

Q3 SYMBOL

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Q3`

## AmplitudeEmbedding

NEW IN 13.3

`AmplitudeEmbedding`  $[[\{x_1, x_2, \dots, x_{2^n}\}, \{s_1, s_2, \dots, s_n\}]]$

returns a quantum state on qubits  $\{s_1, s_2, \dots, s_n\}$ , the amplitudes of which encode classical input data  $\{x_1, x_2, \dots, x_{2^n}\}$ .

### ▾ Details and Options

- The *amplitude embedding* is mapping,  $\{x_1, x_2, \dots, x_{2^n}\} \mapsto \sum_{k=1}^{2^n} |k-1\rangle x_k$ , where  $|a\rangle := |a_1\rangle \otimes |a_2\rangle \otimes \dots \otimes |a_n\rangle$  and  $a := (a_1 a_2 \dots a_n)_2$  is the binary-digit representation of integer  $a$ .

### ▾ Examples (1)

```
In[1]:= Needs["Q3`"]
```

#### ▾ Basic Examples (1)

```
In[1]:= Let[Qubit, S]
```

We consider a quantum register of  $n$  qubits.

```
In[2]:= $n = 4;
kk = Range[$n];
SS = S[kk, $]
```

```
Out[2]= {S1, S2, S3, S4}
```

Suppose that we have a classical input data.

```
In[3]:= Let[Real, x]
xx = x[Range@Power[2, $n]]
```

```
Out[3]= {x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16}
```

The amplitude embedding constructs the following quantum state.

```
In[4]:= AmplitudeEmbedding[xx, SS]
```

```
Out[4]= |0s1 0s2 0s3 0s4> x1 + |0s1 0s2 0s3 1s4> x2 + |0s1 0s2 1s3 0s4> x3 + |0s1 0s2 1s3 1s4> x4 + |0s1 1s2 0s3 0s4> x5 + |0s1 1s2 0s3 1s4> x6 +
|0s1 1s2 1s3 0s4> x7 + |0s1 1s2 1s3 1s4> x8 + |1s1 0s2 0s3 0s4> x9 + |1s1 0s2 0s3 1s4> x10 + |1s1 0s2 1s3 0s4> x11 +
|1s1 0s2 1s3 1s4> x12 + |1s1 1s2 0s3 0s4> x13 + |1s1 1s2 0s3 1s4> x14 + |1s1 1s2 1s3 0s4> x15 + |1s1 1s2 1s3 1s4> x16
```

On an actual quantum computer, the above quantum state must be achieved through a unitary gate. This is represented by the `AmplitudeEmbedding` function.

```
In[5]:= op = AmplitudeEmbeddingGate[xx, SS]
```

```
Out[5]= AmplitudeEmbeddingGate[{x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16}, {S1, S2, S3, S4}]
```

The above unitary gate is decomposed into two uniformly controlled rotations. See [AmplitudeEmbeddingGate](#) and [UniformlyControlledRotation](#).



### See Also

[AmplitudeEmbeddingGate](#) ▪ [UniformlyControlledRotation](#) ▪ [BasisEmbedding](#) ▪ [BasisEmbeddingGate](#)



### Tech Notes

- [Multi-Control Unitary Gates](#)
- [Quantum Information Systems with Q3](#)
- [Quick Quantum Computing with Q3](#)
- [Q3: Quick Start](#)



### Related Guides

- [Quantum Information Systems](#)
- [Q3](#)

### Related Links

- M. Nielsen and I. L. Chuang (2022) , [Quantum Computation and Quantum Information](#) (Cambridge University Press, 2011).
- Mahn-Soo Choi (2022) , [A Quantum Computation Workbook](#) (Springer, 2022).

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## AmplitudeEmbeddingGate

NEW IN 13.3

`AmplitudeEmbeddingGate`  $\{ \{x_1, x_2, \dots, x_{2^n}\}, \{s_1, s_2, \dots, s_n\} \}$ represents the gate to encode classical input data  $\{x_1, x_2, \dots, x_{2^n}\}$  into the amplitudes of a quantum state of qubits  $s_1, s_2, \dots, s_n$ .

## ▾ Details and Options

- The *amplitude embedding* is mapping,  $\{x_1, x_2, \dots, x_{2^n}\} \mapsto \sum_{k=1}^{2^n} |k-1\rangle x_k$ , where  $|a\rangle := |a_1\rangle \otimes |a_2\rangle \otimes \dots \otimes |a_n\rangle$  and  $a := (a_1 a_2 \dots a_n)_2$  is the binary-digit representation of integer  $a$ .

## ▾ Examples (1)

`In[1]:= Needs["Q3`"]`

## ▾ Basic Examples (1)

`In[1]:= Let[Qubit, S]`Consider a system of  $n$  qubits.

```
In[2]:= $n = 3;
        $N = Power[2, $n];
        kk = Range[$n];
        SS = S[kk, $]
```

`Out[2]= {S1, S2, S3}`

We want to embed a classical input data of the form.

`In[3]:= xx = Normalize@RandomVector[$N]`

```
Out[3]= {-0.259904 - 0.427261 i, 0.444914 - 0.227347 i, -0.140951 + 0.0979235 i, 0.122408 + 0.459967 i,
        -0.116309 + 0.123089 i, -0.020741 + 0.0522983 i, -0.314258 + 0.153372 i, -0.288961 - 0.081422 i}
```

We assume that the data is normalized.

`In[4]:= Norm[xx]``Out[4]= 1.`

This is the desired quantum state embedding the classical data above.

```
In[5]:= vec = AmplitudeEmbedding[xx, SS]
```

```
Out[5]= (-0.259904 - 0.427261 i) |0S10S20S3⟩ + (0.444914 - 0.227347 i) |0S10S21S3⟩ -
(0.140951 - 0.0979235 i) |0S11S20S3⟩ + (0.122408 + 0.459967 i) |0S11S21S3⟩ - (0.116309 - 0.123089 i) |1S10S20S3⟩ -
(0.020741 - 0.0522983 i) |1S10S21S3⟩ - (0.314258 - 0.153372 i) |1S11S20S3⟩ - (0.288961 + 0.081422 i) |1S11S21S3⟩
```

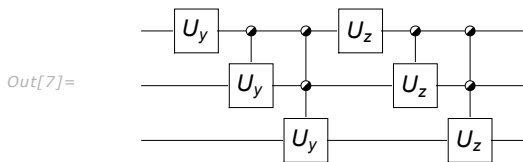
On a quantum machine, the above quantum state must be achieved through a unitary gate starting from the usual initial state  $|0\rangle := |0\rangle^{\otimes n}$ .

```
In[6]:= op = AmplitudeEmbeddingGate[xx, SS]
```

```
Out[6]= AmplitudeEmbeddingGate[{-0.259904 - 0.427261 i, 0.444914 - 0.227347 i,
-0.140951 + 0.0979235 i, 0.122408 + 0.459967 i, -0.116309 + 0.123089 i,
-0.020741 + 0.0522983 i, -0.314258 + 0.153372 i, -0.288961 - 0.081422 i}, {S1, S2, S3}]
```

In quantum circuit, it is depicted as follows, which consists of two uniformly controlled rotations.

```
In[7]:= qc = QuantumCircuit[op]
```



Obtain the output state from the above unitary gate.

```
In[8]:= out = qc ** Ket[]
```

```
Out[8]= (-0.468873 - 0.173953 i) |0S10S20S3⟩ + (0.207993 - 0.454285 i) |0S10S21S3⟩ -
(0.04983 - 0.164235 i) |0S11S20S3⟩ + (0.381304 + 0.284887 i) |0S11S21S3⟩ - (0.0148924 - 0.168692 i) |1S10S20S3⟩ +
(0.0161672 + 0.0538881 i) |1S10S21S3⟩ - (0.151385 - 0.31522 i) |1S11S20S3⟩ - (0.277167 - 0.115353 i) |1S11S21S3⟩
```

Apparently, it looks different from the desired state `vec` shown above. However, this is because of a physically irrelevant global phase. Indeed, the fidelity between the two states are unity.

```
In[9]:= Fidelity[out, vec]
```

```
Out[9]= 1.
```

As already mentioned, the amplitude embedding gate consists of two uniformly controlled rotations. This is made explicitly using the `Expand` function.

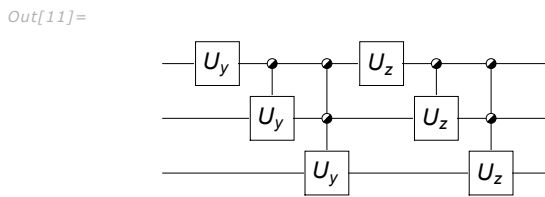
```
In[10]:=
```

```
Expand[op]
```

```
Out[10]=
```

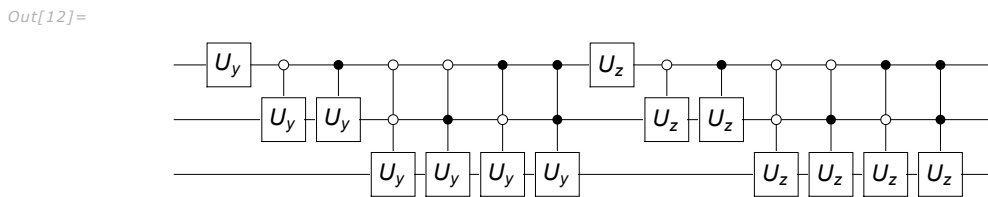
```
Sequence[Rotation[1.03387, {0, 1, 0}, S1],
UniformlyControlledRotation[{S1}, {1.24243, 2.40276}, {0, 1, 0}, S2],
UniformlyControlledRotation[{S1, S2}, {1.56986, 2.44945, 0.641499, 1.41884}, {0, 1, 0}, S3],
Rotation[0.710362, {0, 0, 1}, S1], UniformlyControlledRotation[{S1}, {3.21741, -2.22781}, {0, 0, 1}, S2],
UniformlyControlledRotation[{S1, S2}, {1.6449, -1.22371, -0.379524, -5.55449}, {0, 0, 1}, S3]]
```

```
In[11]:=
qc = QuantumCircuit[Expand@op]
```



The uniformly controlled rotations themselves may be decomposed further into products of more elementary gates as follows (notice the ExpandAll command).

```
In[12]:=
qc = QuantumCircuit[ExpandAll@op]
```



Check again the above statement is true by examining the output state.

```
In[13]:=
more = qc ** Ket[SS]
```

Out[13]=

$$\begin{aligned}
 & (-0.468873 - 0.173953 i) |0_{S_1} 0_{S_2} 0_{S_3}\rangle + (0.207993 - 0.454285 i) |0_{S_1} 0_{S_2} 1_{S_3}\rangle - \\
 & (0.04983 - 0.164235 i) |0_{S_1} 1_{S_2} 0_{S_3}\rangle + (0.381304 + 0.284887 i) |0_{S_1} 1_{S_2} 1_{S_3}\rangle - (0.0148924 - 0.168692 i) |1_{S_1} 0_{S_2} 0_{S_3}\rangle + \\
 & (0.0161672 + 0.0538881 i) |1_{S_1} 0_{S_2} 1_{S_3}\rangle - (0.151385 - 0.31522 i) |1_{S_1} 1_{S_2} 0_{S_3}\rangle - (0.277167 - 0.115353 i) |1_{S_1} 1_{S_2} 1_{S_3}\rangle
 \end{aligned}$$

```
In[14]:=
Fidelity[more, vec]
```

Out[14]= 1.

Note that the above quantum circuit may be simplified even further. See the examples in UniformlyControlledRotation.



See Also

- [AmplitudeEmbedding](#) ▪ [UniformlyControlledRotation](#) ▪ [BasisEmbedding](#) ▪ [BasisEmbeddingGate](#)



Tech Notes

- [Multi-Control Unitary Gates](#)
- [Quantum Information Systems with Q3](#)
- [Quick Quantum Computing with Q3](#)
- [Q3: Quick Start](#)



Related Guides

- Quantum Information Systems
- Q3

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Related Links

- M. Möttönen *et al.*, *Quantum Information and Computation* 5, 467 (2005), "Transformation of quantum states using uniformly controlled rotations."
- M. Nielsen and I. L. Chuang (2022), *Quantum Computation and Quantum Information* (Cambridge University Press, 2011).
- Mahn–Soo Choi (2022), *A Quantum Computation Workbook* (Springer, 2022).

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## UniformlyControlledRotation

NEW IN 13.3

`UniformlyControlledRotation`[[ $c_1, c_2, \dots, c_n$ ], { $\theta_1, \theta_2, \dots, \theta_{2^n}$ }, { $x, y, z$ },  $s$ ]

represents the uniformly controlled rotation on qubit  $s$  around axis  $\{x, y, z\}$  by angles  $\theta_1, \theta_2, \dots, \theta_{2^n}$  depending on all possible bit sequences of control qubits  $c_1, c_2, \dots, c_n$ .

`UniformlyControlledRotation`[[ $c_1, c_2, \dots, c_n$ ], { $\theta_1, \theta_2, \dots, \theta_n$ },  $s[\dots, k]$ ]

uses the  $k$ -axis as the rotation axis.

### ▾ Details and Options

- In general, `UniformlyControlledRotation`[[ $c_1, c_2, \dots, c_n$ ], { $\theta_1, \theta_2, \dots, \theta_{2^n}$ }, { $x, y, z$ },  $s$ ] is a product of  $2^n$  two-level matrices.
- Note also that `UniformlyControlledRotation`[[ $c_1, c_2, \dots, c_n$ ], { $\theta_1, \theta_2, \dots, \theta_{2^n}$ }, { $x, y, z$ },  $s$ ] equals to  $R_v(\theta_1) \oplus R_v(\theta_2) \oplus \dots \oplus R_v(\theta_{2^n})$ , where  $v \equiv \{x, y, z\}$ , and hence is block diagonal.
- An arbitrary multi-qubit unitary matrix  $U$  can be decomposed into a product of uniformly controlled rotations; see Möttönen *et al.* (2004).

### ▾ Examples (4)

```
In[1]:= Needs["Q3`"]
```

#### ▾ Basic Examples (2)

```
In[1]:= Let[Qubit, S, T]
```

```
In[2]:= $n = 3;
$N = Power[2, $n];
kk = Range[$n];
SS = S[kk, $]
```

```
Out[2]= {S1, S2, S3}
```

Define a series of rotations on qubit  $\tau_{[1, \$]}$ .

```
In[3]:= aa = RandomReal[{0, 1}, $N] * Pi
```

```
Out[3]= {1.81603, 3.04719, 2.86853, 0.160244, 1.0859, 1.78237, 2.22677, 0.516285}
```

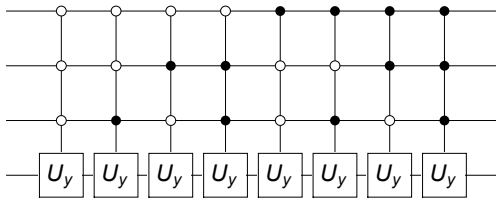
```
In[4]:= op = UniformlyControlledRotation[SS, aa, T[2]]
```

```
Out[4]= UniformlyControlledRotation[{S1, S2, S3},
{1.81603, 3.04719, 2.86853, 0.160244, 1.0859, 1.78237, 2.22677, 0.516285}, {0, 1, 0}, T]
```





```
In[11]:=
qc = QuantumCircuit[Expand@op]
Out[11]=
```



For more examples, see the Scope section below.

▼ Scope (2)

▼ Dagger (1)

```
In[1]:= Let[Qubit, S, T]
```

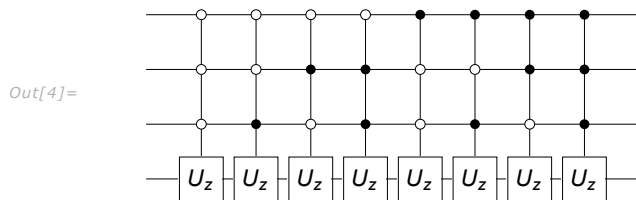
```
In[2]:= $n = 3;
$N = Power[2, $n];
kk = Range[$n];
SS = S[kk, $]
```

```
Out[2]= {S1, S2, S3}
```

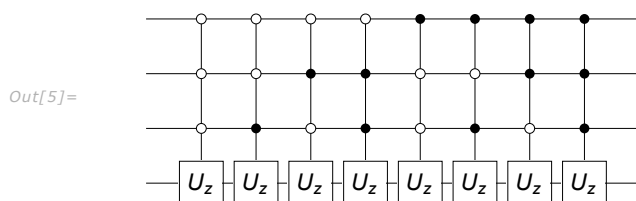
```
In[3]:= aa = RandomReal[{0, 1}, $N] * Pi
```

```
Out[3]= {2.36348, 2.92257, 1.13025, 2.12854, 1.40961, 1.17361, 2.45454, 1.58282}
```

```
In[4]:= op = UniformlyControlledRotation[SS, aa, T[3]];
qc = QuantumCircuit[Expand@op]
```

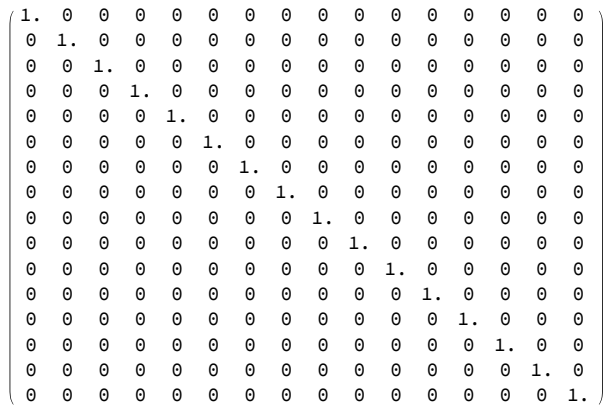


```
In[5]:= new = QuantumCircuit[Expand@Dagger@op]
```



```
In[6]:= Matrix[qc].Matrix[new] // Chop // MatrixForm
```

Out[6]//MatrixForm=



▼ Simplification (1)

As long as the rotation axis is *not* parallel to the x-axis, uniformly controlled rotation may be reduced to a simpler and more efficient gate sequence.

```
In[1]:= Let[Qubit, S, T]
```

```
In[2]:= $n = 3;
$N = Power[2, $n];
kk = Range[$n];
SS = S[kk, $]
```

Out[2]= {S<sub>1</sub>, S<sub>2</sub>, S<sub>3</sub>}

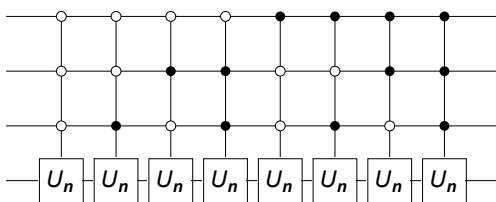
```
In[3]:= aa = RandomReal[{0, 2}, $N] * Pi
```

Out[3]= {5.77497, 0.816812, 3.64532, 1.99685, 1.32353, 5.2546, 5.48638, 3.91312}

Note that the rotation axis is not parallel to the x-axis.

```
In[4]:= op = UniformlyControlledRotation[SS, aa, {0, 1, 1}, T];
qc = QuantumCircuit[Expand@op]
```

Out[4]=



```
In[5]:= new = QuantumCircuit[GateFactor@op]
```

Out[5]=

