# **Spintronic Transport and Kondo Effect in Quantum Dots**

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Received 20 August 2004; accepted 29 September 2004

We investigate the spin-dependent transport properties of quantum-dot based structures where Kondo correlations dominate the electronic dynamics. The coupling to ferromagnetic leads with parallel magnetizations is known to give rise to nontrivial effects in the local density of states of a single quantum dot. We show that this influence strongly depends on whether charge fluctuations are present or absent in the dot. This result is confirmed with numerical renormalization group calculations and perturbation theory in the on-site interaction. In the Fermi-liquid fixed point, we determine the correlations of the electric current at zero temperature (shot noise) and demonstrate that the Fano factor is suppressed below the Poissonian limit for the symmetric point of the Anderson Hamiltonian even for nonzero lead magnetizations. We discuss possible avenues of future research in this field: coupling to the low energy excitations of the ferromagnets (magnons), extension to double quantum dot systems with interdot antiferromagnetic interaction and effect of spin-polarized currents on higher symmetry Kondo states such as SU(4).

**KEY WORDS:** Spintronics; Kondo effect; quantum dot; ferromagnet.

## **1. INTRODUCTION**

The study of spin-polarized transport across interfaces is a subject of long history [1,2]. The recent advent of semiconductor-based electronic devices at the nanoscale has revived the interest in transferring, controlling and detecting spin currents. This research area has been termed *spintronics* due to the exciting possibility of future, successful spin-based electronic technology [3], Nevertheless, spintronics is interesting as well for fundamental physics, both experimentally and theoretically, as its basic constituent—the spin—is of quantum nature only.

The most simple building block of spintronic transport systems is probably the magnetic tunnel junction. It comprises two ferromagnetic electrodes sandwiching a paramagnetic layer. Vertical transport, where current flows across the interfaces, is characterized by the tunneling magnetoresistance (TMR), which measures the relative change in the junction resistance when the contacts' magnetizations is changed from parallel to antiparallel alignment [4]. The TMR is also important in investigating the properties of spintronic resonant tunneling diodes. Remarkably, such devices have been recently built in all-semiconductor heterostructures, taking advantage of diluted magnetic semiconductors made of III–V [5–7] (hole-like transport) and II–VI compounds [8] (electron-like transport).

When the size of the paramagnetic island in a magnetic tunnel junction becomes comparable to the carrier Fermi wavelength, the system behaves effectively as zero-dimensional. Then, quantum effects arise from the quasi-localized nature of electrons and from the phase-coherent transport. The ultimate miniaturization limit is just a single resonant level coupled to a Fermi sea of itinerant electrons, which may be regarded as an artificial realization of the quantum *impurity problem* [9]. Extensive studies of the impurity problem have been performed in semiconductor *quantum dots* [10], where the island (an electron droplet) is formed by means of a constriction in a two-dimensional electron gas. Both

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the discrete energy levels and the tunneling couplings may be tuned almost at will. Very recently, a few experimental works have begun to deal with spin polarized leads [11,12].

In this paper, we consider spintronic transport through quantum dots with strong correlations. It is well known that the electron-electron interaction interaction plays a dominant role in the low temperature transport through quantum dots [10]. In the Coulomb blockade regime, the electron dynamics can be described in terms of single-electron tunneling plus mean-field charging effects. Between two Coulomb blockade peaks, transport is blockaded and the electron number in the dot does not fluctuate. When the electron number is odd, the topmost resonant level is singly occupied. However, when temperature approaches the energy scale  $T \sim T_{\rm K}$ , the spin of the localized electron becomes screened by an antiferro-magnetic interaction with the conduction band electrons. The resulting strong correlations arise from the interplay of higher order tunneling processes and the on-site Coulomb interaction. As a consequence, the many-body state in the T = 0limit is a singlet formed between the quantum impurity and the continuum electrons. The impact in the transport properties of the system is strong; e.g., the temperature-dependent conductance results in a universal function of  $T/T_{\rm K}$ , achieving the quantum limit  $(e^2/h \text{ per spin})$  at T = 0. This is the celebrated Kondo effect [13] in quantum dots [14–16], which was observed several years ago [17].

Now, an important condition for the Kondo effect to take place is the degeneracy (between spins up and down) of the ground state of the dot. Such spin degeneracy may be broken with an external magnetic field (Zeeman splitting) and is well understood [18,19]. But will the Kondo effect be preserved when the spin transfer across the tunnel barrier is spindependent? How will the conventional picture of the Kondo resonance in quantum dots be affected within a spin-polarized medium? To answer these questions various theoretical groups have lately contributed [20–31]. Although these works differ in some predictions due to the range of applicability of the distinct approaches used therein and their limitations, the analysis of the problem with numerical renormalization group [26,27] have led to the conclusion that the Kondo state is robust enough (though with a lower  $T_{\rm K}$ ) against nonzero spin polarizations in the leads when particle-hole symmetry is not broken and real charge fluctuations are completely suppressed [27]. Note that particle-hole asymmetry may be induced in the dot with nearby electric gates, shifting the resonant level away from the symmetric point (see below) [32]. We predict that this would give rise to a sharp decrease of the linear-response conductance. Since the phenomenon is absent when the magnetizations of the electrodes point along opposite directions (antiparallel alignment), we propose [27] the TMR as a possible experimental signature of this *spintronic Kondo effect*.

Another important element of many theories of spintronic transport is the description of intrinsic "spin relaxation" mechanisms that allow for nonequilibrium spin populations to relax. Long spin coherence times  $\tau_{sf}$  have been reported in semiconductor quantum wells [33] and dots [34], The effect of spin relaxation is known to reduce the TMR for a Coulomb-blockaded quantum dot [35] and it leads to a suppression of the Fano factor (shot noise) in the antiparallel configuration [36]. At lower temperatures  $(T < T_{\rm K})$ , spin decoherence causes the destruction of the Kondo effect due the failure of the formation of the many-body singlet state. One could also think about more coherent "spin-flip" process, e.g., arising from the potential spin-orbit coupling which causes the rotation of electron spin in the dot (this effect of spin-orbit coupling for the localized electron should be distinguished from that of the spin-orbit coupling of conduction electrons, which due to time-reversal symmetry has no influence on the Kondo effect [37]). More specifically, when the amplitude of spin flip scattering rate is larger than the Kondo temperature,  $h/2\tau_{\rm sf} \gtrsim T_{\rm K}$ , the density of states (DOS) at the impurity site is expected to develop a splitting and thus a decrease of the linear conductance. This prediction has been confirmed with equation-of-motion technique [21], slave-boson mean-field theory [23,30] and numerical renormalization group [27]. Therefore, spin-flip processes, coherent or incoherent, tend to suppress the Kondo effect.

# 2. HAMILTONIAN AND THEORETICAL APPROACHES

We model the quantum dot as a single discrete level with energy  $\varepsilon_{d,\sigma}$  containing an unpaired spin-1/2 electron with  $\sigma = \{\uparrow, \downarrow\}$  and charging energy *U*. Therefore, the dot is an electronic impurity tunnelling coupled to continuum electrons with a model Hamiltonian given by the Anderson Hamiltonian:

$$\mathcal{H} = \mathcal{H}_{\text{leads}} + \mathcal{H}_{\text{dot}} + \mathcal{H}_{\text{coupling}}, \qquad (1)$$



**Fig. 1.** (Color online). Schematic picture of the quantum dot attached to ferromagnetic leads. The dot is a single resonant level which may be shifted through a capacitative coupling to a gate. In (a) we show the parallel configuration. Dashed arrows denote the majority spins and solid arrows are for minority ones. Case (b) corresponds to the antiparallel alignment.

where (see Fig. 1)

$$\mathcal{H}_{\text{leads}} = \sum_{k\alpha\sigma} \varepsilon_{k\alpha\sigma} c^{\dagger}_{k\alpha\sigma} c_{k\alpha\sigma}, \qquad (2)$$

$$\mathcal{H}_{\rm dot} = \sum_{\sigma} \varepsilon_{\rm d,\sigma} \hat{n}_{\sigma} + U \hat{n}_{\uparrow} \hat{n}_{\downarrow} + (R d_{\uparrow}^{\dagger} d_{\downarrow} + \text{H.c.}), \quad (3)$$

$$\mathcal{H}_{\text{coupling}} = \sum_{k\alpha\sigma} (V_{k\alpha\sigma} c^{\dagger}_{k\alpha\sigma} d_{\sigma} + \text{H.c.}), \qquad (4)$$

are written in terms of the creation and annihilation operators in the dot  $d_{\sigma}^{\dagger}$ ,  $d_{\sigma}$  (the occupation number is defined as  $\hat{n}_{\sigma} = d_{\sigma}^{\dagger} d_{\sigma}$  and in the leads  $c_{k\alpha\sigma}^{\dagger}, c_{k\alpha\sigma}$ with k the wavevector and  $\alpha$  labeling left ( $\alpha = L$ ) and right ( $\alpha = R$ ) reservoirs. Tunneling of electrons from the dot to the leads is described by the hopping parameter  $V_{\alpha k\sigma}$ . In  $\mathcal{H}_{dot}$  included is an internal spin-flip process with rate  $\tau_{sf}^{-1} \sim 2R/\hbar$  [35,36]. Notice that in this framework the spin-flip process is purely coherent, and precisely speaking it does not account for incoherent spin relaxation processes. It may originate either from the transverse component of an applied magnetic field or from a tunable spin-orbit coupling of the Rashba-Dresselhaus type in the dot [3] (see the introduction and compare with Ref. [37]). Since the spin-flip processes, coherent or incoherent, have similar influence on Kondo effect, we leave the term in R in  $\mathcal{H}_{dot}$  phenomenological. What is important here is that R lifts the degeneracy of the discrete level and that it cannot be eliminated with a unitary transformation since the lead magnetizations already mark a privileged spin direction. For R = 0 and  $p \neq 0$ , the SU(2) symmetry is broken and the spin symmetry of the problem is U(1) whereas in the presence of both

spin flip scattering and ferromagnetic electrodes, the U(1) spin symmetry is explicitly broken.

Due to coupling to the leads, the electron in the dot becomes quasilocalized with a escaping rate related to the hybridization broadening,  $\Gamma_{\alpha\sigma}(\omega) = \pi \sum_k |V_{k\alpha\sigma}|^2 \delta(\omega - \varepsilon_{k\alpha\sigma})$ . This is the imaginary part of the hopping self-energy, which is spindependent because tunneling is spin-dependent. This can be achieved by coupling the dot to ferromagnetic leads. We take constant tunneling coefficients  $V_{\alpha}$ and equal tunnel barriers (symmetric couplings:  $V_L = V_R$ ). In the wide-band limit, the energy dependence of  $\Gamma_{\alpha\sigma}(\omega)$  is unimportant (which is a good approximation for low voltages). Moreover, we assume that the degree of spin polarization on lead  $\alpha$  is given by

$$p_{\alpha} = \frac{\Gamma_{\alpha\uparrow} - \Gamma_{\alpha\downarrow}}{\Gamma_{\alpha\uparrow} + \Gamma_{\alpha\downarrow}},\tag{5}$$

Notice that Eq. (5) is already a gross simplification as it might well be that  $p_{\alpha}$  has little to do with the real magnetization of the reservoir. In fact, various definitions for p are possible depending on the experiment [38]. In addition, we neglect proximity effects such as stray fields coming from the ferromagnets and consider that the bandwidth D is spin independent. [We prefer not to delve into details since already the simple form of Eq. (5) gives rise to nontrivial effects which can be directly measured].

We consider collinear magnetizations, both in parallel (P) and antiparallel (AP) configurations. With the approximations discussed above, we have for the P case  $(p_L = p_R \equiv p)\Gamma_{L\uparrow} = \Gamma_{R\uparrow} = (1 + p)\Gamma_0/2$  and  $\Gamma_{L\downarrow} = \Gamma_{R\downarrow} = (1 - p)\Gamma_0/2$ , where  $\Gamma_0 \equiv \Gamma_{\alpha\uparrow} + \Gamma_{\alpha\downarrow}$ , whereas the AP case  $(p_L = -p_R \equiv p)$ yields  $\Gamma_{L\uparrow} = \Gamma_{R\downarrow} = (1 + p)\Gamma_0/2$  and  $\Gamma_{L\downarrow} = \Gamma_{R\uparrow} = (1 - p)\Gamma_0/2$ .

Let us first provide an intuitive picture of the influence of spin-polarized transport in the Kondo resonance at R = 0. We take  $E_F = 0$ . For P alignment in the fully polarized case ( $p_L = p_R = 1$ ), the singlet state cannot form due to the lack of spin down electrons. Hence, we expect a decrease of the Kondo temperature ( $T_{\rm K}$  is roughly the binding energy of the singlet state) with increasing p. In the AP configuration ( $p_L = -p_R = 1$ ), however, the Kondo effect survives since an spin up (down) localized electron may be screened by the right (left) electrods. Of course, the conductance would be zero unless a vanishingly small R is allowed to come into play. Now, in the P case there may arise an exchange field [24] acting on the dot as an effective Zeeman splitting [39]. Is the Kondo effect robust against this exchange field? The answer is yes! [27]. When the gate voltage is tuned in such a way that  $\varepsilon_d = -U/2$  (the symmetric Anderson model), charge fluctuations become suppressed. Only when particle-hole symmetry is broken ( $\varepsilon_d \neq -U/2$ ) do we find a splitting in the Kondo peak of the local DOS.

We briefly review now the different theoretical methods employed to solve Eq. (1). The equationof-motion technique [40] is useful to study nonequilibrium situations (for finite bias) at relatively "high" temperatures ( $T \gtrsim T_{\rm K}$ ). Although it reproduces qualitatively the DOS peaks, it fails to describe properly the strong coupling regime, where Kondo physics completely quenches the impurity spin. When applied to our problem, it predicts the exchange field induced splitting but not its disappearance at  $\varepsilon_d =$ -U/2. On the other hand, slave-boson mean-field theory [41] correctly accounts for the Fermi-liquid fixed point of the Kondo problem at T = 0. As a result, it is only valid when the particle-hole symmetry is not broken (no splitting). The noncrossing approximation [42] is another slave-boson based approach and offers a consistent picture of the Kondo effect at  $T \sim T_{\rm K}$ . However, it does not take into account vertex corrections and produces spurious peaks at  $E_{\rm F}$  in the presence of a magnetic field. Finally, a numerical renormalization group calculation [43] encompasses the whole regime but remains valid only at equilibrium.

In the following, we report results using the interpolative U-finite perturbation theory [44] since it gives a good description of the dynamical properties of Eq. (1) for a wide range of parameters. It can describes both, the Kondo regime and the mixed-valence regime (where the dot level is close to  $E_{\rm F}$ ,  $-\Gamma_0 \leq \varepsilon_{\rm d} \leq 0$ ). However, in this approach the width of the Kondo resonance decreases algebraically instead of having an exponentional decay. Then, we elaborate as well on a numerical renormalization group analysis, which leads to *nonperturbative* results for all the regimes listed above.

### 3. DOS SPLITTING AND TMR

As indicated above, the Kondo resonance for a quantum dot coupled to two ferromagnets with parallel magnetizations, splits away from the symmetric case ( $\varepsilon_d \neq U/2$ ) where charge fluctuations are important. Figure 2 shows our results using the interpolative U-finite perturbation theory including magnetic leads. The DOS for the symmetric Anderson model



**Fig. 2.** (Color online). (a) Density of states for the symmetric Anderson model,  $\varepsilon_d = -U/2$  at T = 0 with  $U/\pi\Gamma_0 = 2.5$ . Solid line for p = 0, and dashed line for  $p = 0.6(\Gamma_{\uparrow}\rho_{\uparrow} = \Gamma_{\downarrow}\rho_{\downarrow})$ . (b) Density of states for the asymmetric Anderson model corresponding to the mixed valence regime,  $\varepsilon_d = -\Gamma_0$  at T = 0. The nonmagnetic case corresponds to the solid line. Spin up and down density of states for p = 0.6 are shown with the dashed lines.

is plotted in Fig. 2(a) for the unpolarized case p = 0and for nonzero polarization p = 0.6 in the P configuration. For unpolarized leads, the DOS shows the usual Kondo resonance reaching the *unitary limit* and two broad peaks at  $\pm U/2$  corresponding to the two mean-field (electron-like and hole-like) peaks. The main effect of the polarized reservoirs is to make the Kondo resonance narrower but keeping the same DOS height at  $E_{\rm F}$ ; i.e., for  $\varepsilon_{\rm d} = -U/2$  the lead magnetizations preserve the unitary limit.

The physical scenario changes dramatically when charge fluctuations are important as in the mixed valence regime. The solid line in Fig. 2(b) corresponds to  $\varepsilon_d = -\Gamma_0$  for p = 0. The DOS displays a strongly renormalized level by the charge fluctuations close to  $E_{\rm F}$  with no evidence of the meanfield peaks. For a finite spin polarization p = 0.6, Fig. 2(b) depicts both the spin up  $\rho_{\uparrow}(\omega)$  and the spin down  $\rho_{\perp}(\omega)$  contributions to the local DOS  $\rho(\omega) =$  $\rho_{\uparrow}(\omega) + \rho_{\downarrow}(\omega)$ . Here,  $\rho_{\downarrow(\uparrow)}(\omega)$ , moves toward positive (negative) frequencies. As a result,  $\rho(\omega)$  shows a splitting at low frequencies and the quantum occupations per spin change appreciably:  $\langle \hat{n}_{\uparrow}(p=0) \rangle <$  $\langle \hat{n}_{\uparrow}(p=0.6) \rangle$  and  $\langle \hat{n}_{\downarrow}(p=0) \rangle > \langle \hat{n}_{\downarrow}(p=0.6) \rangle$ . This demonstrates the sensitivity of the spintronic Kondo effect to variations of the external gate voltage.

We now use a numerical renormalization group calculation to investigate both the linear conductance and the splitting in the total DOS as a function of the gate voltage  $\varepsilon_d$  and the polarization of the leads *p*. Figure 3(a) shows the splitting  $\delta$  of the Kondo peak as a function of the gate voltage [27]. It increases linearly from zero as the gate moves away from the



**Fig. 3.** (a) Splitting  $\delta$  of the Kondo peak as a function of  $\varepsilon_d$  for p = 0.25 and U = 0.4D (*D* is the continuum bandwidth). (b)  $\delta$  versus *p* for  $\varepsilon_d = -0.1D$  and U = 0.45D. (c) TMR versus *p* for  $\varepsilon_d = -0.1D$  and U = 0.4D, (d) TMR as a function of  $\varepsilon_d$  for p = 0.25 and U = 0.4D. In all cases  $\Gamma_0 = 0.02D$ .

symmetric point  $\varepsilon_d = -U/2$ . In terms of the lead polarization p (the arrangement is parallel),  $\delta$  is linear as well [see Fig. 3(b)].

In Fig. 3(d) we plot the TMR defined as

$$TMR = \frac{G_{\rm P} - G_{\rm AP}}{G_{\rm AP}},\tag{6}$$

where  $G_P(G_{AP})$  is the linear conductance in the P (AP) case. For the symmetric Anderson model, the Kondo effect survives even for a finite value of polarization |p| < 1. Then, at  $\varepsilon_d = -U/2$  we find that  $TMR = p^2/(1-p^2)$ , in excellent agreement with the numerical result. Away from the symmetric point, i.e.,  $\varepsilon_d \neq -U/2$ ,  $g^P$  gets strongly suppressed as p increases. Then, the system exhibits a strong negative TMR [see Fig. 3(c)]. As a result, we predict a sharp peak of the TMR by varying the gate potential [see Fig. 3(d)]. The origin of this peak is exclusively due to the particularities of the spintronic Kondo effect.

We have so far discussed the case of a quantum dot symmetrically coupled to the leads, i.e.,  $\Gamma_L = \Gamma_R = \Gamma_0$ . As we have seen, the P configuration leads to nontrivial effects in the transport properties of the dot for  $\varepsilon_d \neq -U/2$  since  $\varepsilon_{d,\uparrow}$  and  $\varepsilon_{d,\downarrow}$  are not equally coupled to the leads whereas for the AP configuration both  $\varepsilon_{d,\uparrow}$  and  $\varepsilon_{d,\downarrow}$  are renormalized by the Kondo correlations in the same manner. This scenario is modified when we take an asymmetric quantum dot,  $\Gamma_L \neq \Gamma_R$ . In this case, both configurations (P and AP) give rise to a split DOS since  $\varepsilon_{d\uparrow}$  is coupled to the leads with  $\Gamma_{L\uparrow} + \Gamma_{R\uparrow}$  unlike  $\varepsilon_{d\downarrow}$  (with  $\Gamma_{L\downarrow} + \Gamma_{R\downarrow}$ ). In general, there will be splitting provided  $\Gamma_{L\uparrow} + \Gamma_{R\uparrow} \neq \Gamma_{L\downarrow} + \Gamma_{R\downarrow}$ .

#### 4. SHOT NOISE

The shot noise are the dynamical fluctuations of the current (current-current correlations) that appear in electric conductors due to the quantization of the charge. Research on shot noise in mesoscopic physics has developed into a fruitful area of research [45]. Nevertheless, there have hitherto been very few attempts to investigate shot noise in Kondo impurities [23,46–51].

Here, we consider the noise power (the Fourier transform of the time correlator of the electric current) at zero frequency:

$$S_{\alpha\beta}(\omega=0) = 2 \int d\tau \langle \{\delta \hat{I}_{\alpha}(\tau), \delta \hat{I}_{\beta}(0)\} \rangle$$
$$= 2 \int d\tau [\langle \{\hat{I}_{\alpha}(\tau), \hat{I}_{\beta}(0)\} \rangle - \langle \hat{I}_{\alpha} \rangle \langle \hat{I}_{\beta} \rangle], \quad (7)$$

where  $\delta \hat{I}_{\alpha} = \hat{I}_{\alpha} - I_{\alpha}$  describes the fluctuations of the current away from its average value  $I_{\alpha} = \langle \hat{I}_{\alpha} \rangle$ . We shall work at T = 0 so that the current will fluctuate due to quantum fluctuations only (we disregard thermal fluctuations).

Within slave-boson mean-field theory, the shot noise in a two-terminal geometry is shown [50] to have the well known expression  $S \sim \tilde{T}(1 - \tilde{T})$ , i.e., the conventional result for the partition noise but with *renormalized* transmissions  $\tilde{T}$ . This is valid as long as we restrict ourselves to the Fermi-liquid fixed point of the Kondo problem.

It is customary to define the Fano factor:

$$\gamma = \frac{S(0)}{2e\langle I \rangle}.$$
(8)

Since we are dealing with a two-terminal system, we have dropped the lead indices. Now, for a classical conductor with no correlations, the Fano factor equals 1 (Poissonian limit). Deviations of this limit are usually due to the application of Pauli principle or to the effect of strong electron-electron interactions as those giving rise to the Kondo effect. In Fig. 4(a), we plot the influence of spin flips in  $\gamma$ . The polarization in the leads is taken as  $p_L = p_R = 0.5$ . At low bias,  $\gamma$  behaves as  $1 - \tilde{T}(E_F)$  [45]. Since for R = 0 the Kondo resonance achieves the unitary limit at zero bias, the Fano factor is completely suppressed down to zero. As R increases, spin flips induce decoherence in the correlated motion of the electrons which leads to the singlet formation. Hence, the transmission at  $E_{\rm F}$  departs from its unitary limit and, as a consequence,  $\gamma$  increases at zero bias ( $V_{dc} = 0$ ). For larger



**Fig. 4.** Fano factor  $\gamma$  versus applied dc voltage  $V_{dc}$  of a quantum dot in the Kondo regime attached to ferromagnetic leads and in the presence of spin flip scattering with rate *R*. Voltage is units of the Kondo temperature  $k_B T_K^0 = D \exp(-\pi |\varepsilon_d|/2\Gamma)$ . We take  $\varepsilon_d = -6\Gamma$  and  $D = 100\Gamma$ . (a) Lead polarizations are  $p_L = p_R = 0.5$ . (b) R = 0 and  $p = p_L = p_R$  (parallel alignment).

voltages, notice that at R = 0 we recover the limit  $\gamma = 1/2$  of a double-barrier resonant system [45]. For R > 0 the behavior of  $\gamma$  at larger  $V_{dc}$  depends on the particular two-peak structure of the transmission [23].

In Fig. 4(b), we calculate the Fano factor for R = 0 and different values of the lead magnetizations (P configuration,  $p = p_L = p_R$ ). We find that  $\gamma$  increases with the polarization p at a given voltage bias (except at  $V_{dc} = 0$ ). This is caused by the suppression of the Kondo effect for  $V_{dc} > 0$ . For AP alignments (not shown here),  $\gamma$  increases less rapidly due to the independence of  $T_{\rm K}$  on the lead polarization.

Notice that in slave-boson mean-field theories the fluctuations of the boson field are neglected. However, we do not expect large deviations from the results reported here when  $T \ll T_{\rm K}$ . The boson fluctuations will evidently become important as temperature approaches  $T_{\rm K}$ .

### 5. POSSIBLE FUTURE ADVANCES

We have demonstrated that rich physics appears when the formation of the Kondo state in a quantum dot competes with the presence of spin-polarized tunneling currents and spinflip processes. We discuss now possible extensions of the theory that could dramatically alter the effects exposed above. We are confident that the field will still offer unexpected results and that, consequently, future calculations and experiments will be full of rewards.

#### 5.1. Magnon-Assisted Transport

We have thus far considered metallic freeelectron ferromagnets as the injecting and receiving contacts. In reality, transition-metal electrodes are described by exchange Hamiltonians. In these models, it is assumed that electrical conduction is carried by itinerant *s*-electrons while (insulator) magnetism is caused by a different group: localized *d*-electrons. Interaction between free electrons and localized moments gives rise to *electron-magnon* coupling [52].

It has been suggested that magnon-assisted tunneling in magnetic tunnel junctions may lower the TMR as a function of the bias voltage [53,54], giving rise to a zero-bias anomaly in nonlinear currentvoltage characteristics [55] (see, e.g., Ref. [56] for a more detailed review on the subject). The peak width is given by the energy involved in the spin excitations, which is of the order of the Curie temperature  $(T_C)$  of the metal. As  $T_C \gg T_K$ , one would naively expect that the Kondo effect will be always destroyed by emission and absorption of magnons via spin-flip processes. Within the tunneling Hamiltonian formalism, we replace  $\mathcal{H}_{\text{leads}}$  in Eq. (1) with

$$\mathcal{H}_{\text{leads}} = \sum_{k\alpha\sigma} \varepsilon_{k\alpha\sigma} c^{\dagger}_{k\alpha\sigma} c_{k\alpha\sigma} - J_{\text{dd}} \sum_{\langle i,j \rangle \alpha} \vec{S}_{\alpha i} \cdot \vec{S}_{\alpha j} - J_{\text{sd}} \sum_{i} \psi^{\dagger}_{\alpha\sigma} (\vec{S}_{\sigma\sigma'} \cdot \vec{S}_{\alpha i}) \psi_{\alpha\sigma'}, \qquad (9)$$

where the first term describes the conduction band electrons, the second term is the Heisenberg interaction between localized moments  $\vec{S}_{\alpha i}$ ,  $\vec{S}_{\alpha j}$  at neighboring sites *i* and *j* of lead  $\alpha$  and the third term is the interaction between a localized moment at site *i* and an itinerant electron with creation operator  $\psi_{\alpha\sigma}^{\dagger} = \sum_{\vec{k}} e^{-ik\alpha\cdot\vec{r}\alpha i} c_{k\alpha\sigma}^{\dagger}$ . After applying the Holstein-Primakoff transformation, the ferromagnet low-energy spin excitations can be written in terms of a bosonic collective bath (magnons), each carrying a magnetic moment  $e\hbar/mc$ . Thus, electron-magnon interaction  $\mathcal{H}_{em}$ at the interface induces spin mixing [53]:

$$\mathcal{H}_{\rm em} = -\tilde{J}_{\rm sd} \sum_{k,k',q} [c^{\dagger}_{k\alpha\uparrow} c_{k\alpha\downarrow} (a^{\dagger}_q + a_q) + \text{H.c.}], \quad (10)$$

where the  $\tilde{J}_{sd}$  is a renormalized coupling constant (which is taken as momentum independent for simplicity) and  $a_q^{\dagger} \sim N^{-1/2} \sum_{\alpha i} \exp(-i\vec{q} \cdot \vec{r}_{\alpha i}) S_i^{-}$  is the magnon creation operator.

In Eq. (10), we have written down only the spinflip part of the electron-magnon interaction. It will lead to nontrivial correlations when combined with the Hamiltonian  $\mathcal{H}_{dot}$  of a Coulomb-blockade quantum dot in Eq. (1). In fact,  $\mathcal{H}_{em}$  involves spin-flip inelastic transitions (the term proportional to *R* in Eq. (3) is simply elastic), inducing decoherence in the

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Kondo resonance by means of emission (dominant at low T) and absorption of magnons of energy  $\hbar\omega_q$ . (At low temperatures,  $\omega_q$ , depends approamately on q in a quadratic way,  $\omega_q \sim q^2$ ). Furthemore, subtle out of equilibrium effects such as interlayer exchange interaction [57] may arise as well.

We should mention that magnon excitations may also act as a *dissipative* bath. Since the standard spectral density for a three-dimensional (cubic) ferromagnet goes as  $\omega^{1/2}$  [54], we would deal with a *subohmic* bath. (The spectrum of the bath is cut off by a maximum magnon frequency due, e.g., to an anisotropy energy). For comparison, the DOS of magnons in antiferromagnetic two-dimensional systems varies as  $\omega$ , which amounts to an *ohmic* bath.

#### 5.2. Double Quantum Dots

Double-quantum-dot (DQD) systems have recently attracted much attention since that they form the simplest artificial systems showing molecule-like correlations at the nanoscale. As a consequence, a DQD has been proposed as a basic constituent of a solid-state quantum computer [3].

Two quantum dots can be coupled either in series or in parallel, allowing for tunneling and capacitive couplings in between. As far as Kondo physics is concerned, a DQD may be regarded as an artificial realization of the *two-impurity Kondo problem* [58–62]. It consists of two Kondo impurities with spin  $\vec{S}_1$  and  $\vec{S}_2$  interacting via an antiferromagnetic (AF) exchange coupling,  $J_{AF}\vec{S}_1 \cdot \vec{S}_2$ :

$$\mathcal{H}_{\rm AF} = J_{\rm AF} \sum_{\sigma\sigma'} d^{\dagger}_{1\sigma} d_{1\sigma'} d^{\dagger}_{2\sigma'} d_{2\sigma}, \qquad (11)$$

where  $d_{1\sigma}^{\dagger}(d_{2\sigma'}^{\dagger})$  creates an electron at dot 1 (2) with spin  $\sigma(\sigma')$ . It has been shown that the ratio  $J_{\rm JF}/T_{\rm K}$ determines the ground state of the system. In particular, when  $J_{\rm AF} \gg T_{\rm K}$  the two dots are locked into a antiferromagnetic singlet state whereas for  $J_{\rm AF} \ll$  $T_{\rm K}$  each dot forms its own Kondo state with continuum electrons in the leads. The critical value at which the transition from the Kondo state (KS) to the AF phase takes place can be obtained by comparing their ground state energies. Thus, the critical point depends on the Kondo temperature for each dot  $(T_{\rm K}^{\rm T}, T_{\rm K}^{\rm T})$  as follows

$$\left(\frac{J_{\rm AF}}{T_K^{\rm l}}\right)_c = \frac{4}{\pi} \left(1 + \frac{T_K^2}{T_K^{\rm l}}\right). \tag{12}$$



**Fig. 5.** (Color online). Schematic picture of the parallel double quantum dot attached to ferromagentic leads.  $J_{AF}$  is the interdot exchange coupling between the spin of the dots. (a) Parallel configuration of the leads polarization. Dashed arrows correspond to the majority spins and solid arrows are for minority ones. (b) Antiparallel alignment of the leads polarization.

For a symmetrically coupled DQD with a common gate  $\varepsilon_1 = \varepsilon_2$  one has  $(J_{AF}/T_K) = 8/\pi$ . In general for  $0 \le T_K^2 \le T_K^1$  we have  $4/\pi \le (I/T_K^1)_c \le 8/\pi$ . Since  $T_K^1$ , and  $T_K^2$  depend exponentially on the tunneling couplings and the level positions, a small asymmetry between these parameters induces a huge change in the ratio  $T_K^2/T_K^1$ .

Let us consider a parallel DQD (see Fig. 5) connected to two ferromagnetic leads. The Kondo temperature for the dot  $i \in \{1, 2\}$  depends on the configuration of the polarization of the leads (parallel or antiparallel). To simplify, we take the same polarization for the leads p > 0 (P alignment) and identical dots  $T_K^2(p) = T_K^1(p) = T_K(p)$ . Now the transition from KS  $\rightarrow$  AF singlet state is achieved more easily by increasing p. We keep the antiferromagnetic coupling fixed  $J_{AF} \ll T_{K}(p=0)$  and vary p. In this way,  $T_{\rm K}(p)$  becomes smaller leading to a weaker Kondo effect. Here, the width of the zero-bias anomaly (ZBA) decreases with p and the conductance reaches the unitary limit as in the case of a single dot [23,27]. By further increasing p, the AF coupling is much stronger than the Kondo scale leading to the transition Kondo  $\rightarrow$  AF when  $J_{AF}/T_{K}(p) > 8/\pi$ .

#### 5.3. Higher Symmetry Kondo States

In DQD systems with a strong interdot Coulomb interaction, the total charge allowed in both dots at the same time is just one electron. As a consequence, there are four ground states with the same energy, namely,  $\{1 \uparrow, 1 \downarrow, 2 \uparrow, 2 \downarrow\}$ . Quantum fluctuations between these states due to coupling to the leads yield, in the low temperature limit, a highly correlated state with SU(4) symmetry [63]. Note that these fluctuations do not involve only spin flips in the DQD (spin Kondo effect) but also flips in the *orbital* sector [64]. To describe these new processes, we define the *pseudospin* as a fictitious spin that points along +(-)z when the electron lies at the dot 1(2). Then, it is shown that the spin Kondo state becomes intermingled with a pseudospin Kondo state, giving rise to a complete entanglement between the spin and charge degrees of freedom.

The spin-pseudospin entanglement develops from the correlated tunneling that involves a flip of the spin *and* the pseudospin of the DQD system at the same time. Technically, it arises from the Schrieffer-Wolff transformation [13] which maps the DQD Hamiltonian into an effective exchange coupling between the localized spin and pseudospin and the conduction electrons. In the DQD Hamiltonian, one replaces  $\mathcal{H}_{dot}$  in Eq. (1) with the Hamiltonian of two dots plus a charging energy  $U_{12}$  between them:

$$\mathcal{H}_{\text{inter}} = \sum_{i=1,2} U_{12} \hat{n}_1 \hat{n}_2.$$
(13)

 $\hat{n}_i$  denotes the occupation number on dot *i*. We denote with  $\vec{T}$  the pseudospin operator in the DQD system. The resulting Kondo Hamiltonian formally reads:

$$\mathcal{H}_{K}^{\mathrm{SU}(4)} = J^{\mathrm{SU}(4)} \vec{S} \cdot (\psi^{\dagger} \vec{\sigma} \vec{r} \psi) \cdot \vec{T}, \qquad (14)$$

where  $\psi^{\dagger} = \sum_{k} \psi_{k}$  is the field operator with  $.\psi_{k} = [c_{e,k,\uparrow}^{\dagger}, c_{o,k,\downarrow}^{\dagger}, c_{e,k,\downarrow}^{\dagger}, c_{o,k,\downarrow}^{\dagger}]$  a spinor in the representation of even and odd channels of the lead operators [65]. In Eq. (14)  $J^{\text{SU}(4)}$  is a coupling constant which goes to the strongly fixed point in the flow diagram. We recall that the SU(4) Kondo state takes place provided there are two conduction channels (described by the matrix  $\vec{\tau}$ ). The latter equation explains the entanglement between the spin and the orbital electronic degrees of freedom.

The transport properties of a SU(4) Kondo state strongly differ from the conventional SU(2) Kondo state, both at equilibrium and out of equilibrium [65]. First, the Kondo temperature inferred from Eq. (14) is largely enhanced (around 200 times) as compared to  $T_K$  of a spin Kondo system [66]. This means that the differential conductance peak (which mimics the DQD density of states) becomes greatly broadened in a transition from the SU(2) to the SU(4) Kondo physics. Such a transition can be tuned with a magnetic flux in an Aharonov-Bohm interferometer with one dot at each arm [65]. Second, the Kondo resonance is no longer peaked at  $E_{\rm F}$  but at  $\sim E_{\rm F} + T_{\rm K}$  to fulfill the Friedel-Langreth sum rule [13].

How would spintronic transport modify a SU(4) Kondo resonance? In the presence of ferromagnetic leads and away from the particle-hole symmetry point, we expect the spin part of the Kondo state to slowly vanish with increasing lead polarization. Experimentally, one would see a splitting of the Kondo resonance into three peaks. The centered peak would still correspond to the pseudospin Kondo state, which is not sensitive to the magnetization at the leads. At the same time,  $T_K^{SU(4)}$  decreases but the linear conductance would increase since the SU(2)Kondo resonance associated to the orbital Kondo effect peaks at  $E_{\rm F}$  again. To further destroy the pseudospin Kondo state, two possibilities emerge from an analogy with the spin case. First, one allows for tunneling between the dots, which breaks the fourfold degeneracy favoring the formation of a bonding (symmetric) state between the dots. Then, interdot tunneling acts as an external Zeeman splitting in the spin sector. Second, one could consider asymmetric couplings of the DQD to the leads; e.g., the DQD system may be coupled strongly to the left lead,  $\Gamma_L > \Gamma_R$ . This way, we regard the leads as pseudospin polarized much like the spin-dependent tunneling due to ferromagnetic leads in the spin case. We may even define the pseudospin polarization for each spin species as

$$p_{\sigma} = \frac{\Gamma_{\sigma L} - \Gamma_{\sigma R}}{\Gamma_{\sigma L} + \Gamma_{\sigma R}},\tag{15}$$

[cf. Eq. (5)]. At this point, more calculations are needed to further exploit this analogy between spins and pseudospins, which may give rise to a unifying picture of the influence of real and pseudo-spin polarized leads in the transport through quantum-dot structures.

# 6. CONCLUSIONS AND EXPERIMENTAL RELEVANCE

We have investigated the spintronic properties of a quantum dot in the Kondo regime. We have considered a dot attached to ferromagnetic leads and in the presence of intradot spin flip scattering. Using both perturbation theory in the on-site

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interaction and the NRG method, we have shown that the Kondo effect is not necessarily suppressed by the spin polarizations of the leads: for the symmetric Anderson model, where charge fluctuations are completely suppressed, the Kondo effect is robust even for finite polarizations. For the asymmetric Anderson model, the Kondo peak does split into two. This is due to the presence or absence of particle-hole symmetry. In the presence of particle-hole symmetry, the Kondo peak at the Fermi level remains unsplit even at finite polarizations and the linear conductance achieves the unitary limit. This remains true as long as only spin fluctuations are present in the QD. On the contrary, when particle-hole symmetry is absent, the conductance is suppressed due to the visible splitting of the Kondo peak. Since the Kondo resonance is mostly unaltered for antiparallel magnetizations, we have calculated the TMR in the Kondo, mixed-valence, and empty-level regimes. The TMR shows a characteristic behavior for each of them. In addition, we have shown that the TMR is strongly affected in the presence of spin flip processes.

We have studied the form of the shot noise when charge fluctuations are completely suppressed. We have shown that the Fano factor approaches the Poissonian limit when the spin flip scattering rate is of the order of the Kondo temperature. For parallel arrangements, the Fano factor enhances with increasing lead polarizations.

Moreover, we have suggested and discussed possible new advances in this field such as the influence of magnons in the Kondo state of a quantum dot, the effect of spin-polarized currents on doublequantum-dot systems mimicking the two-impurity problem and on more exotic Kondo states with higher symmetry.

The physics addressed in this paper is realistic and can be visible within the scope of present techniques as the energies we treat are within the Kondo scale. In particular, a change has been detected in the resistivity of a Kondo alloy due to spin-polarized currents [67]. Furthermore, it is already possible to attach ferromagnetic leads to a carbon nanotube [68], and a carbon-nanotube quantum dot has been shown to display Kondo physics below an unusually high temperature [69]. Finally, a quantum dot coupled to ferromagnetic electrodes has been proposed as a promising candidate for spin injection devices, and studied experimentally both in the Coulomb blockade regime [70] and in the Kondo regime [71].

#### ACKNOWLEDGMENTS

We thank R. Aguado, M. Büttiker, T. Costi, L. Glazman, K. Kang, K. Le Hur, G. Platero, P. Simon, and E. V. Sukhorukov for valuable comments and discussions on this subject. D. S. and R. L. were supported by the the EU RTN under Contract No. HPRN-CT-2000-00144, Nanoscale Dynamics, and by the Spanish MECD. M.-S.C. acknowledges supports from the SKORE-A program, and from the eSSC at POSTECH.

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