral edge
mode. Then it is clear that a QPC serves as a BS for helical
edge modes [21,22].

When a superconductor is put on top of helical edge modes,
the edge modes become superconducting due to the proximity
effect [23,24]. The edge modes beneath the superconductor
will be referred to as superconducting edge modes or simply
superconductors. A Cooper pair, itself being a pairing between
time-reversal counterparts, moves with no back scattering
across the (point) interface between superconductor and helical
edge modes. That is, the Andreev reflection is always perfect
even in the presence of interface barrier [25]. This is in sharp
contrast to usual normal-superconductor hybrid structures,
where in reality normal-electron reflection is unavoidable.
Therefore, the helical edge states provide a well-defined
and transparent channel for Cooper pairs [26,27]. Two-
dimensional QSHI has been observed recently in HgTe/CdTe
quantum wells [4] and also expected in InAs/GaSb/AlSb
type-II quantum wells [28].

Several HBT-type experiments of Cooper pairs have been
proposed before [6–9]. In their cases, however, a *Y* junction is
used instead of a true electron BS, and not only Andreev but
also normal-electron reflections are involved [8]. It is shown
below that these differences affect significantly the results. Our
setup is much closer in spirit to the original HBT experiment.
Another closely related work is Ref. [29]: Here the Cooper
pairs were injected to helical edge modes of QSHI through
direct tunneling from spin-singlet superconductors. Unlike in
our setup, the coupling between the helical edge modes and
the superconductor in their setup is weak. As pointed out
by the authors [29], most electrons are thus passing along
the edges without Andreev scattering and rare electrons do
undergo Andreev reflection but scatter to a different edge as
holes. Such a crossed Andreev reflection leads to an almost
perfect positive correlation [29].

Recently, the splitting of Cooper pairs has been demonstrated experimentally [10,11]. However, in these experiments
the scattering of Cooper pairs themselves have been suppressed

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with Coulomb interaction. While the experiments are only conductance measurements as they are, the cross correlation is therefore expected to be manifestly positive in such setups [30].

II. NORMAL-ELECTRON CASE

We consider a stripe of QSHI. The helical edge states at its boundary are described by a one-dimensional Dirac Hamiltonian

$$H = \hbar v \sum_{\ell=1,2} \int dx [\psi_{\ell\downarrow}^{\dagger}(i\,\partial_x - \mu)\psi_{\ell\downarrow} - \psi_{\ell\uparrow}^{\dagger}(i\,\partial_x + \mu)\psi_{\ell\uparrow}],$$
(1)

where $\psi_{\uparrow}$ ($\psi_{\downarrow}$) is the field operator of $\uparrow$ ($\downarrow$) electrons, μ is the chemical potential, and v is the propagation velocity. The edge of a QSHI sample forms a closed loop and can never be terminated in the middle. In Eq. (1) it has been assumed that the lower ($\ell = 1$) and upper ($\ell = 2$) segment of the whole edge are isolated from the rest by putting contact reservoirs A, B, C, D, and other additional contacts (not shown) as in Fig. 1(a).

Constricting the QSHI bar with side gates forms a QPC. The $\uparrow$ electron injected to the QPC from reservoir A, for example, either moves on to reservoir C with probability amplitude t or tunnels to the upper edge going out to reservoir D with probability amplitude r. The QPC thus serves as a tunable BS for electrons. The QPC is thus characterized completely by the scattering matrix Σ

$$\begin{bmatrix} c'_{1\downarrow} \\ c'_{2\downarrow} \\ c'_{1\uparrow} \\ c'_{2\uparrow} \end{bmatrix} = \Sigma \begin{bmatrix} c_{1\uparrow} \\ c_{2\uparrow} \\ c_{1\downarrow} \\ c_{2\downarrow} \end{bmatrix} = \begin{bmatrix} 0 & 0 & t & r \\ 0 & 0 & r & t \\ t & r & 0 & 0 \\ r & t & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{1\uparrow} \\ c_{2\uparrow} \\ c_{1\downarrow} \\ c_{2\downarrow} \end{bmatrix}$$
(2)

FIG. 1. (Color online) The schematic setups for a HBT experiment of Cooper pairs using edge states of QSHI. The light gray represents the QSHI sample, the arrowed thick lines represent the helical edge states. $\uparrow$ ($\downarrow$) electrons move counterclockwise (clockwise). The QPC formed by constriction with side gates serves as an electron beam splitter. A HBT experiment setup (a) for normal electrons and (b) for Cooper pairs. In (c), Cooper pairs are injected from two superconductors.

relating the amplitudes $c'_{\ell\sigma}$ ($\ell = 1,2$ and $\sigma = \uparrow, \downarrow$) at the output ports to $c_{\ell\sigma}$ at the input ports of QPC. We have suppressed the energy (or momentum) dependence of $c_{\ell\sigma}$ for notational simplicity.

We apply a bias voltage V to reservoir C, keeping A, B, and D electrically grounded. With this bias configuration, only $\downarrow$ electrons are injected from C along the lower edge. The quantum statistical properties of the electron beam are characterized by the correlation functions defined by

$$S_{\alpha\beta} = \int_{-\infty}^{\infty} dt \langle \Delta I_\alpha(t) \Delta I_\beta(0) + \Delta I_\beta(0) \Delta I_\alpha(t) \rangle$$
(3)

with $\Delta I_\alpha(t) = I_\alpha(t) - \bar{I}_\alpha$, where $\bar{I}_\alpha$ is the average current *into* reservoir α. Given the scattering matrix in (2), the calculation of $\bar{I}_\alpha$ and $S_{\alpha\beta}$ is a simple application of the Landauer-Büttiker formalism [18]. We focus on the zero-temperature limit, $k_B T \ll eV$. In the present case, the average currents are given by

$$\bar{I}_A = \frac{e^2 V}{2\pi\hbar}|t|^2, \quad \bar{I}_B = \frac{e^2 V}{2\pi\hbar}|r|^2.$$
(4)

Clearly the injected current $\bar{I} \equiv -\bar{I}_C$ flows either to A or B ($I_A + I_B = \bar{I}$), but not to D ($\bar{I}_D = 0$). This partitioning at the BS leads to the well-known results [18]

$$S_{AA} = S_{BB} = -S_{AB} = 2e\bar{I}|rt|^2$$
(5)

for the current correlations, and its negative value is well understood by their antibunching behavior [18]. For later reference, we stress that the contact D is completely mute, i.e., $I_D = 0$ and $S_{AD} = S_{BD} = S_{DD} = 0$.

III. HBT CORRELATIONS OF COOPER PAIRS

Let us now turn to the setup, Fig. 1(b), of our main concern. We replace the normal contact C by a superconducting contact S. A superconducting edge state is achieved by putting an s-wave superconductor on top of the edge. Due to the proximity effect, the edge states beneath the superconductor form a superconducting state with an induced gap $\Delta = \Delta_0 e^{-i\varphi}$ [24]. The Hamiltonian of the superconducting edge modes is then given by [cf. (1)]

$$H_S = \hbar v \int dx [\psi_{1\downarrow}^{\dagger}(i\,\partial_x - \mu)\psi_{1\downarrow} - \psi_{1\uparrow}^{\dagger}(i\,\partial_x + \mu)\psi_{1\uparrow}$$
$$+ \Delta \psi_{1\uparrow}^{\dagger}\psi_{1\downarrow}^{\dagger} + \Delta^* \psi_{1\downarrow}\psi_{1\uparrow}].$$
(6)

We apply bias voltage V to S and keep A, B, and D grounded, effectively injecting Cooper pairs from S along the lower edge channels. The injection of Cooper pairs can be equivalently described by the Andreev reflection process, where, say, an $\uparrow$ electron injected towards superconductor is reflected by a $\downarrow$ hole away from superconductor, see Fig. 2. The Andreev reflection amplitude is given by $a = e^{i\varphi - i\arccos(E/\Delta_0)}$ for incident electrons of energy E ($|E| < \Delta_0$). Note that due to the time-reversal invariance the Andreev reflection probability remains unity, $|a|^2 = 1$, regardless of the imperfections at the edge-superconductor interface [25]. This is in stark contrast to the usual normal-superconductor interface, where interface imperfections suppress Andreev reflections.

Combining the Andreev reflections at the normal-superconductor interface and the normal scattering (2) at the QPC, one obtains the total scattering matrix for electrons and holes:

$$
\begin{bmatrix} c'_{1\downarrow} \\ b'_{1\downarrow} \\ c'_{2\downarrow} \\ b'_{2\downarrow} \\ c'_{2\uparrow} \\ b'_{2\uparrow} \end{bmatrix} = \begin{bmatrix} 0 & a^*|t|^2 & 0 & a^*r^*t & r & 0 \\ a|t|^2 & 0 & art^* & 0 & 0 & r^* \\ 0 & a^*rt^* & 0 & a^*|r|^2 & t & 0 \\ ar^*t & 0 & a|r|^2 & 0 & 0 & t^* \\ r & 0 & t & 0 & 0 & 0 \\ 0 & r^* & 0 & t^* & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{1\uparrow} \\ b_{1\uparrow} \\ c_{2\uparrow} \\ b_{2\uparrow} \\ c_{2\downarrow} \\ b_{2\downarrow} \end{bmatrix},
\tag{7}
$$

where $b_{\ell\sigma}$ and $b'_{\ell\sigma}$ ($\ell = 1, 2$ and $\sigma = \uparrow, \downarrow$) are amplitudes of the holes on the input and output ports. Here $r^* \equiv [r(-E)]^*$, $t^* \equiv [t(-E)]^*$, and $a^* \equiv [a(-E)]^*$ describe the scattering of holes. We ignore weak energy dependence of r and t.

The average currents I_α and the current correlations $S_{\alpha\beta}$ ($\alpha, \beta = A, B, D$) are calculated using the Landauer-Buttiker formalism extended to the normal-superconductor hybrid structure [31,32]. The currents

$$
\bar{I}_A = \frac{2e^2 V}{2\pi\hbar} |t|^2, \quad \bar{I}_B = \frac{2e^2 V}{2\pi\hbar} |r|^2,
\tag{8}
$$

are twice larger than (4), demonstrating perfect Andreev reflections at (or injection of Cooper pairs from) S. The correlation functions are also given exactly in the same form as (5),

$$
S_{AA} = S_{BB} = -S_{AB} = 2e\bar{I}|rt|^2,
\tag{9}
$$

except that the total current $\bar{I} \equiv \bar{I}_A + \bar{I}_B$ is now twice larger. Surprisingly, the cross correlation is negative. (As in the normal case above, the contact D is mute; $I_D = 0$ and $S_{AD} = S_{BD} = S_{DD} = 0$.)

This is surprising because the description in (7) in terms of electrons and holes is equivalent to Cooper pairs injected from S and scattered at QPC. Note that either (i) entire Cooper pairs go to A or B [Figs. 3(a) and 3(c)], or (ii) constituent electrons in each pair split up into A and B [Figs. 3(b) and 3(d)]. Naively, one may expect a positive contribution from case (ii) with one electron at each port A and B simultaneously. Assuming (partial) bosonic nature of Cooper pairs, one may also expect $S_{AD} > 0$ in case (i).

The above expectation fails because it has ignored the two-particle interference [33]. For example, the two process
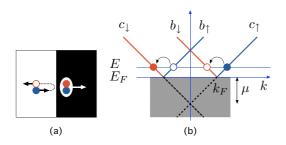
$(A \uparrow$ electron; $B \uparrow$ electron$) \rightarrow (A \downarrow$ hole; $B \downarrow$ hole$)$ [Figs. 3(a) and 3(c)] and $(A \uparrow$ electron; $B \uparrow$ electron$) \rightarrow (B \downarrow$ hole; $A \downarrow$ hole$)$ [Figs. 3(b) and 3(d)] are not distinguishable and interfere with each other. Due to the Fermi-Dirac statistics, the amplitudes for these processes are opposite in sign, hence giving negative cross correlation in (9).

There is another simple way to understand the negative correlation. For example, $\uparrow$ electron from A undergoes either Figs. 3(a) or 3(b). Since the perfect Andreev reflection at the edge-superconductor interface is noiseless, it is nothing but the partitioning of a single hole and thus gives a negative contribution to S_{AB}. Similarly, the partitioning into Figs. 3(c) and 3(d) also give negative contributions.

We thus conclude that Cooper pairs bear no bosonic characters at all, once they get out of the superconductor. It is interesting to note that in recent experiments with quantum dots [10,11] the processes Figs. 3(a) and 3(c) have been suppressed due to Coulomb interactions. In this case, there is no two-particle interference.

IV. BACK-SCATTERING EFFECT

The above conclusion appears contradictory to the previous theoretical works [6–9], where positive correlation was predicted in certain range. As mentioned earlier, in these works a Y junction is used instead of a true electron BS.
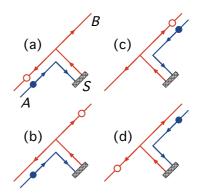

FIG. 3. (Color online) Elementary scattering processes in a Hanbury Brown and Twiss experiment of Cooper pairs, described in electron-hole picture. Blue (red) line represents the edge channels of $\uparrow$ ($\downarrow$) electrons (filled circle) or holes (empty circle). In Cooper pair picture, (a) corresponds to Cooper pairs being injected from S, scattered at the quantum point contact (QPC), and going entirely to A. Analogously, in (c), Cooper pairs entirely go to B. In (c) and (d), constituent electrons in a Cooper pair split up at the QPC.

FIG. 2. (Color online) (a) Andreev reflection at the interface between normal (left) and superconducting (right) helical edge modes, where the $\uparrow$ electron is reflected as a $\downarrow$ hole. (b) The energy-momentum relation of particles (c_σ) and holes (b_σ) with spin $\sigma = \uparrow, \downarrow$.

One crucial effect is the multiple reflections between the junction and the superconductor. Further, not only Andreev but also normal-electron reflections are involved. The latter effect cannot be simulated in our system and is analyzed in the supplementary material.

To simulate the former effects, let us now replace both C and B by superconductors, S_1 and S_2, with phases φ_1 and φ_2, respectively. We apply bias voltage V on both S_1 and S_2 and keep A and D grounded. The scattering of electrons and holes is governed by the scattering matrix (ignoring corrections of order $E^2/\Delta_0^2 \ll 1$)

$$
\begin{bmatrix} c'_{1\downarrow} \\ b'_{1\downarrow} \\ c'_{2\uparrow} \\ b'_{2\uparrow} \end{bmatrix} = \begin{bmatrix} 0 & \frac{a_1^*|t|^2}{z^*} & \frac{w^*r}{z^*} & 0 \\ \frac{a_1|t|^2}{z} & 0 & 0 & \frac{wr^*}{z} \\ \frac{wr}{z} & 0 & 0 & \frac{a_2^*|t|^2}{z} \\ 0 & \frac{w^*r^*}{z^*} & \frac{a_2|t|^2}{z^*} & 0 \end{bmatrix} \begin{bmatrix} c_{1\uparrow} \\ b_{1\uparrow} \\ c_{2\downarrow} \\ b_{2\downarrow} \end{bmatrix}, \quad (10)
$$

where $a_1 = -e^{i\varphi_1 + i2k_F L_1}$, $a_2 = -e^{i\varphi_2 - i2k_F L_2}$, $k_F = \mu/\hbar v$, and L_1 and L_2 are the distances from the QPC to the superconducting contacts S_1 and S_2, respectively. The *multiple reflections* between QPC and superconductors are manifested through the factors $z = 1 - |r|^2 e^{i\theta}$ and $w = 1 - e^{i\theta}$, where $\theta = (\varphi_1 - \varphi_2) + 2k_F(L_1 + L_2)$ is the phase accumulation during one cycle of the multiple reflections. This immediately leads to the resonance behavior of the average currents,

$$
\bar{I}_A = \bar{I}_D = \frac{2e^2 V}{2\pi\hbar} \frac{|t|^4}{1 + |r|^4 - 2|r|^2 \cos\theta} \quad (11)
$$

and the correlations

$$
S_{AA} = S_{DD} = S_{AD} = 2e\bar{I} \frac{|r|^2(1 - \cos\theta)}{1 + |r|^4 - 2|r|^2 \cos\theta}. \quad (12)
$$

For $\theta = 2\pi n$ with n an integer, the currents take maximum and the noises vanish.

We note that the sign of the cross correlation S_{AD} is now positive, in contrast to (9), which is always negative. This difference is ascribed to the multiple reflections between QPC and superconductors. As noted by Baym [34], the intensity correlation $\langle I(t + \tau)I(t) \rangle$ is equivalent to the relative probability to observe two particles at two points separated by a distance $v\tau$, where v is the flight velocity. If the HBT experiment is done with a true BS as in Fig. 1(b), then the intensity correlation is entirely due to the spatial distribution and the quantum statistical property of the particles in the beam itself. To the contrary, if the BS is replaced by a Y junction [6–9] and multiple reflections occur between the source and the junction, then the cross correlation is not directly related to the spatial distribution in the beam alone but affected significantly by the successive interactions with the source. Therefore, the positive cross correlations in Refs. [6–9] do not represent entirely the quantum statistical properties of Cooper pairs *after* being emitted from a superconductor (away by a distance larger than the superconducting coherence length).

APPENDIX: COMPARISON OF THE CHAOTIC QUANTUM DOT AND THE HELICAL LIQUID

Previous theoretical works [8,35] considered a chaotic quantum dot coupled to two normal leads 2 and 1 (corresponding to A and B in our case) and one superconducting reservoir. They found both positive and negative cross correlations in a wide range of parameter values, and provide clear interpretations of the relevant microscopic processes. Therefore, it will be useful for a deeper understanding of the physics behind the cross correlation to compare more closely the processes in their and our setup.

The crucial differences are (i) the multiple reflections between the dot-superconductor interface and the Y junction (dot-normal-metal interface), and (ii) the normal-electron reflections (other than Andreev reflections). With the former effect discussed in the main text of the paper, here we focus on the latter effect.

Here we follow the scattering-matrix approach in Ref. [8], which provides a clear picture of the two-particle interference [33] and hence direct comparison of their system to ours. A semiclassical analysis is given in Ref. [35], which is more useful for interpretation in terms of partitioning noise.

We first discuss the case with the transparency Γ_S of the dot superconductor is perfect ($\Gamma_S = 1$). The normal and superconducting contacts supports N and M channels, respectively.

For small $2N/M \ll 1$, the dominant process is the (local) Andreev reflections at the normal contact-dot interface, due to the gap in the dot spectrum induced by the superconductor. For finite (but small) $2N/M$, there are finite probabilities of normal reflections. The cross Andreev reflections (CARs) from one normal reservoir to the other is still negligible; i.e., the terms such as (S_{12}^{eh}) can be ignored. In this limit, the cross correlation is given by

$$
\frac{P_{12}}{4e^3 V/2\pi\hbar} \approx \frac{P_{12}^{eh} + P_{12}^{he}}{4e^3 V/2\pi\hbar} \approx 2\,\mathrm{Tr}\left[\left(S_{12}^{ee}\right)^\dagger \left(S_{11}^{eh}\right)\left(S_{21}^{hh}\right)^\dagger \left(S_{22}^{he}\right)\right].
$$
$$(A1)$$

This term arises from the two-particle interference of the two processes (a) and (b) in Fig. 4: The two processes cannot be distinguished and therefore the amplitudes (not probability) should be summed. The interference gives *positive* cross correlation [33] since it involves the exchange of electron and hole, namely, different species of particles. Therefore, the normal scattering processes (either electron or hole) in Fig. 4 is crucial to the positive cross correlation in this regime. In our case, the process of Fig. 4 is prohibited because of the helical (chiral) nature of the edge states.

In the limit of $2N/M \gg 1$, Ref. [8] obtained negative cross correlation. In this limit, there are finite probabilities of the CAR processes. According to Ref. [8], the main contributions
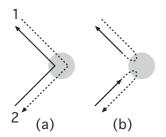
FIG. 4. Equation (A1) corresponds to the interference of the two processes (a) and (b). The interference is positive since it only involves the exchange of electron and hole, i.e., different species of particles.

come from P_{12}^{ee} and P_{12}^{hh}, where P_{12}^{ee} is expressed explicitly as

$$- \frac{P_{12}^{ee}}{4e^3 V/2\pi\hbar} = \mathrm{Tr}\left[\left(S_{11}^{ee}\right)^\dagger\left(S_{11}^{eh}\right)\left(S_{21}^{eh}\right)^\dagger\left(S_{21}^{ee}\right)\right.$$
$$\left. + \left(S_{11}^{ee}\right)^\dagger\left(S_{12}^{eh}\right)\left(S_{22}^{eh}\right)^\dagger\left(S_{21}^{ee}\right)\right]$$
$$+ \mathrm{Tr}\left[\left(S_{12}^{ee}\right)^\dagger\left(S_{11}^{eh}\right)\left(S_{21}^{eh}\right)^\dagger\left(S_{22}^{ee}\right)\right.$$
$$\left. + \left(S_{12}^{ee}\right)^\dagger\left(S_{12}^{eh}\right)\left(S_{22}^{eh}\right)^\dagger\left(S_{22}^{ee}\right)\right]. \quad (A2)$$

For example, the third term $\mathrm{Tr}[(S_{12}^{ee})^\dagger(S_{11}^{eh})(S_{21}^{eh})^\dagger(S_{22}^{ee})]$ corresponds to the two-particle interference of the two processes, Figs. 5(a) and 5(b). The interference is *negative* in this case, because it involves the exchange of two electrons (after the Andreev reflection). Again, both processes, Figs. 5(a) and 5(b), include normal-electron reflections, which are forbidden in our case. Other terms also involve similar normal-electron reflections. The above analysis shows that the system considered
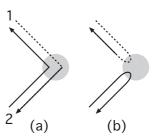

FIG. 5. The third term in Eq. (A2), $\mathrm{Tr}(S_{12}^{ee})^\dagger(S_{11}^{eh})(S_{21}^{eh})^\dagger(S_{22}^{ee})$, corresponds to the interference of the two processes (a) and (b). The interference is negative since it involves electrons (i.e., the electrons after the AR).

in Refs. [8,35] is in a clear distinction from ours, which involves only Andreev reflections (scattering of Cooper pairs).

Let us now turn to the case with $\Gamma_S < 1$. It turns out that for $\Gamma_S < 1/2$, the cross correlation becomes positive in the limit $2N/M\Gamma_S \gg 1$. This is ascribed to the additional fluctuations at the dot-superconductor interface due to imperfect Andreev reflections, which are forbidden in our system due to the time-reversal symmetry and makes another difference.

It will be useful to simulate this latter effect in our system by applying a magnetic field, which effectively introduces a mass gap into the helical liquid [24].

In conclusion, the close comparisons above reveal where the discrepancy between their and our results arise. While in our setup the cross correlation involves only the scattering of Cooper pairs (or Andreev reflections), in Ref. [8] it involves normal-electron reflections as well as Andreev reflections.

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[5] Strictly speaking, the simple association of bosons with positive and fermions with negative cross correlation is not always true. Nevertheless, the large negative correlation in our results is strongly counterintuitive.

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[25] In the presence of strong (so as for Luttinger liquid parameter to be less than 1/4) electron-electron interaction, the interface barrier can suppress the Andreev reflection. This effect is described elsewhere.

[26] In helical edge modes, the spin↑ (spin↓) electron moves only to the right (left). Cooper-pair injection is still possible: When a Cooper pair is injected, it has a finite center-of-mass momentum, say q, so that the momenta of two constituting electrons are $q + k$ and $q - k$. One should also recall the p-wave nature of the pairing in helical edge modes originating from strong spin-orbit coupling.

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