Solid-state implementation of quantum teleportation and quantum dense coding

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We consider three generic types of coupling between two quantum bits (qubits), which are typically found in solid-state qubits proposed in the literature. We show that proper choice of non-local Bell states can simplify significantly the implementation of quantum teleportation and quantum dense coding.

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Entanglement is one of the distinguishing properties of quantum mechanics, and plays a central role in quantum information processing. Two examples are quantum teleportation [1] and quantum dense coding [2]. While both of them have already been demonstrated experimentally using polarization-entangled photon pairs [3,4], neither has been realized in solid-state quantum bits (qubits). The latter is important when one wants eventually to perform large-scale information processing.

A severe problem to overcome is decoherence since the influence of the environment is stronger in solid-state devices, especially when it comes to quantum entanglement. Experiments have confirmed that coherence can be maintained long enough to perform single-qubit operations on Josephson qubits [5,6] and quantum-dot qubits [7]. Schemes to prepare an entangled state and detect it on quantum dots have been proposed theoretically [8,9]. Nevertheless, it is still challenging to demonstrate experimentally two-qubit unitary gate operations, such as quantum dense coding (QDC) and quantum teleportation (QT), on solid-state qubits. Given the limit of the decoherence time, it is very necessary to reduce the number of required operations. A reduced number of steps also helps to avoid errors from imperfect gate operations, which are inevitable in real experimental circumstances.

The schemes for quantum teleportation and quantum dense coding have been well established in terms of universal gates and, in principle, can apply to any qubits. However, given a specific form of Hamiltonian, in particular coupling between two qubits, actual realizations can be quite complicated. In fact, even in Refs. [3,4], the success probability of teleportation and dense coding was less than 1, because of the peculiar properties of the Bell-state analyzer for photon pairs. In the present work, we consider a few already proposed solid-state qubits (or simplified versions of them) and show that particular choices of Bell states (the orthogonal basis for a two-qubit system; see below) allow us to reduce significantly the number of steps required to perform quantum teleportation and quantum dense coding.

Quantum dense coding and quantum teleportation. To establish the notation, we first briefly review the quantum dense coding and quantum teleportation schemes. Quantum dense coding uses a preshared entangled pair of qubits to transmit two bits of classical information by sending only one qubit. It consists of three steps. (i) An entangled state of two qubits is prepared and shared between the two parties, say, Alice (A) and Bob (B). It can be one of the four maximally entangled Bell states

$$|\phi^{\pm}\rangle=\frac{1}{\sqrt{2}}(|00\rangle\pm|11\rangle), \tag{1a}$$

$$|\psi^{\pm}\rangle=\frac{1}{\sqrt{2}}(|01\rangle\pm|10\rangle), \tag{1b}$$

where $|ij\rangle=|i\rangle_A\otimes|j\rangle_B$ with $i,j=0,1$, and $|0\rangle$ and $|1\rangle$ compose an orthogonal basis of each qubit. (ii) When Bob wants to send a message to Alice, he encodes his message on his qubit of the preshared entangled pair by performing one of the four unitary operations $\mathbf{1}$, σ^x, σ^y, and σ^z (two bits of information). Each of these operations transforms the entangled state uniquely to one of the Bell states (1); e.g., if the preshared entangled state was $|\phi^+\rangle$, then $\mathbf{1}_B|\phi^+\rangle=|\phi^+\rangle$, $\sigma_B^x|\phi^+\rangle=|\psi^+\rangle$, $\sigma_B^y|\phi^+\rangle=i|\psi^-\rangle$, and $\sigma_B^z|\phi^+\rangle=|\phi^-\rangle$. Bob then sends his qubit to Alice. (iii) Finally, Alice (the receiver) performs a *nonlocal* Bell measurement to see in which of the four Bell states the two qubits are. This reveals precisely which operation Bob performed.

Quantum teleportation utilizes a classical channel to transmit quantum information. It also consists of three steps. (i) Alice and Bob prepare and share an entangled pair, say, $|\phi^+\rangle_{AB}$ in Eq. (1). (ii) Bob takes his qubit (B) of the pair and puts it together with a third qubit C in the unknown state $|\psi\rangle_C=a|0\rangle_C+b|1\rangle_C$ that he wants to "teleport" to Alice. Up to this point, the total wave function $|\Psi\rangle_{ABC}=|\phi^+\rangle_{AB}\otimes|\psi\rangle_C$ of the three qubits can be recast as

$$\begin{aligned}|\Psi\rangle_{ABC}=&\frac{1}{2}|\psi\rangle_A\otimes|\phi^+\rangle_{BC}+\frac{1}{2}\{\sigma^x|\psi\rangle_A\}\otimes|\psi^+\rangle_{BC}\\&+\frac{1}{2}\{-i\sigma^y|\psi\rangle_A\}\otimes|\psi^-\rangle_{BC}\\&+\frac{1}{2}\{\sigma^z|\psi\rangle_A\}\otimes|\phi^-\rangle_{BC}.\end{aligned} \tag{2}$$

Bob carries out a Bell measurement on the two qubits B and C and informs Alice of his measurement result (two bits of classical information) through a classical channel. (iii) Finally, Alice performs the proper operation ($\mathbf{1}$, σ^x, σ^y, or σ^z) on her qubit A, depending on the classical information that she receives from Bob, to get the desired state $|\psi\rangle$.

Quantum dense coding and quantum teleportation have three technical parts in common, whereas the former uses a quantum communication channel and the latter a classical channel. The three common parts are *Bell preparation* [step (i) of QDC and QT above], *Bell measurement* [(iii) of QDC and (ii) of QT], and *Bell transformation* [(ii) of QDC and (iii) of QT]. The communication channel does not depend on the choice of Bell states. Further, in terms of universal gates, the nonlocal Bell measurement is achieved simply by the inverse procedure of the Bell preparation [14]. Accordingly, in what follows, we will focus our discussion on two technical parts, Bell preparation and Bell transformation.

For comparison, we note that quantum optical realizations such as those in Refs. [4] and [3] are different from the conceptual schemes described in terms of universal gate operations [2,1,14]. In the quantum optical realizations, the Bell preparation was achieved by generating polarization-entangled photon pairs using parametric down-conversion, which is not described as a unitary gate operation. At the Bell transformation step, a half-wave retardation plate and a quarter-wave plate were used for changing the polarization and for the polarization-dependent phase shift, respectively. The Bell measurement step was achieved by a coincidence analysis between four single-photon detectors located at the four output channels of a beam splitter followed by two two-channel polarizers. It should be noted that in this scheme the Bell measurement bears no relation to the Bell preparation, in contrast to the scheme in terms of universal gates. More importantly, this coincidence analysis is not enough to distinguish all four Bell states from one another.

New Bell states. The choice of Bell states is not unique; Eq. (1) is merely one choice. As we will see below, a particular choice can be especially convenient for one system and others for different systems. This depends most strongly on the way the two qubits interact in a given quantum machine. In this work, we first consider the isotropic coupling

$$H_{\rm int}=\frac{1}{2}\vec{\sigma}_A\cdot\vec{\sigma}_B=\sigma_A^+\sigma_B^-+\sigma_A^-\sigma_B^++\frac{1}{2}\sigma_A^z\sigma_B^z \tag{3}$$

and the planar coupling

$$H_{\rm int}=\sigma_A^+\sigma_B^-+\sigma_A^-\sigma_B^+ \tag{4}$$

between a given pair of qubits A and B. Later we will also consider Ising coupling [Eq. (12) below]. In Eqs. (3) and (4), the Hamiltonian is normalized by the coupling energy scale characteristic of a particular realization of the qubits. The isotropic type of coupling (3) arises, e.g., in quantum computers using spins on quantum dots [10]. In this case, (antiferromagnetic) Heisenberg exchange interaction $J=4t^2/U$ (where U is the on-site repulsion on the dot and t is the hopping amplitude between the dots) determines the coupling energy scale. The planar type of coupling (4) is found, for example, in Josephson qubits [11,12]. The coupling energy scales are determined by the Josephson coupling energy E_J [11,12]. A proper choice of Bell states should also take into account to what extent single-qubit rotations are allowed. For example, for Josephson qubits in the charging model [11], whereas rotations around the x and z axes can be directly implemented, rotation around the y axis is achieved only in combinations of x- and z-axis rotations. To make our argument as versatile as possible, we will consider this restricted case, i.e., the single-qubit Hamiltonian has the form

$$H_0=\sum_j\;(h_j^x\sigma_j^x+h_j^z\sigma_j^z). \tag{5}$$

Therefore, we are given the natural two-qubit operation

$$U_{\rm int}(\theta)=\exp(-i\theta H_{\rm int}) \tag{6}$$

directly related to Eq. (3) or (4), and single-qubit rotations

$$R_x(\theta)=\exp(-i\theta\sigma^x),\quad R_z(\theta)=\exp(-i\theta\sigma^z) \tag{7}$$

related to Eq. (5).

Now let us consider the following choice of Bell states, a nonlocal orthonormal basis in two-qubit Hilbert space:

$$|B_{00}\rangle=\frac{1}{\sqrt{2}}(|01\rangle+i|10\rangle), \tag{8a}$$

$$|B_{01}\rangle=\frac{1}{\sqrt{2}}(|00\rangle+i|11\rangle), \tag{8b}$$

$$|B_{10}\rangle=\frac{1}{\sqrt{2}}(|11\rangle+i|00\rangle), \tag{8c}$$

$$|B_{11}\rangle=\frac{1}{\sqrt{2}}(|10\rangle+i|01\rangle). \tag{8d}$$

We will show that for the isotropic or planar type of coupling, Eq. (3) or (4), the Bell states (8) have a substantial advantage in quantum teleportation and quantum dense coding compared with the original Bell states (1). To see this, we first think of the preparation of an entangled pair of qubits in one of the Bell states. Starting from the logical basis $|ss'\rangle$ $(s,s'=0,1)$, any of the Bell states (8) can be obtained by $|B_{ss'}\rangle=U_{\rm Bell}|ss'\rangle$ where the *Bell operator* $U_{\rm Bell}$ is given by [13]

$$\begin{aligned}U_{\rm Bell}&=U_{\rm int}(\pi/4)[R_x(-\pi)\otimes\mathbf{1}]U_{\rm int}(\pi/4)\\&=\frac{1}{\sqrt{2}}\begin{pmatrix}0&1&i&0\\1&0&0&i\\i&0&0&1\\0&i&1&0\end{pmatrix}.\end{aligned} \tag{9}$$

Notice that the Bell operator (9) is implemented in a simple manner by means of only the natural two-qubit gate $U_{\rm int}$ and single-qubit rotation R_x [Eqs. (6) and (7)]. The original Bell states (1), on the other hand, can be prepared in combinations of the Hadamard gate and the exclusive-OR (XOR) gate. However, with isotropic or planar coupling, the implementation of XOR involves a complicated combination of two-qubit operations and single-qubit rotations [10–12]. (In quantum optical implementations [3,4], the polarization-entangled photon pairs are directly prepared by the parametric down-conversion method, but the efficiency is relatively low.)

Since a nonlocal Bell measurement corresponds to the inverse of the preparation procedure [14], i.e., transforming $|B_{ss'}\rangle \mapsto |ss'\rangle = U_{\rm Bell}^{-1}|B_{ss'}\rangle$ and measuring on the logical basis $|ss'\rangle$, one gets the same advantage mentioned above in the Bell measurement step also.

To check that it is indeed possible to use the Bell states (8) for quantum dense coding and quantum teleportation, we examine the transformation properties of Bell states (8). For example, $|B_{00}\rangle$ transforms as

$$\begin{aligned} \mathbf{1}\otimes R_x(\pi)&:|B_{00}\rangle \mapsto -i|B_{01}\rangle, \\ \mathbf{1}\otimes R_y(\pi)&:|B_{00}\rangle \mapsto +i|B_{10}\rangle, \\ \mathbf{1}\otimes R_z(\pi)&:|B_{00}\rangle \mapsto |B_{11}\rangle, \end{aligned} \tag{10}$$

where $R_y(\pi)=R_z(\pi)R_x(\pi)$ [Eq. (7)]. Therefore by choosing one of four rotations $\mathbf{1}$, $R_x(\pi)$, $R_y(\pi)$, and $R_z(\pi)$, Bob can encode two bits of information in one qubit to be transmitted to Alice. Similarly, we can check for teleportation. Suppose that Bob wants to "teleport" an unknown state $|\psi\rangle = a|0\rangle + b|1\rangle$ to Alice by using, say, the Bell state $|B_{00}\rangle_{AB}$ preshared between Alice and Bob. One can show that the total wave function $|\Psi\rangle = |B_{00}\rangle_{AB}\otimes|\psi\rangle_C$ can be decomposed into

$$\begin{aligned} |\Psi\rangle =& \frac{1}{2}\{R_z(\pi)|\psi\rangle_A\}\otimes|B_{00}\rangle_{BC} + \frac{1}{2}\{iR_y(\pi)|\psi\rangle_A\}\otimes|B_{01}\rangle_{BC} \\ &+ \frac{1}{2}\{iR_x(\pi)|\psi\rangle_A\}\otimes|B_{10}\rangle_{BC} + \frac{1}{2}|\psi\rangle_A\otimes|B_{11}\rangle_{BC}. \end{aligned} \tag{11}$$

Therefore, if Bob obtains, say, the result $|B_{00}\rangle_{BC}$ from the Bell measurement performed on qubits B and C, he informs Alice of the result, and Alice performs $R_z(\pi)$ on her qubit A to restore the desired state $|\psi\rangle_A$.

Ising Coupling. Next we discuss the case of Ising coupling:

$$H_{\rm int} = \sigma_A^z \sigma_B^z. \tag{12}$$

Such an interaction describes, e.g., capacitively coupled Josephson qubits [15,16]. In this case, it turns out that the choice of Bell states is [17]

$$|B_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + i|11\rangle), \tag{13a}$$

$$|B_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - i|10\rangle), \tag{13b}$$

$$|B_{10}\rangle = \frac{1}{\sqrt{2}}(|10\rangle - i|01\rangle), \tag{13c}$$

$$|B_{11}\rangle = \frac{1}{\sqrt{2}}(|11\rangle + i|00\rangle). \tag{13d}$$

The Bell operator generating these states [cf. Eq. (9)] is given by

$$\begin{aligned} U_{\rm Bell} =& [R_x(-\pi/2)\otimes R_x(-\pi/2)]U_{\rm int}(\pi/4)[R_x(\pi/2) \\ &\otimes R_x(\pi/2)] = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ i & 0 & 0 & 1 \end{pmatrix}. \end{aligned} \tag{14}$$

Following the same lines as in the above discussion, one infers that for the Ising type of coupling the Bell states (13) form a more convenient basis.

In conclusion, we have considered three generic types of interaction that commonly arise in solid-state qubits. We showed that a particular choice of Bell states for each type significantly simplifies the implementation of quantum teleportation and quantum dense coding.

[1] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
[2] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[3] D. Bouwmeester *et al.*, Nature (London) **390**, 575 (1997); **394**, 841 (1998); S. L. Braunstein and H. J. Kimble, *ibid.* **394**, 841 (1998).
[4] K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, Phys. Rev. Lett. **76**, 4656 (1996).
[5] Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).
[6] C. H. van der Wal *et al.*, Science **290**, 773 (2000).
[7] J. M. Kikkawa and D. D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998).
[8] D. Loss and E. Sukhorukov, Phys. Rev. Lett. **84**, 1035 (2000).
[9] M.-S. Choi, C. Bruder, and D. Loss, Phys. Rev. B **62**, 13 569 (2000).
[10] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
[11] Y. Makhlin, G. Schön, and A. Shnirman, Nature (London) **398**, 305 (1999).
[12] J. E. Mooij *et al.*, Science **285**, 1036 (1999); T. P. Orlando *et al.*, Phys. Rev. B **60**, 15 398 (1999).
[13] When two-qubit operations are much harder than single-qubit rotations, one can consider another choice. For example, given a planar coupling Hamiltonian, one can choose the following Bell states:

$$|B_{00}\rangle = \frac{i}{\sqrt{2}}(|\phi^+\rangle - i|\phi^-\rangle),$$

$$|B_{01}\rangle = \frac{-1}{\sqrt{2}}(|\psi^+\rangle - i|\psi^-\rangle),$$

$$|B_{10}\rangle = \frac{-1}{\sqrt{2}}(|\psi^+\rangle + i|\psi^-\rangle),$$

$$|B_{11}\rangle = \frac{i}{\sqrt{2}}(|\phi^+\rangle + i|\phi^-\rangle),$$

where $|\psi^\pm\rangle$ and $|\phi^\pm\rangle$ are the original Bell states Eq. (1). The corresponding Bell matrix [the counterpart of Eq. (9)]

$$U_{\rm Bell} = \frac{i}{2}\begin{pmatrix} 1-i & 0 & 0 & 1+i \\ 0 & 1+i & -1+i & 0 \\ 0 & -1+i & 1+i & 0 \\ 1+i & 0 & 0 & 1-i \end{pmatrix}$$

is implemented by

$$U_{\rm Bell} = [R_x(\pi/2)\otimes R_x(\pi/2)]U_{\rm int}(\pi/2)[R_x(\pi/2)\otimes R_x(\pi/2)].$$

The two-qubit gate $U_{\rm int}$ is required only once.

[14] J. Preskill, http://www.theory.caltech.edu/people/preskill/ph229/
[15] G. Falci *et al.*, Nature (London) **407**, 355 (2000).
[16] M.-S. Choi, M. Y. Choi, T. Choi, and S.-I. Lee, Phys. Rev. Lett. **81**, 4240 (1998).
[17] I. H. Nahm (unpublished) proposed to use the same Bell states (13) in order to avoid an additional phase shift in dense coding.