Robust Quantum State Transfer through Strong Coupling to Bosonic Bath

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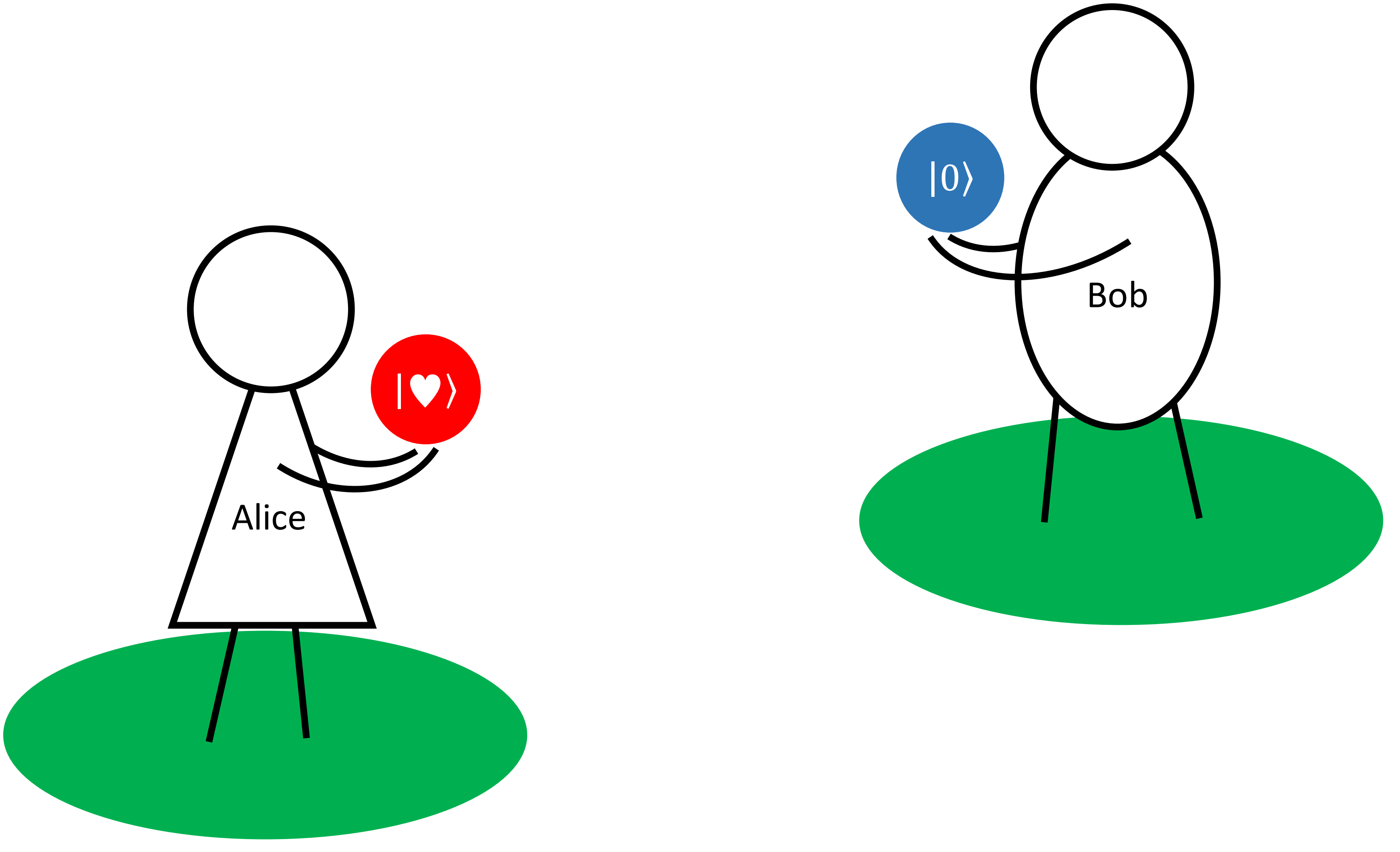
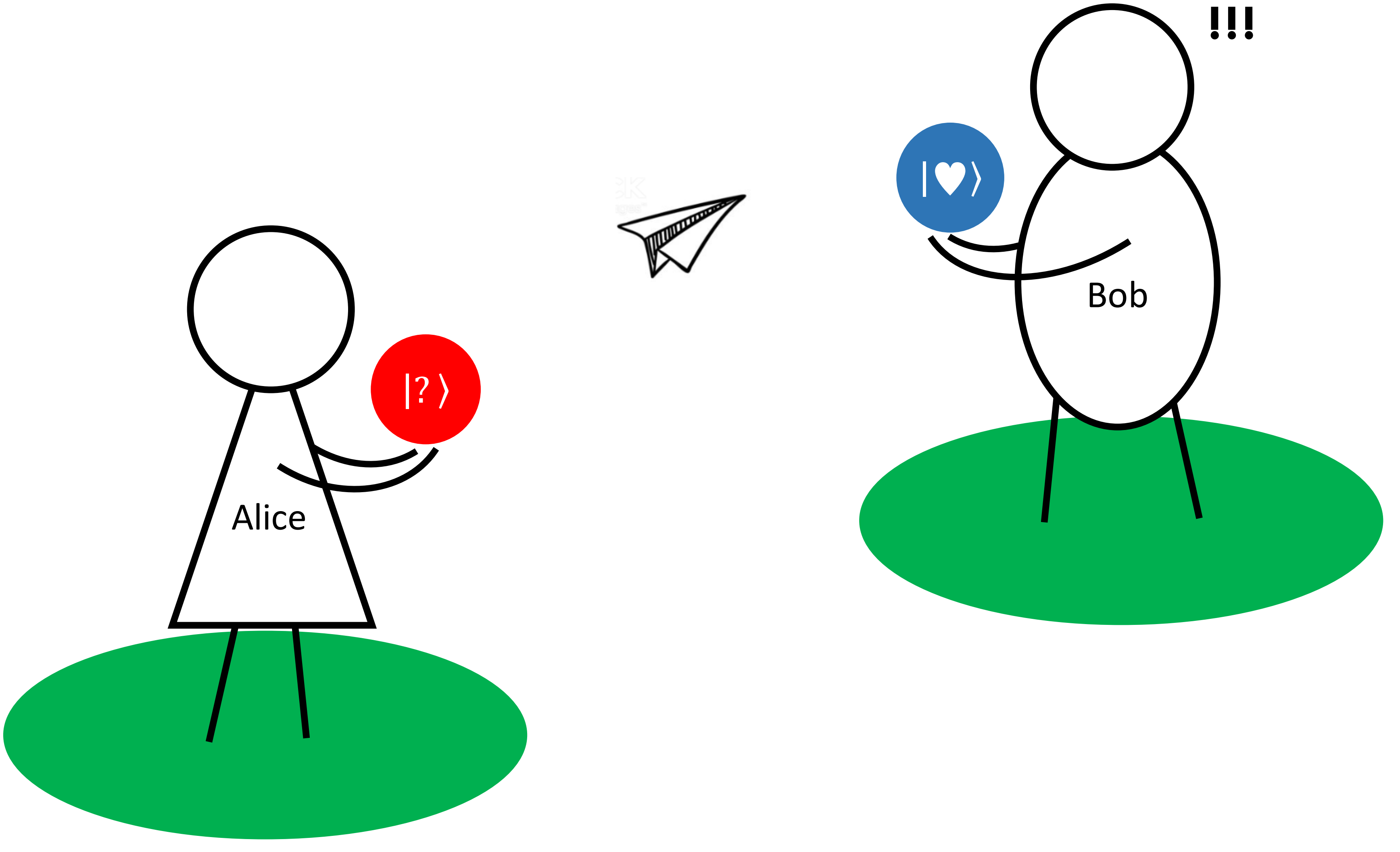
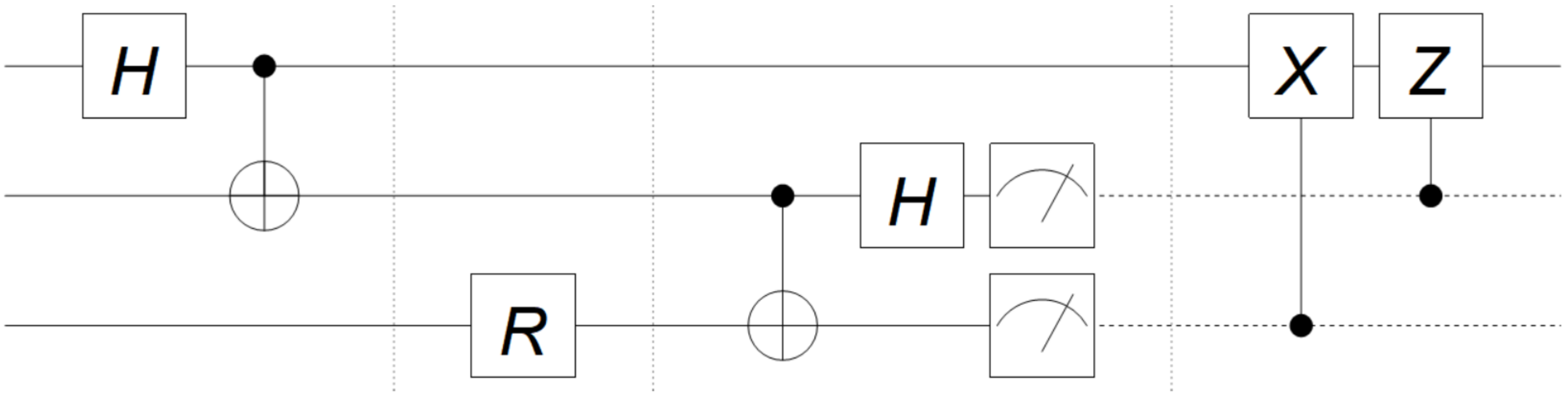
$t = 0$



$t = T$

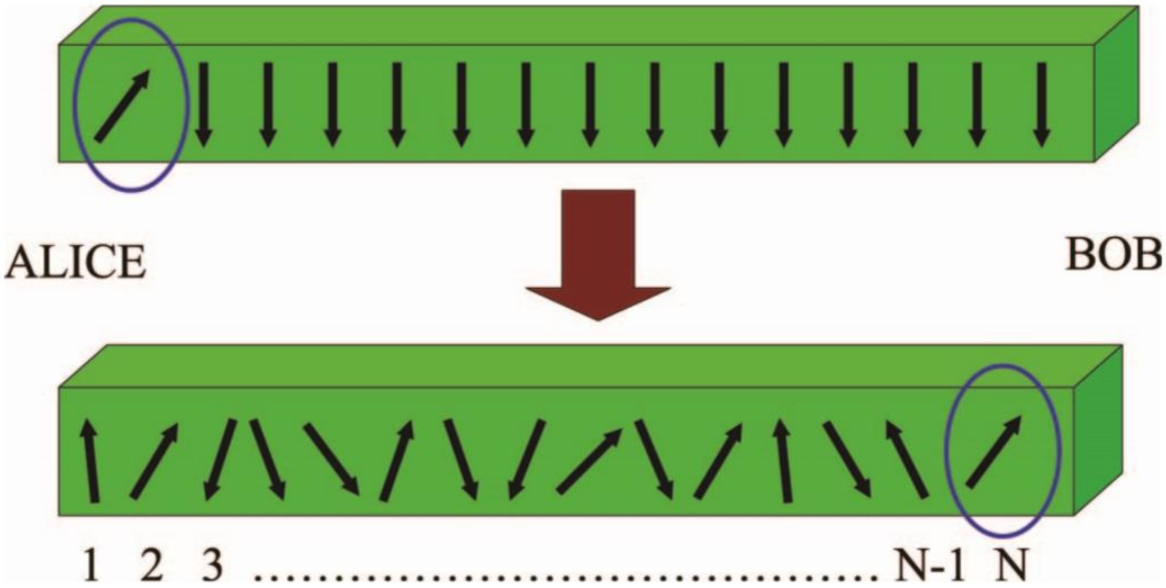


Quantum Communication - Teleportation



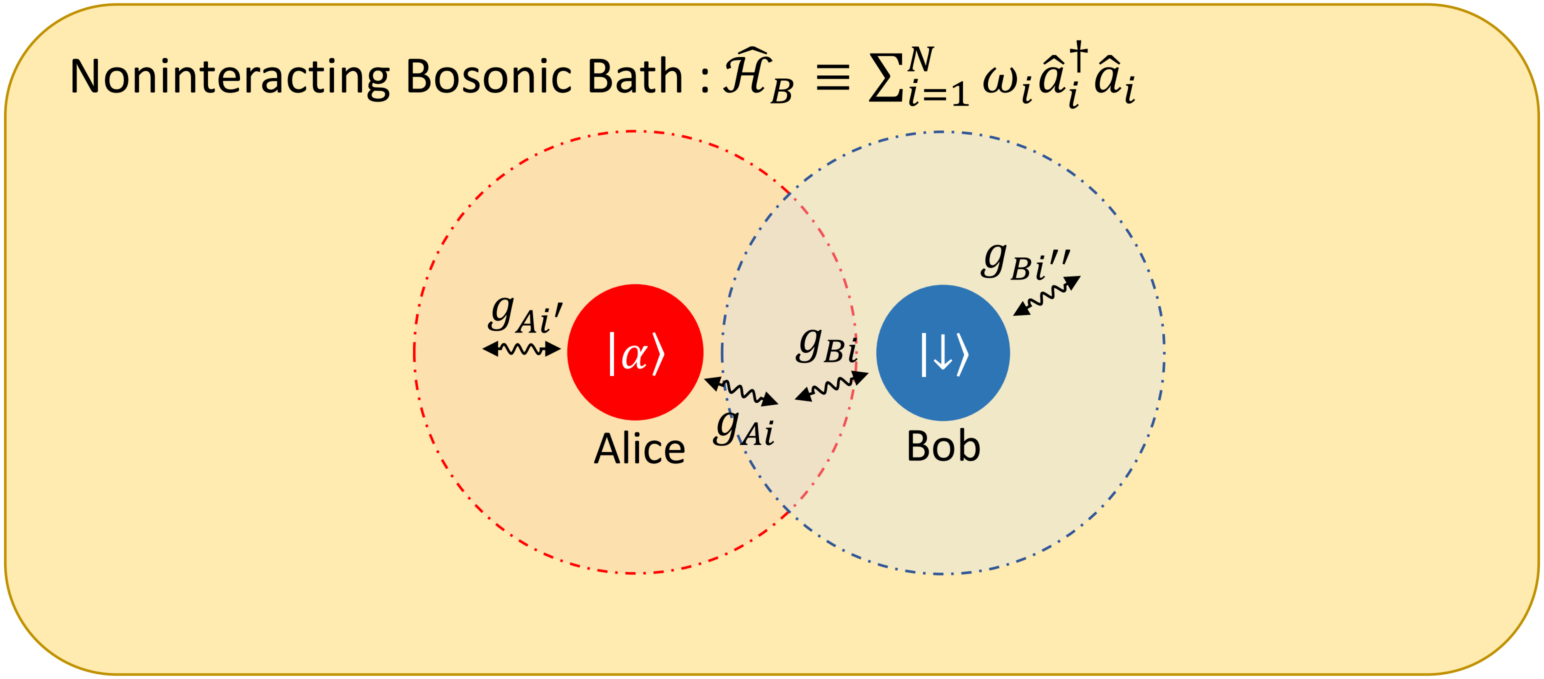
$t = 0$

$t = T$



Bose, S., Contemporary Physics, 48(1), 13-30

Model & Goal



- Strong JC interaction between qubits and nearby bosons
- No interaction between qubits and bosons

"State of Alice's qubit can be transferred to Bob's qubit through the bath"

Degenerated multimodes Jaynes Cummmings model is defined :

$$\hat{\mathcal{H}} \equiv \frac{\omega_q}{2}\hat{\sigma}_z + \omega\sum_{i=1}^{N}\hat{a}_i^\dagger\hat{a}_i + \sum_{i=1}^{N} g_i\left(\hat{a}_i^\dagger\sigma^- + \hat{a}_i\sigma^+\right)$$

$\hat{a}_i^\dagger$: real space boson creation operator
$\hat{\sigma}_i$: qubit's Pauli operator
ω : boson modes degenerate frequency
ω_q : two levels frequency difference
g_i : coupling strength

Transform hamiltonian as only a single mode interact with qubit,

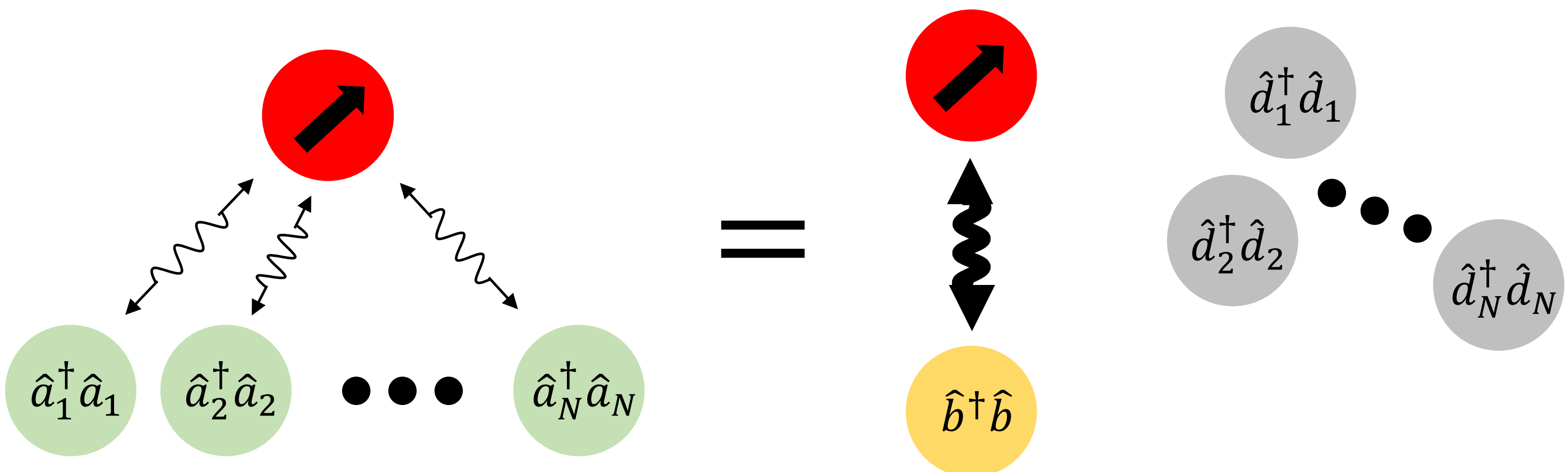
$$\hat{\mathcal{H}} = \frac{\omega_q}{2}\hat{\sigma}_z + \omega\hat{b}^\dagger\hat{b} + g_b(\hat{b}^\dagger\sigma^- + \hat{b}\sigma^+) + \omega\sum_{i=1}^{N-1}\hat{d}_i^\dagger\hat{d}_i$$

$\hat{b}^\dagger$: bright mode boson creation operator
$\hat{d}_i^\dagger$: i^{th} dark mode boson creation operator
$g_b = \sqrt{\sum_{i=1}^{N} g_i^2}$

Nuovo Cimento. D 19.5 (1997): 749-752

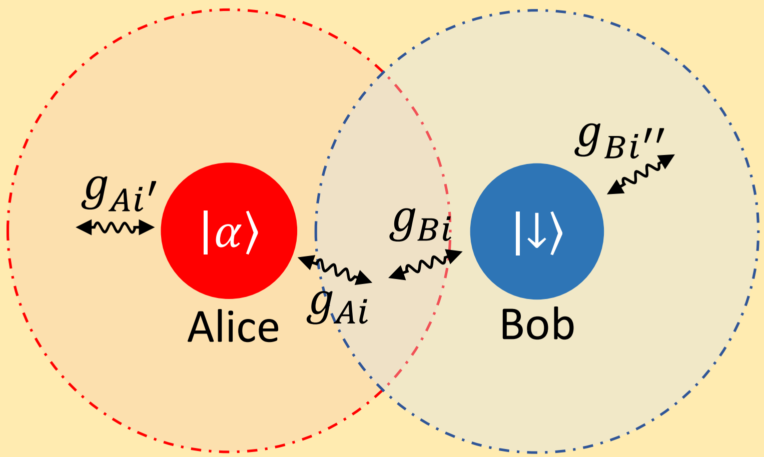
Basic Theory

This effective Hamiltonian shows only single bright mode interact with qubit.



Extend to Our Situation

Noninteracting Bosonic Bath : $\widehat{\mathcal{H}}_B \equiv \sum_{i=1}^{N} \omega_i \hat{a}_i^\dagger \hat{a}_i$



- Nondegenerated modes
- Two qubits

Bright Modes Approximation

The Hamiltonian we can have in real space is

$$\hat{\mathcal{H}} = \sum_{i,j=1}^{N} \hat{c}_i^{\dagger} h_{ij} \hat{c}_j + \sum_{i \in \mathcal{S}_A} g'_{Ai}\left(\hat{c}_i^{\dagger}\sigma_A^{-} + \hat{c}_i\sigma_A^{+}\right) + \sum_{i \in \mathcal{S}_B} g'_{Bi}\left(\hat{c}_i^{\dagger}\sigma_B^{-} + \hat{c}_i\sigma_B^{+}\right)$$

However it can always be transformed to eigenmodes of bosonic system,

$$\hat{\mathcal{H}} = \sum_{k=1}^{N} \omega_k \hat{a}_k^{\dagger} \hat{a}_k + \sum_{k=1}^{N} g_{Ak}\left(\hat{a}_k^{\dagger}\sigma_A^{-} + \hat{a}_k\sigma_A^{+}\right) + \sum_{k=1}^{N} g_{Bk}\left(\hat{a}_k^{\dagger}\sigma_B^{-} + \hat{a}_k\sigma_B^{+}\right)$$

$$g_{ak} \equiv \sum_{i \in \mathcal{S}_a} g'_{ai} U_{ik} \ \ (a = A, B) \ \ , \omega_k = \sum_{ij=1}^{N} U_{ik}^{*} h_{ij} U_{jk}$$

At regime g_{Ak} , $g_{Bk} \gg \delta\omega_k$, the hamiltonian can be approximate to nondegenerated multimodes JC model.

Bright Modes Approximation

$$\hat{\mathcal{H}} = \sum_{k=1}^{N} \omega_k \hat{a}_k^\dagger \hat{a}_k + \sum_{k=1}^{N} g_{Ak}\left(\hat{a}_k^\dagger \sigma_A^- + \hat{a}_k \sigma_A^+\right) + \sum_{k=1}^{N} g_{Bk}\left(\hat{a}_k^\dagger \sigma_B^- + \hat{a}_k \sigma_B^+\right)$$

But $g_{Ak} \neq g_{Bk}$ prevents to express with single bright mode. Instead we can express with two bright mode.

We define,

$$g_b = \sqrt{\sum_{k=1}^{N} g_{Ak}^2} = \sqrt{\sum_{k=1}^{N} g_{Bk}^2}$$

$$g_{P,k} = g_b \frac{g_{B,k} - \kappa g_{A,k}}{\sqrt{\sum_{j=1}^{N} \left|g_{B,j} - \kappa g_{A,j}\right|^2}} \qquad ; \kappa = \sum_{j=1}^{N} g_{A,j}^* g_{B,j} \,/ g_b^2$$

$$\Gamma = \frac{1}{g_b^2} \sum_{k=1}^{N} g_{A,k}^* g_{P,k}$$

Bright Modes Approximation

$$\hat{\mathcal{H}} \simeq \Omega_b^{(1)} \hat{b}_1^\dagger \hat{b}_1 + \Omega_b^{(2)} \hat{b}_2^\dagger \hat{b}_2$$ ⋯ Bright modes

$$+g_b\left(\hat{b}_1^\dagger \sigma_A^- + \hat{b}_1 \sigma_A^+\right)$$ ⋯ Interaction between Alice and first bright mode

$$+g_b\left(\Gamma \hat{b}_1^\dagger \sigma_B^- + \Gamma^* \hat{b}_1 \sigma_B^+\right)$$ ⋯ Interaction between Bob and first bright mode

$$+g_b\sqrt{1-|\Gamma|^2}\left(\hat{b}_2^\dagger \sigma_B^- + \hat{b}_2 \sigma_B^+\right)$$ ⋯ Interaction between Bob and second bright mode

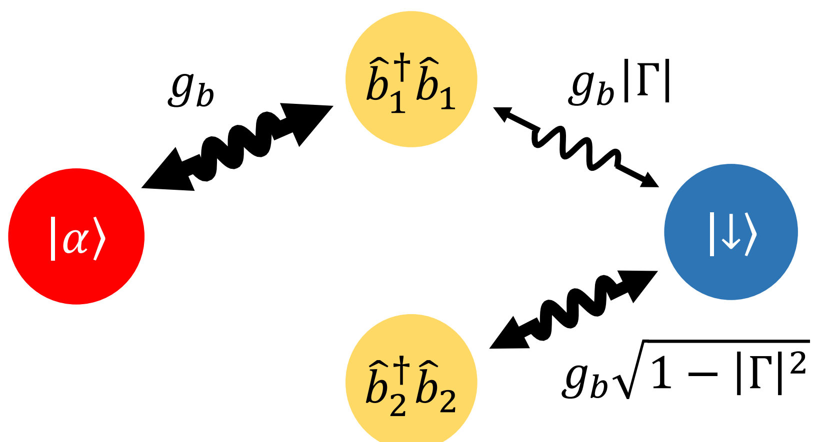
$$+\sum_{k=1}^{N-2} \Omega_d^{(k)} \hat{d}_k^\dagger \hat{d}_k$$ ⋯ Dark modes

$$\Omega_b^{(1)} = \frac{1}{g_b^2} \sum_{k=1}^{N} \omega_k \left|g_{A,k}\right|^2$$

$$\Omega_b^{(2)} = \frac{1}{g_b^2} \sum_{k=1}^{N} \omega_k \left|g_{P,k}\right|^2$$

$$g_b = \sqrt{\sum_{k=1}^{N} g_{Ak}^2} = \sqrt{\sum_{k=1}^{N} g_{Bk}^2}$$

$$\Gamma = \frac{1}{g_b^2} \sum_{k=1}^{N} g_{A,k}^* g_{P,k}$$; *Overlapped factor*



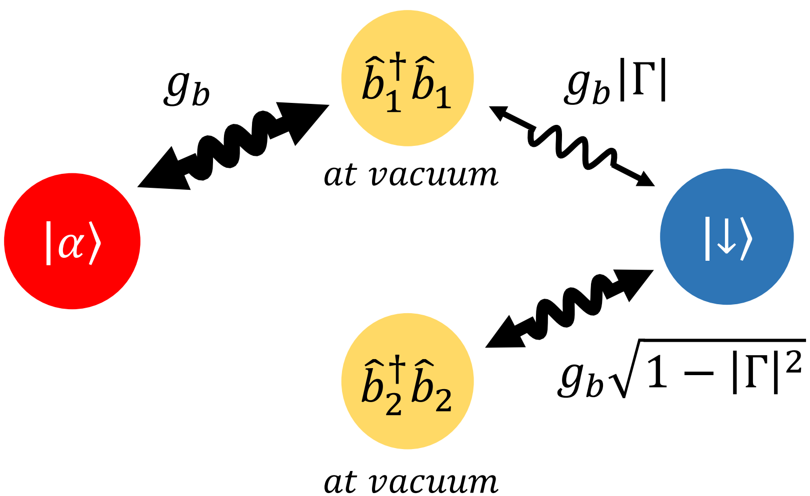
Bright Modes Approximation

The expectation number of bright modes is defined

$$\langle \hat{b}_1^\dagger \hat{b}_1 \rangle = \frac{1}{g_b^2} \sum_{k=1}^{N} |g_{A,k}|^2 \langle \hat{a}_k^\dagger \hat{a}_k \rangle \xrightarrow{T, \frac{1}{N} \to 0} 0$$

$$; \; g_b^2 = \sum_{k=1}^{N} g_{Ak}^2 = \sum_{k=1}^{N} g_{Bk}^2$$

$$\langle \hat{b}_2^\dagger \hat{b}_2 \rangle = \frac{1}{g_b^2} \sum_{k=1}^{N} |g_{p,k}|^2 \langle \hat{a}_k^\dagger \hat{a}_k \rangle \xrightarrow{T, \frac{1}{N} \to 0} 0$$



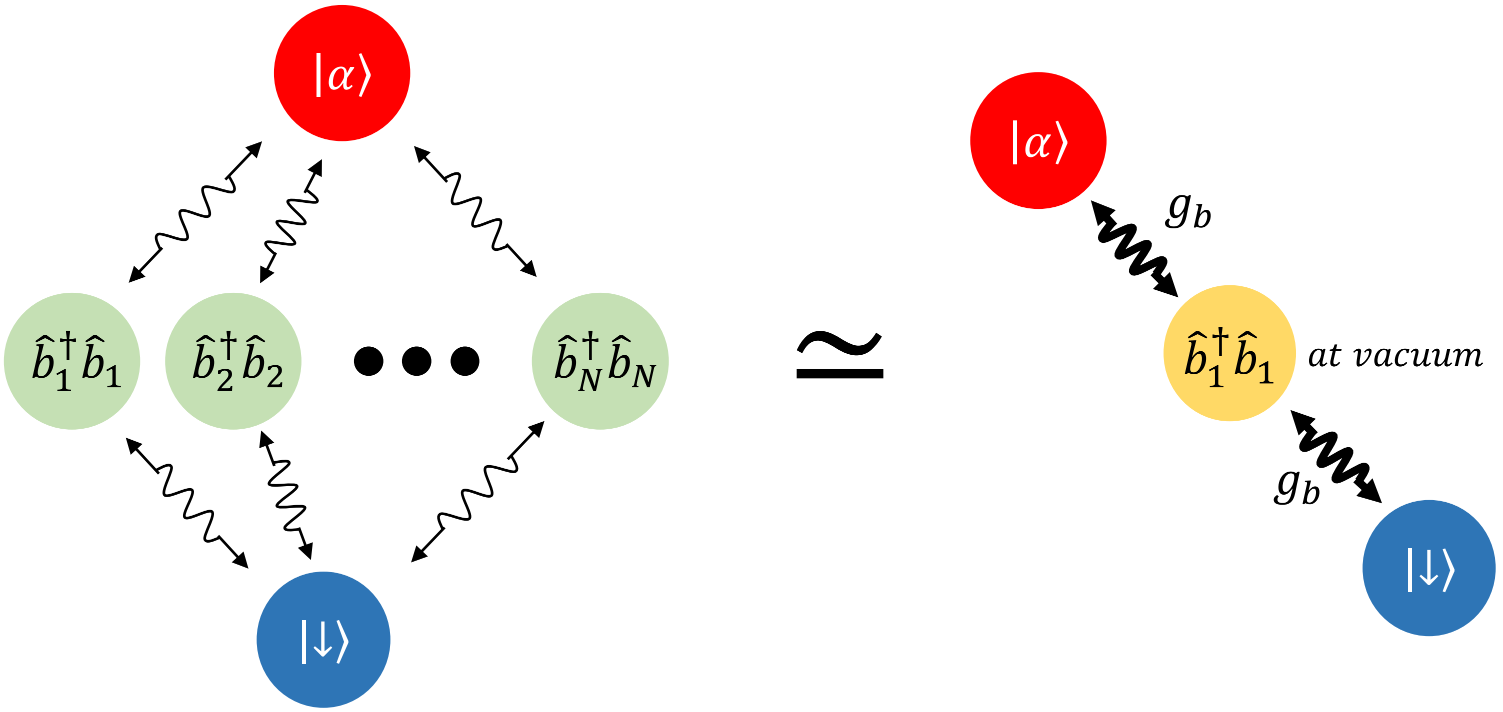
Only five states participate :
$\{|\downarrow, 0,0, \downarrow\rangle\}$ and
$\{|\uparrow, 0,0, \downarrow\rangle, |\downarrow, 1,0, \downarrow\rangle, |\downarrow, 0,1, \downarrow\rangle, |\downarrow, 0,0, \uparrow\rangle\}$

Extreme Regime : $\Gamma = 1$

As most of natural systems have ohmic spectral density, we choose a situation with

$$J(\omega) \sim \omega$$

$$\omega_i = \omega_0 i \, , g_i^{A(B)} = g_0 \sqrt{i}$$



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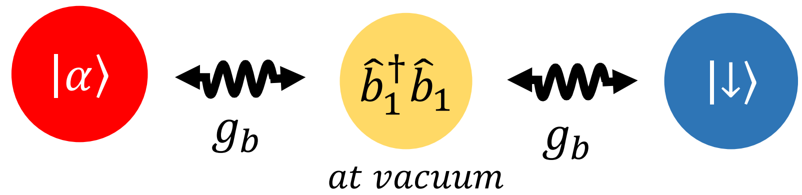

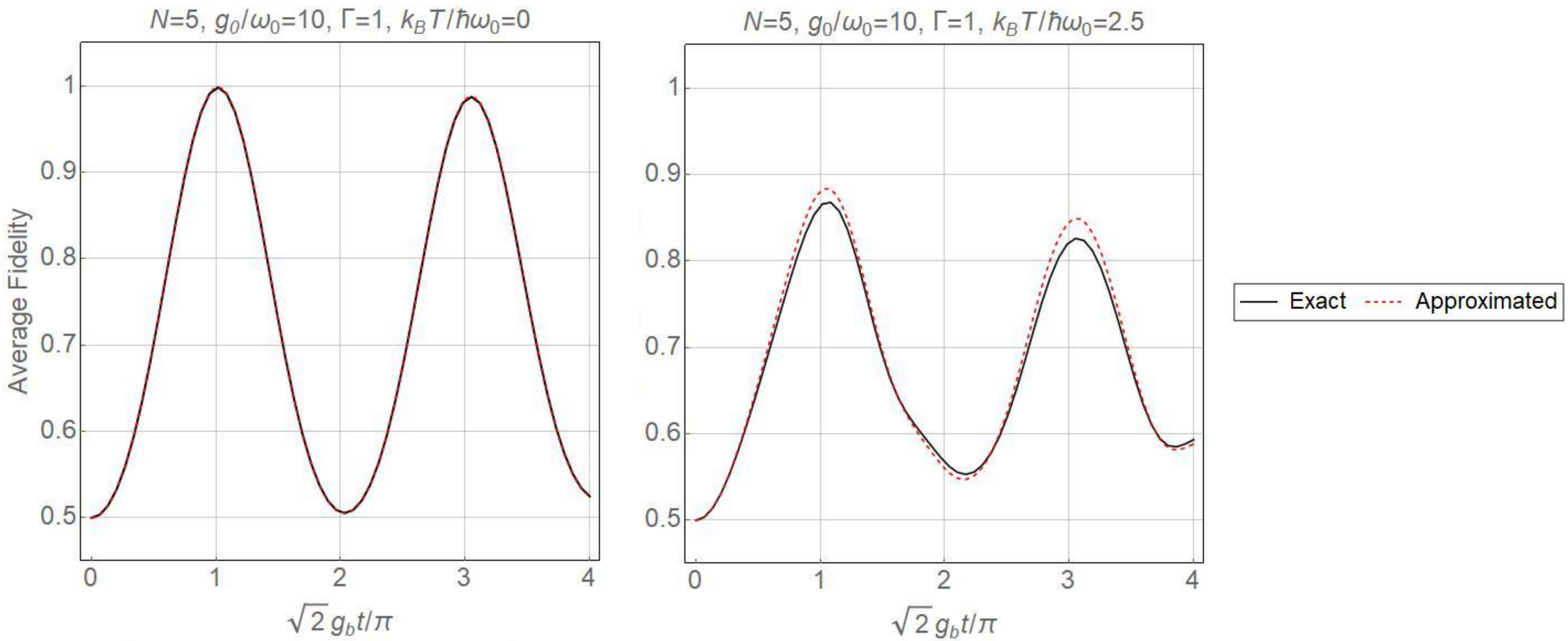

Well approximated with single bright mode interaction.

Extreme Regime : $\Gamma = 1$

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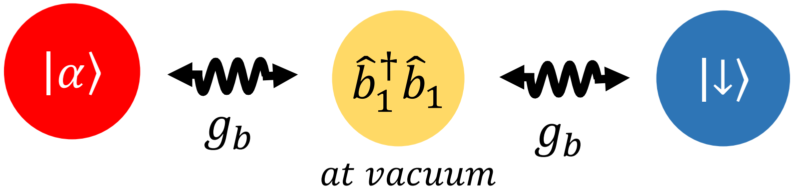
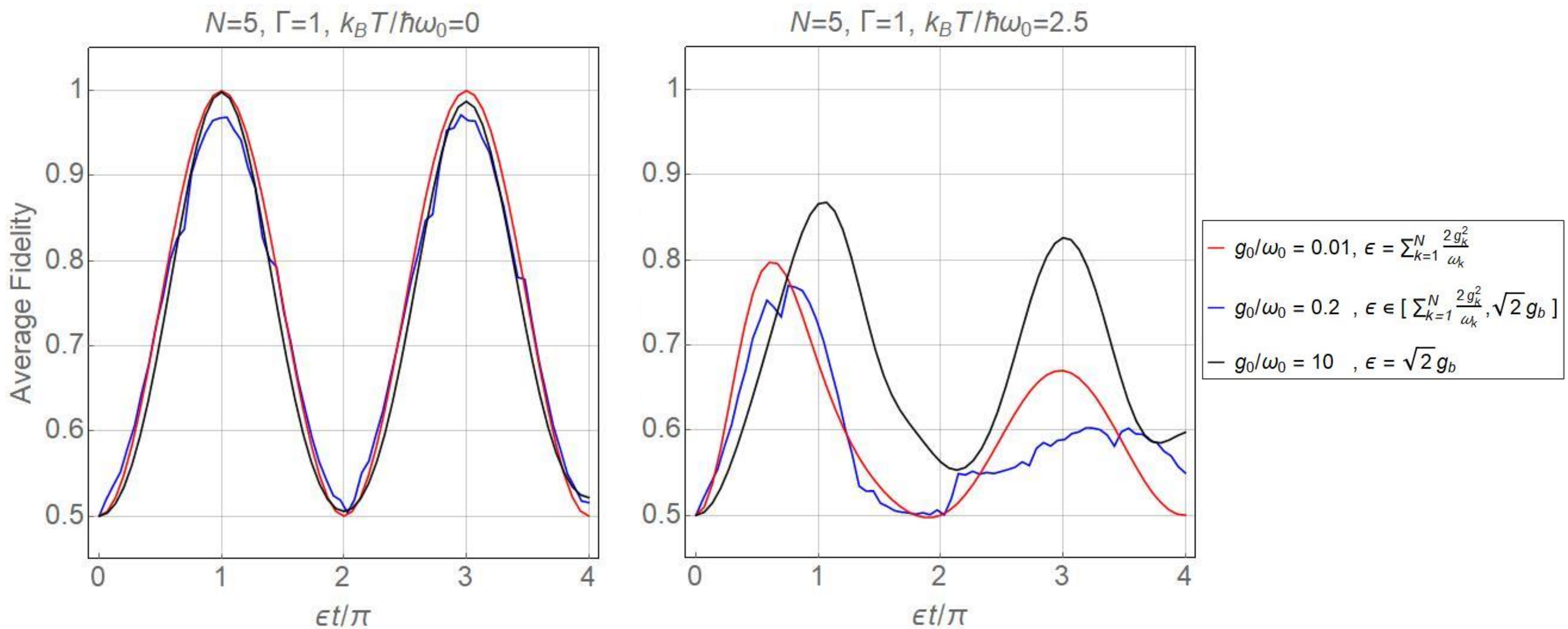
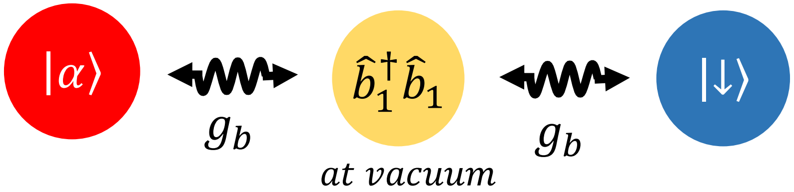


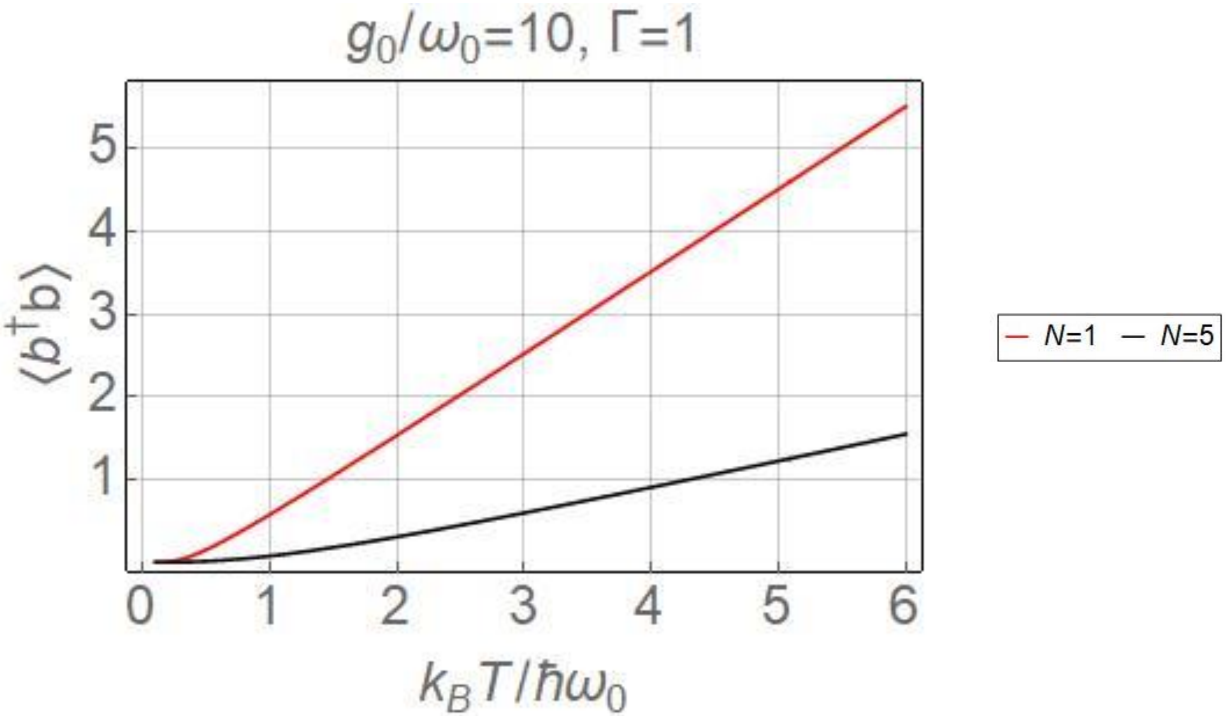


More protected to the temperature than other g_0/ω_0 values.

Extreme Regime : $\Gamma = 1$

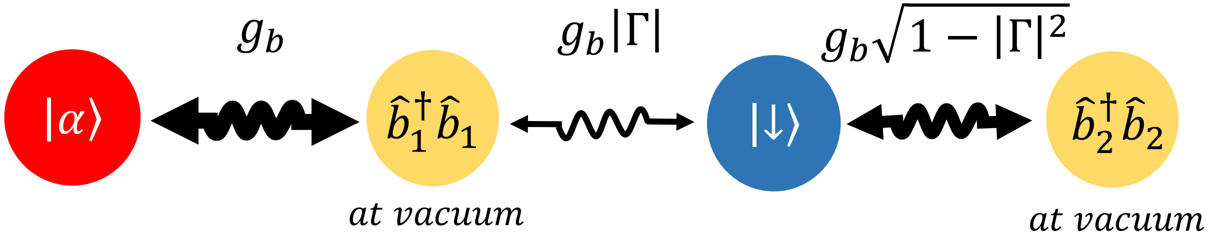
$$\omega_i = \omega_0 i \,, g_i^{A(B)} = g_0\sqrt{i}$$

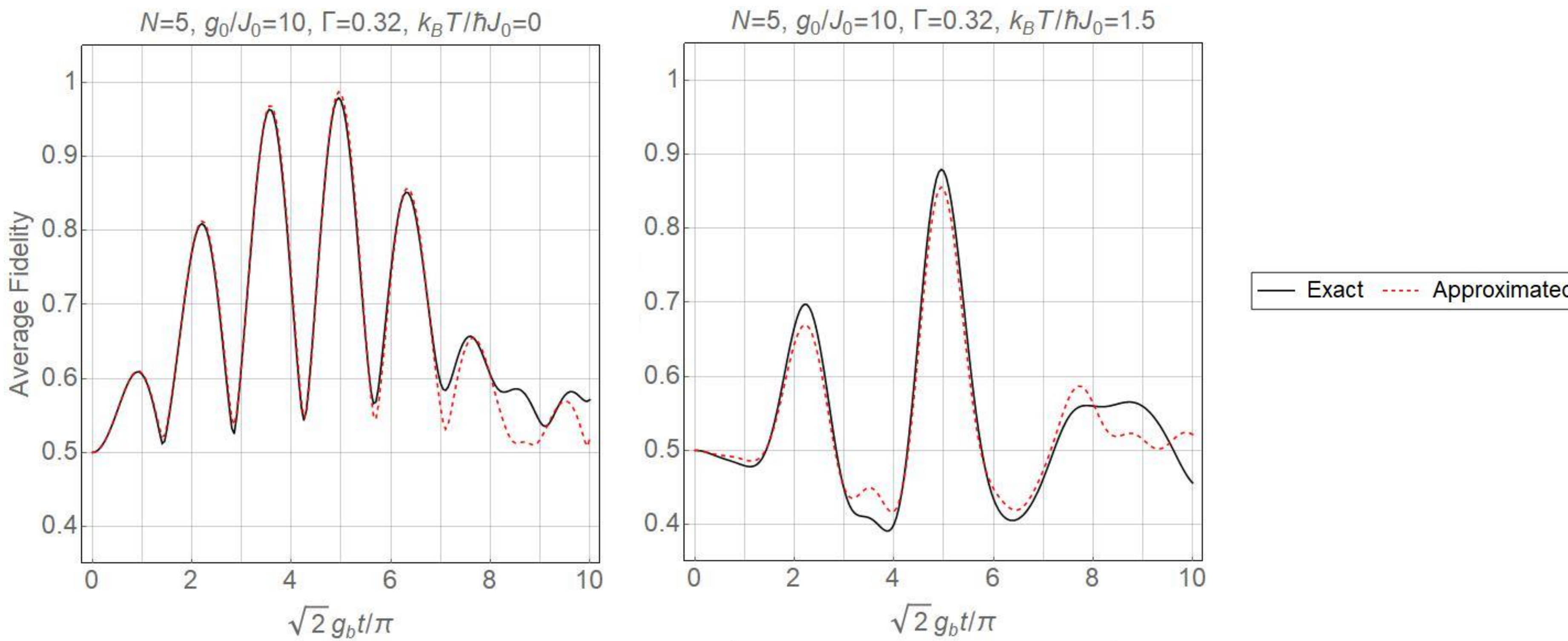




More interacting number of bath particles, more protected transfer.

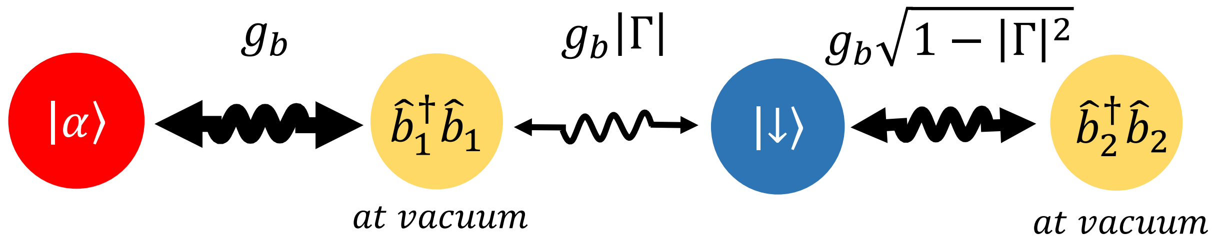
$\Gamma \neq 1$

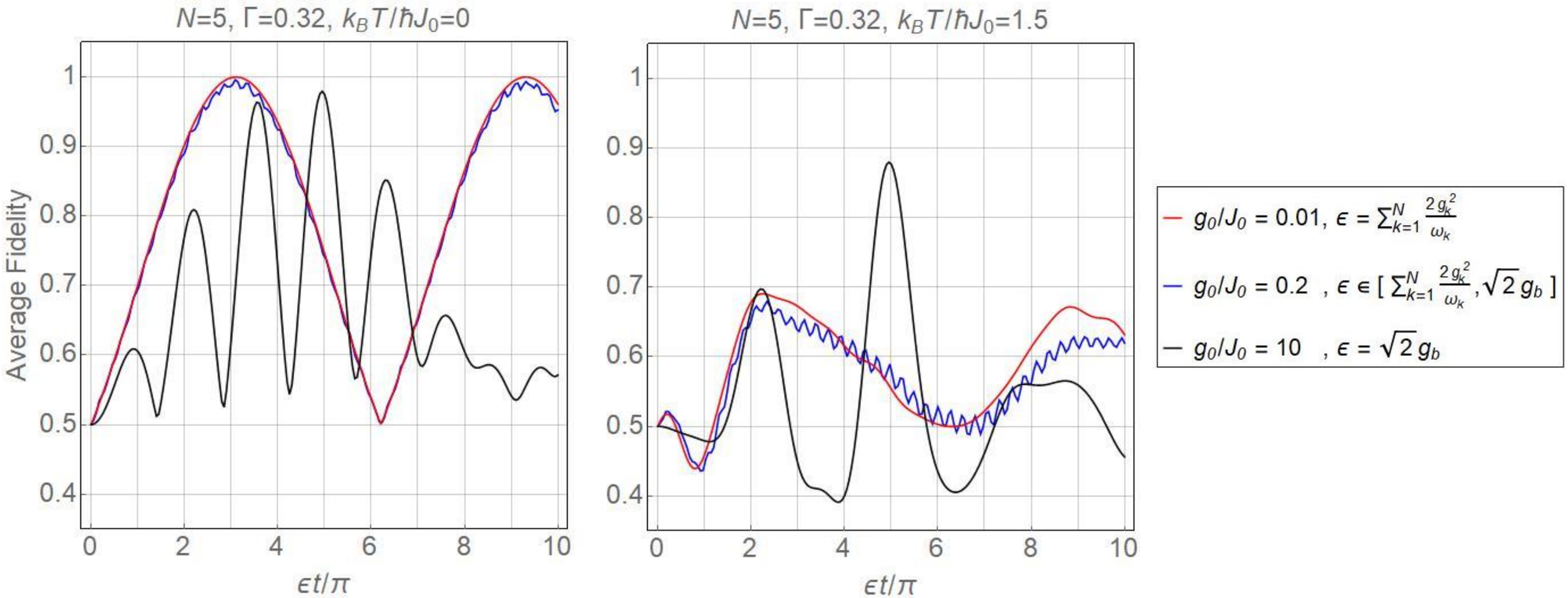




Well approximated with two bright mode interaction.

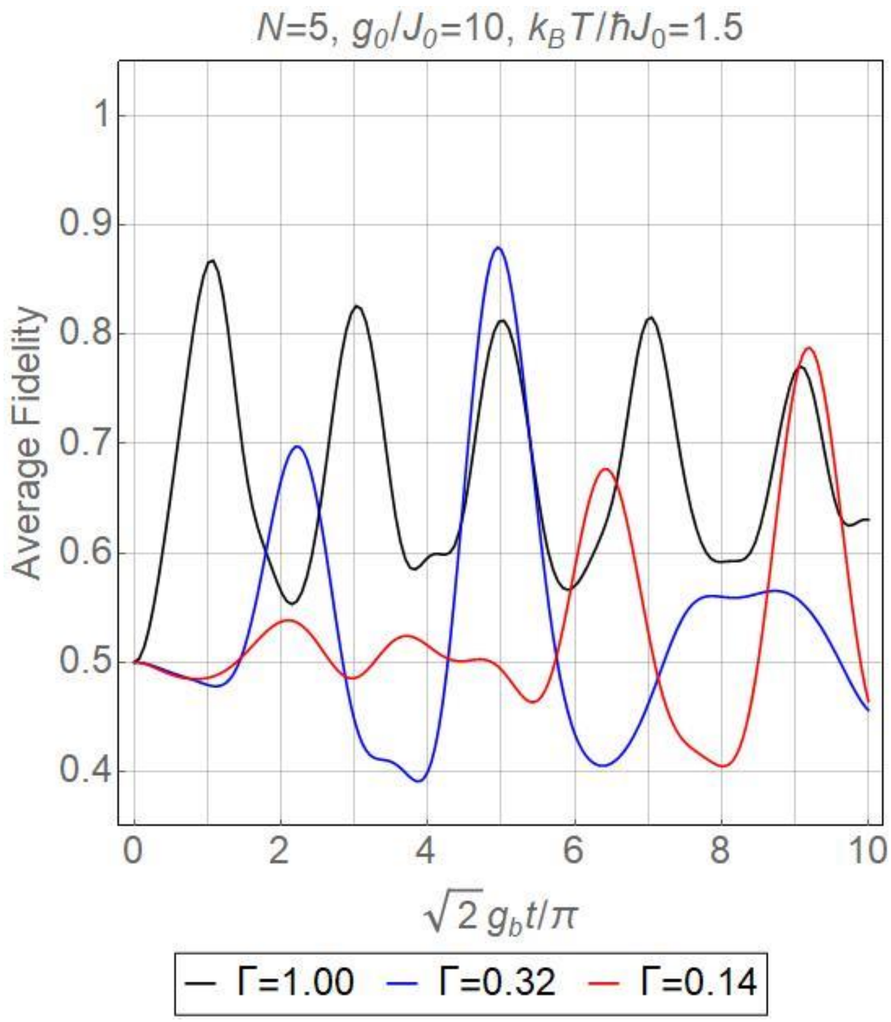
$\Gamma \neq 1$





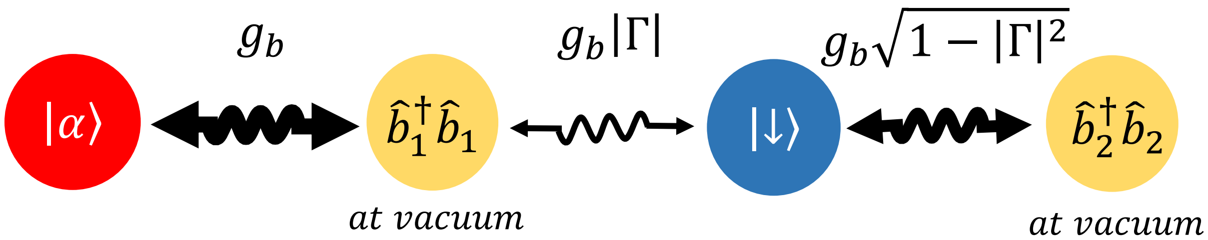
More protected to the temperature than other g_0/ω_0 values.

Γ Dependence



$\Gamma \gtrsim 0.2$ to get high fidelity on the similar time scale of $\Gamma = 0.2$.

Summary



- When the coupling between the eigenmodes and qubits are stronger than the difference of eigenmodes, the model can be approximated to two bright modes interacting with qubits.

- At this regime, state transfer is protected not only T=0 but also at higher temperature depending on system particles number *N*.

- When the overlapped factor Γ is more than 0.2, state transfers with high fidelity at $1/g_b$ time scale.