Brief Introduction to Topological Insulators

- Seungju Han

Hall Effect

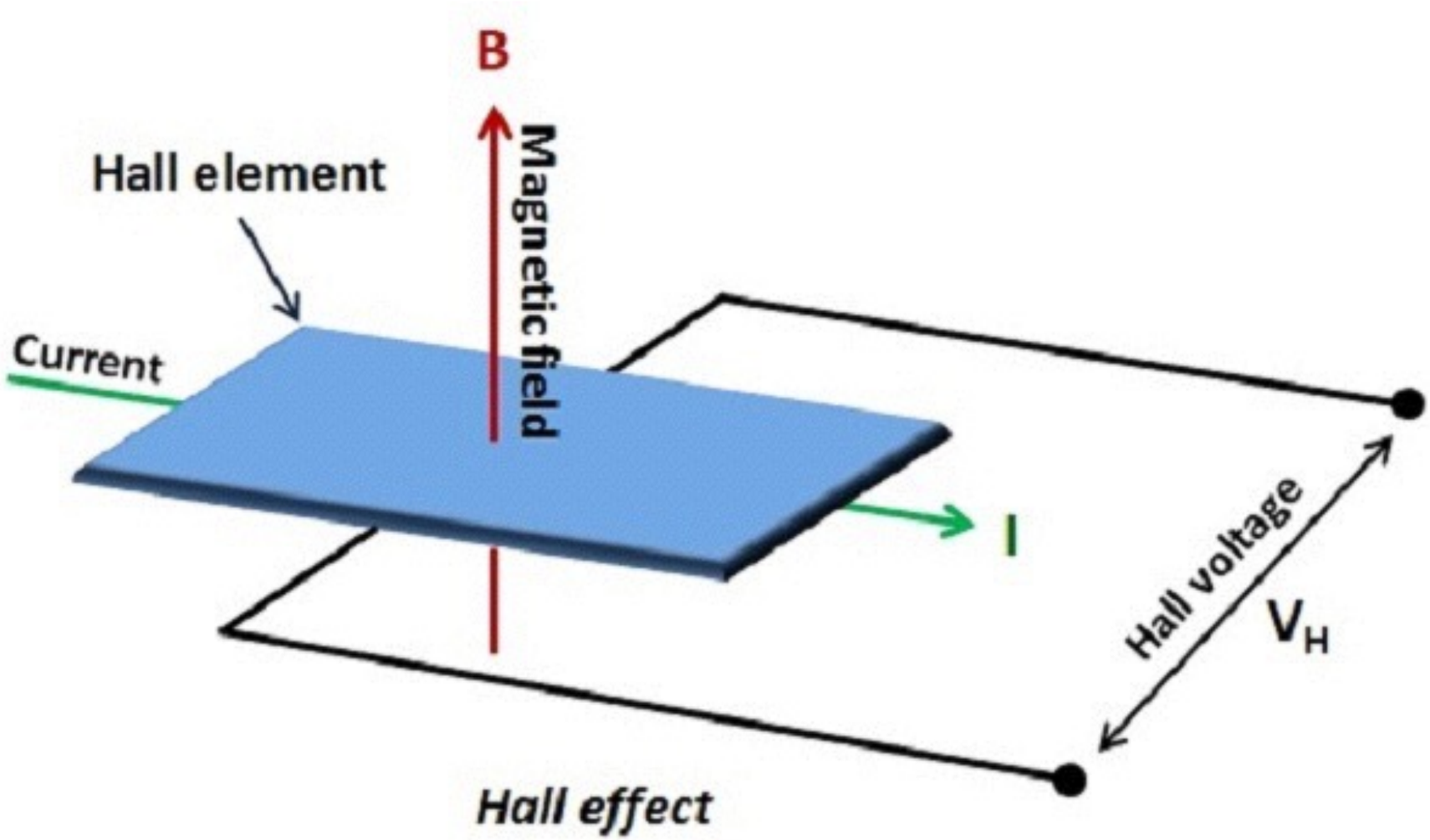
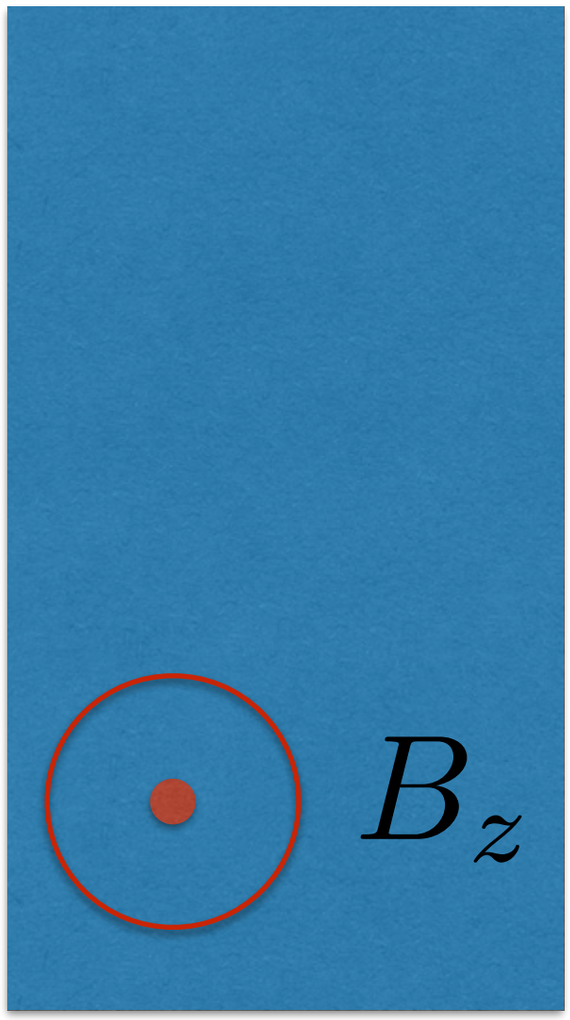
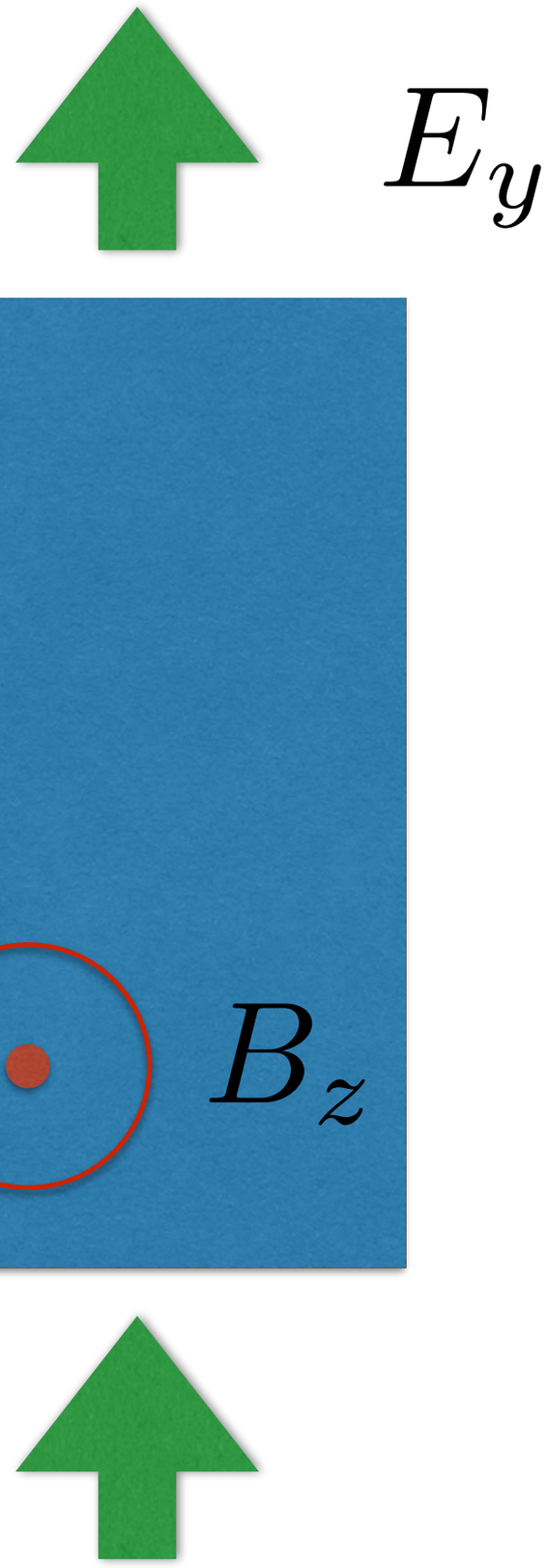


Image :: http://dangerousprototypes.com/blog/2012/01/27/interfacing-a-pic-microcontroller-with-the-acs712-hall-effect-current-sensor/

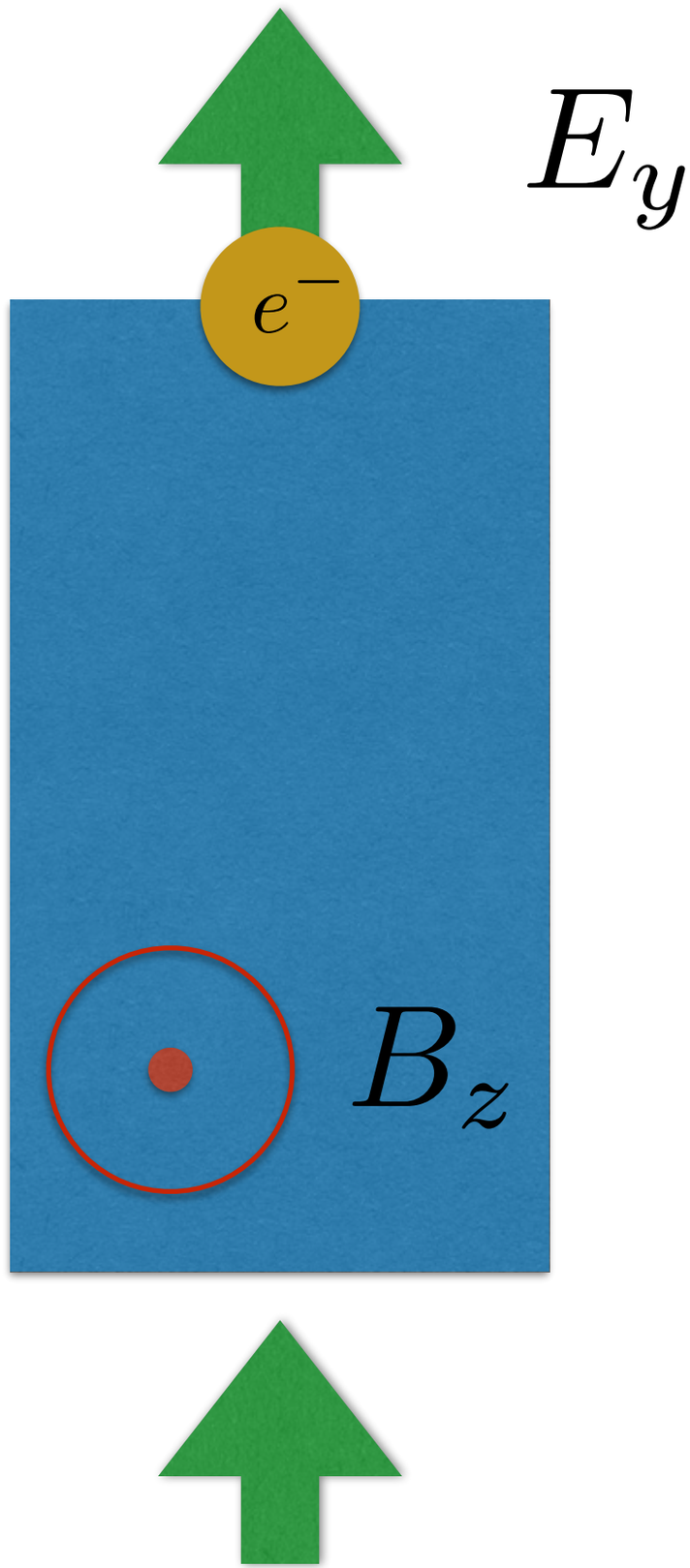
Hall Effect



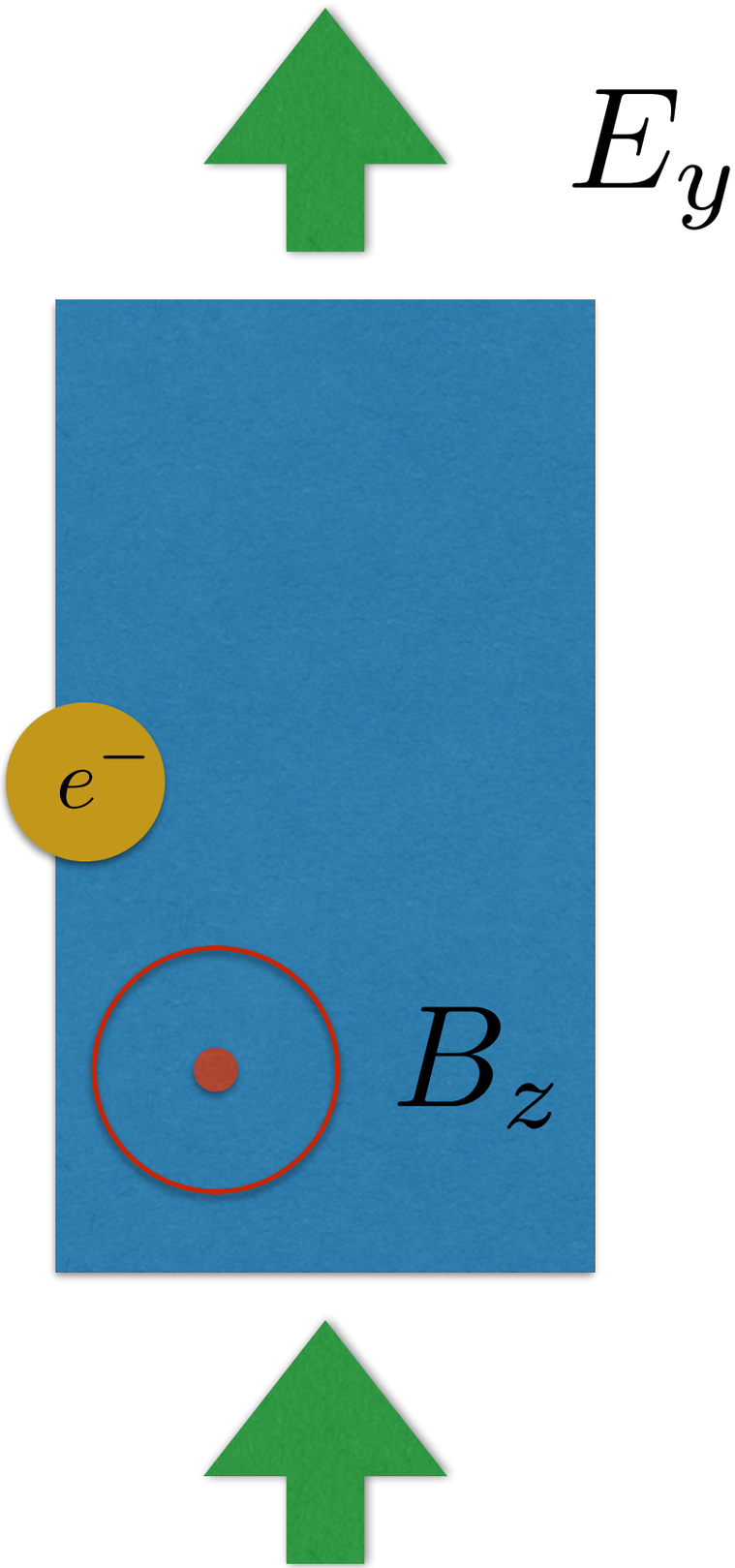
Hall Effect



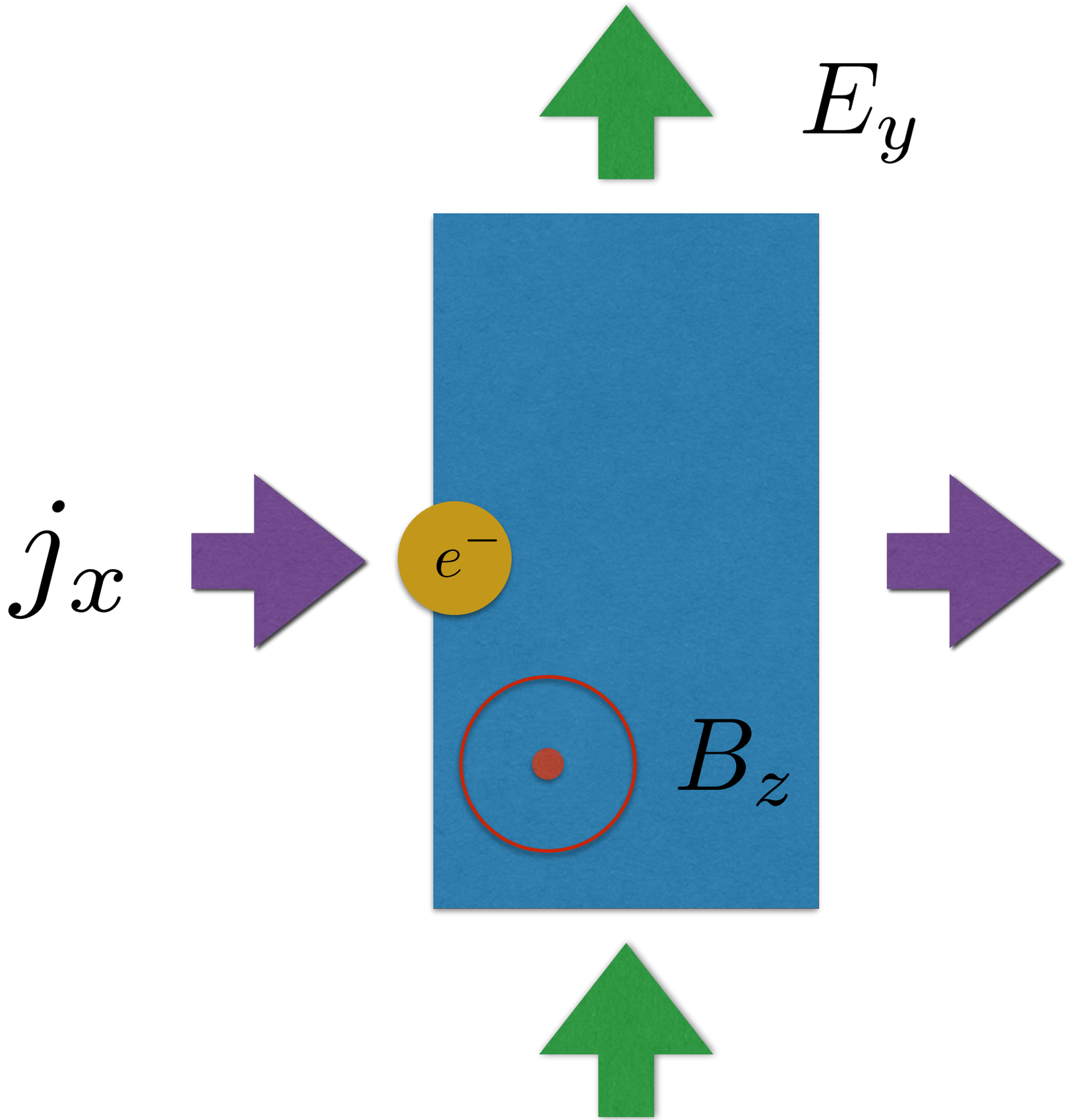
Hall Effect



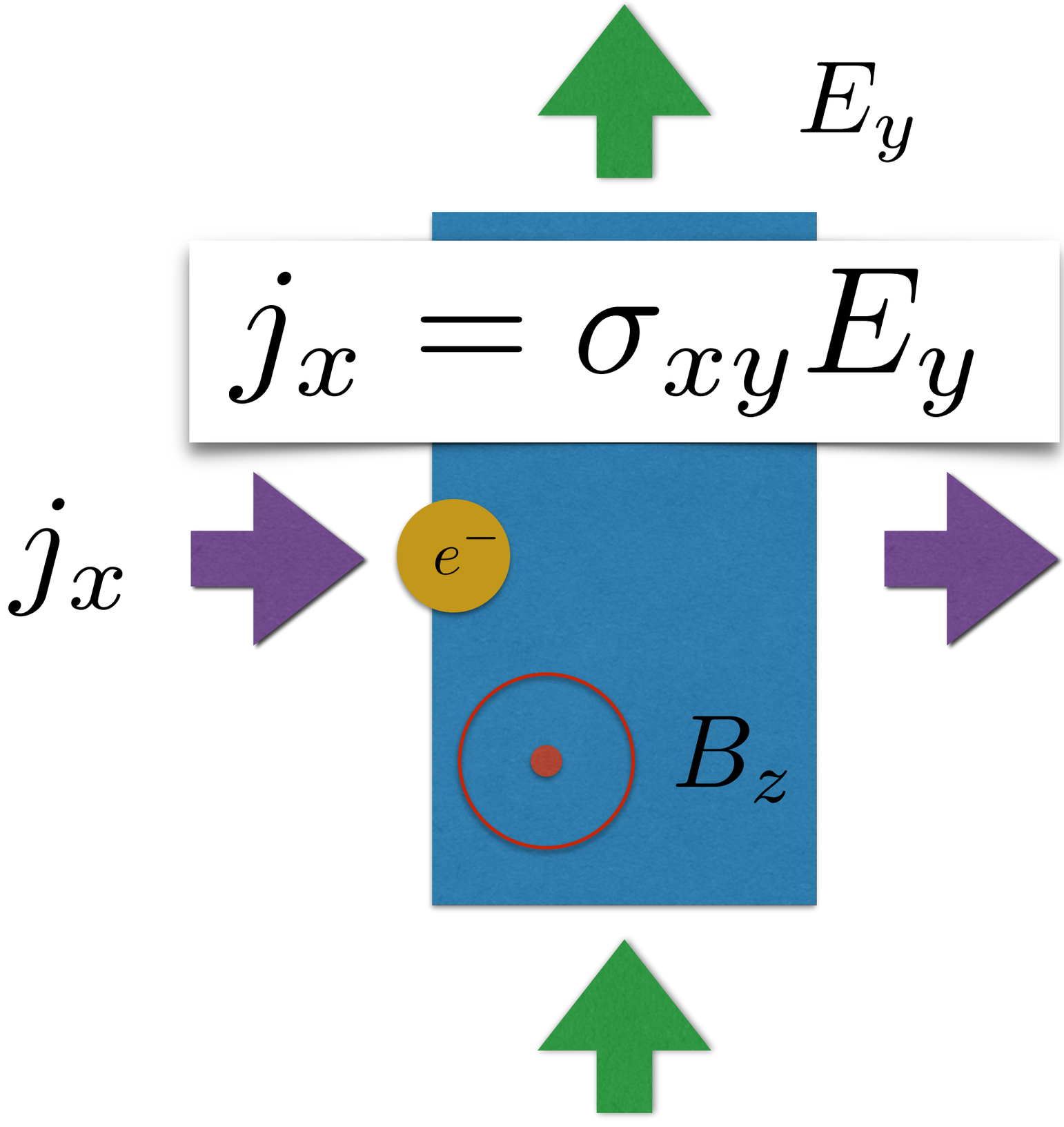
Hall Effect



Hall Effect



Hall Effect



Integer Quantum Hall Effect

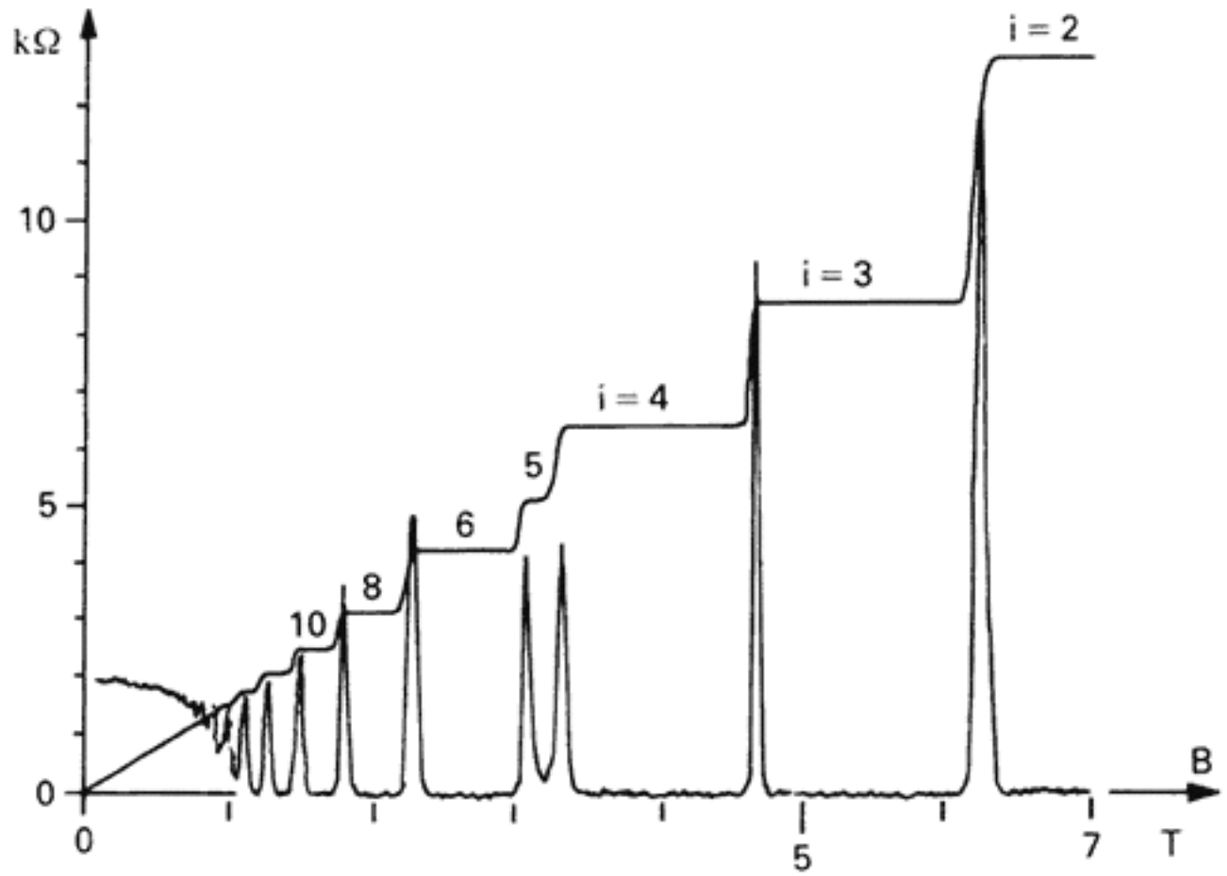
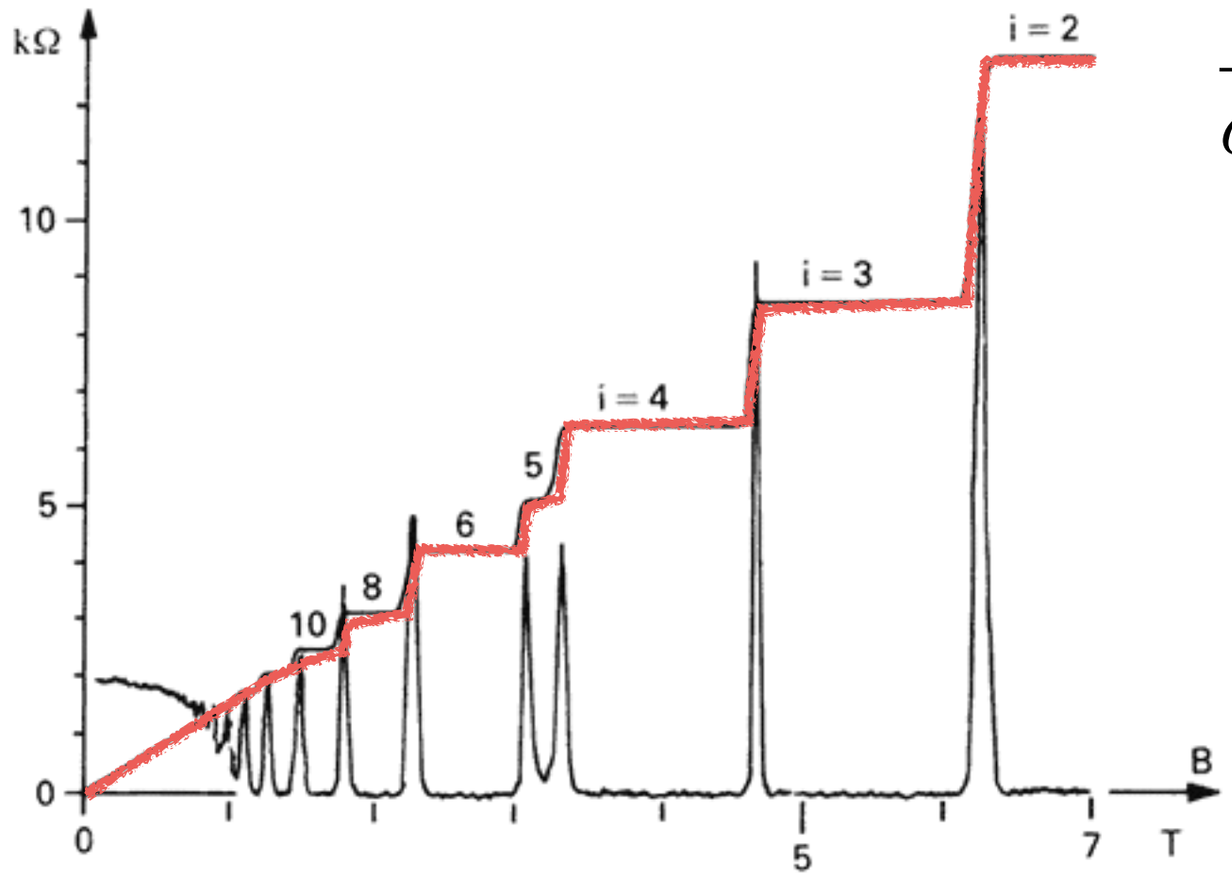


Image :: http://www.nobelprize.org/nobel_prizes/physics/laureates/1998/press.html

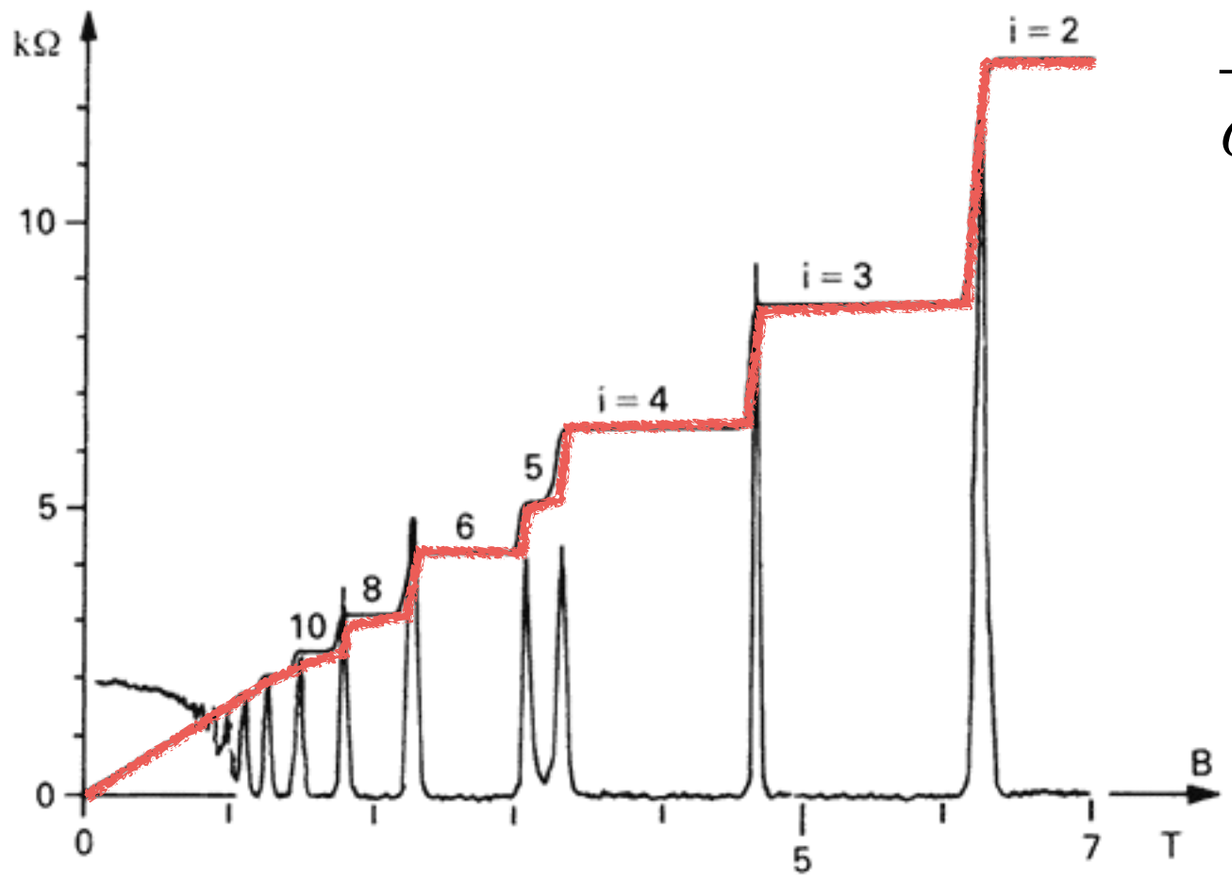
Integer Quantum Hall Effect



$$\frac{1}{\sigma_{xy}}$$

Image :: http://www.nobelprize.org/nobel_prizes/physics/laureates/1998/press.html

Integer Quantum Hall Effect

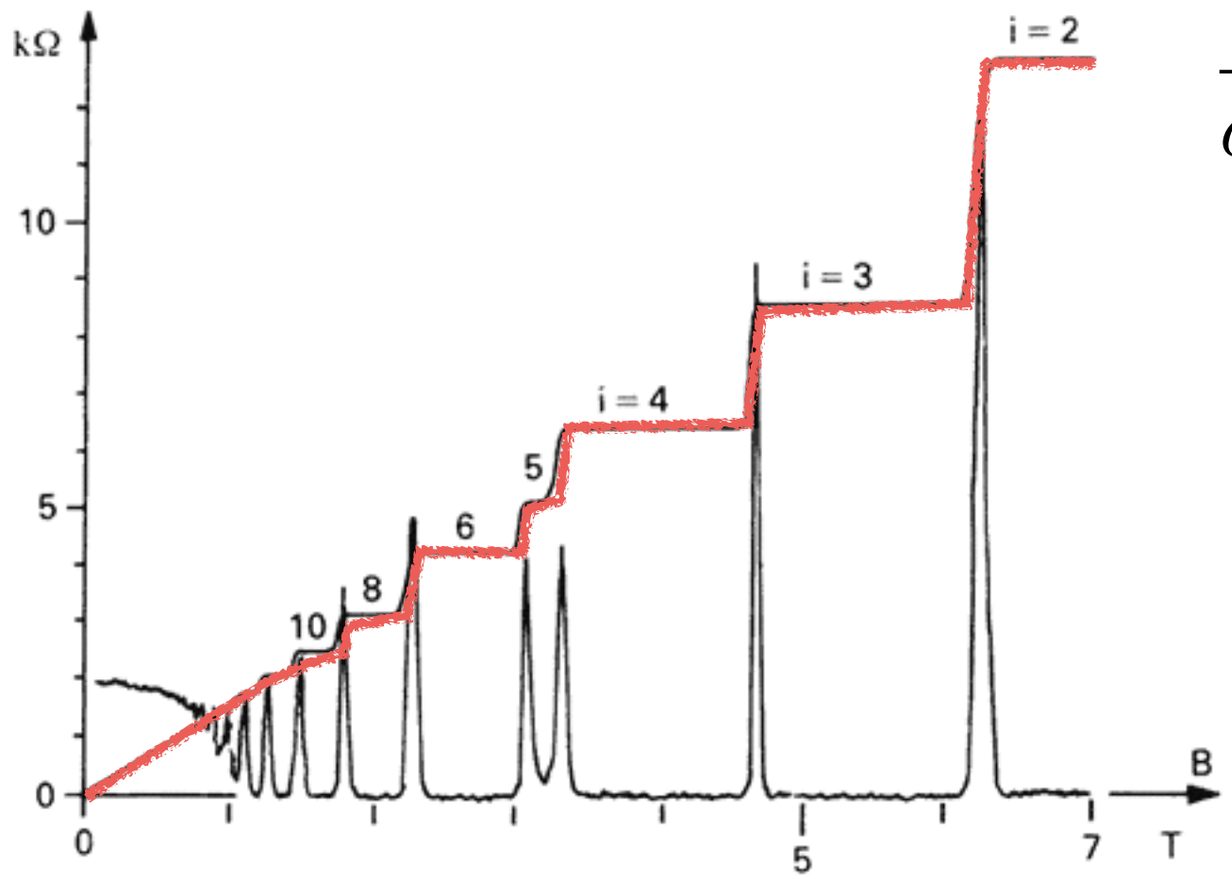


$$\frac{1}{\sigma_{xy}}$$

$$\sigma_{xy} = \frac{e^2}{h} N_c \quad \text{(Precise)}$$

Image :: http://www.nobelprize.org/nobel_prizes/physics/laureates/1998/press.html

Integer Quantum Hall Effect



$$\frac{1}{\sigma_{xy}}$$

$$\sigma_{xy} = \frac{e^2}{h} N_c \quad \text{(Precise)}$$

Conductance quantum * Integer

Image :: http://www.nobelprize.org/nobel_prizes/physics/laureates/1998/press.html

Integer Quantum Hall Effect

$$\sigma_{xy} = \frac{e^2}{h} N_c$$

$$N_c = \sum_n \int_{\mathrm{BZ}} \frac{\mathrm{d}^2 \vec{k}}{2\pi} F_n(\vec{k})$$

$$F(\vec{k}) = i \langle \vec{\nabla}_{\vec{k}} u_n(\vec{k}) | \times | \vec{\nabla}_{\vec{k}} u_n(\vec{k}) \rangle$$

Integer Quantum Hall Effect

$$\sigma_{xy} = \frac{e^2}{h} N_c$$

Integer Quantum Hall Effect

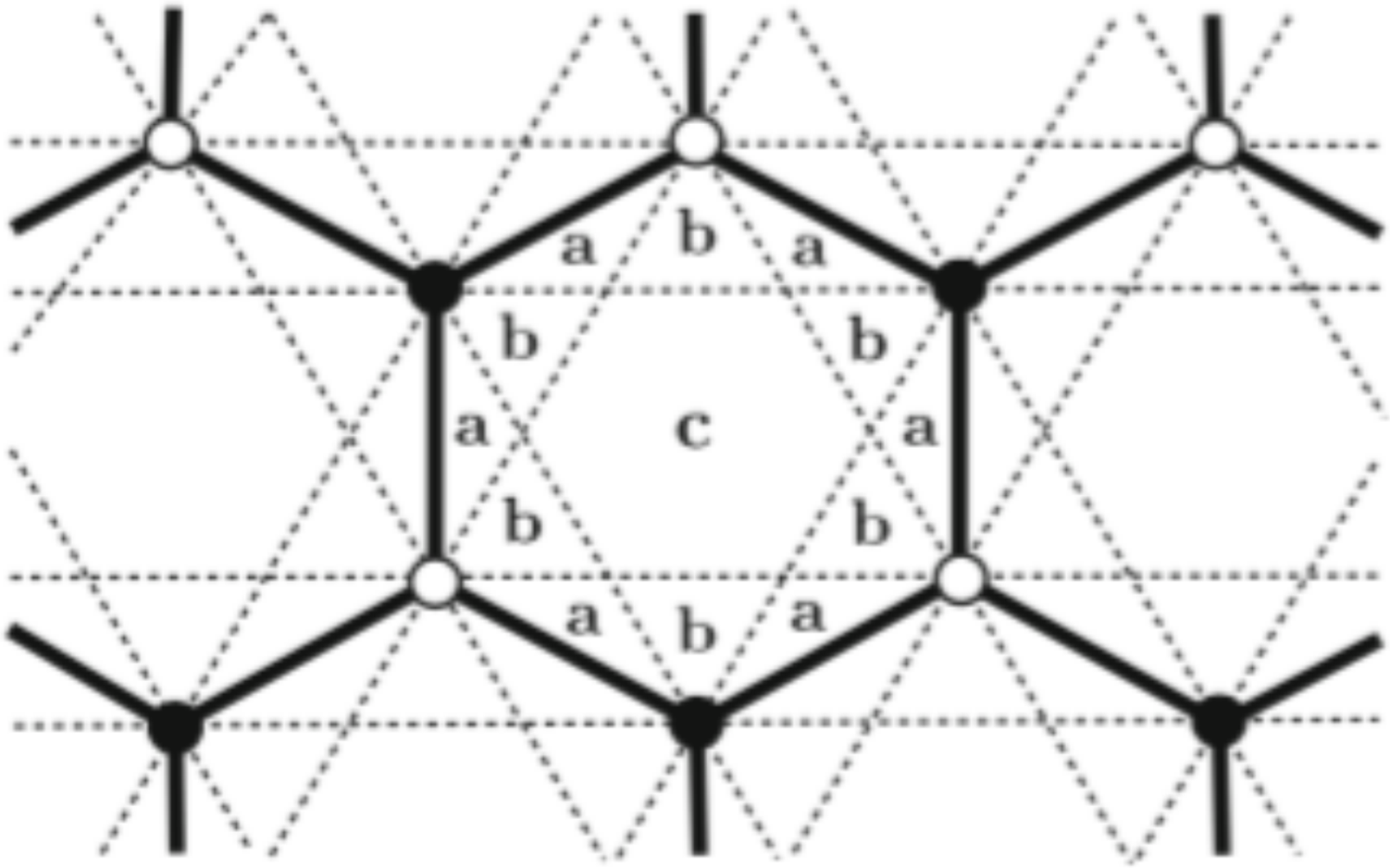
$$\sigma_{xy} = \frac{e^2}{h} N_c$$

One channel for each Chern number

Chern Number ::
Topological invariant
Always Integer when we calculate it on closed manifold
Cannot be changed without band touching

Haldane Honeycomb Model

Simple example



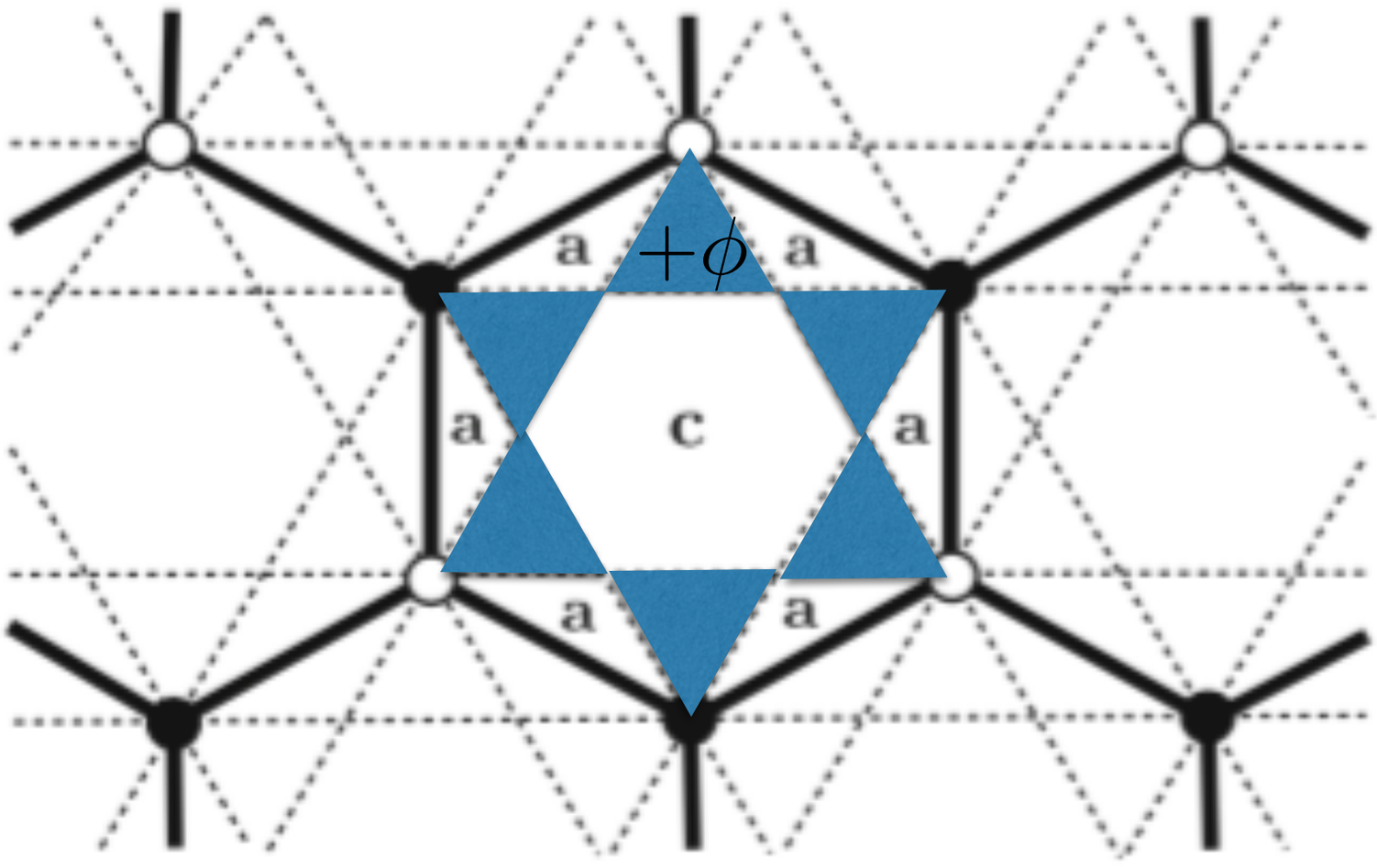
Spinless Fermions

$$H = \epsilon(\vec{k})\ 1_{2\times 2} + \vec{d}(\vec{k}) \cdot \vec{\sigma}$$

Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.1

Haldane Honeycomb Model

Simple example



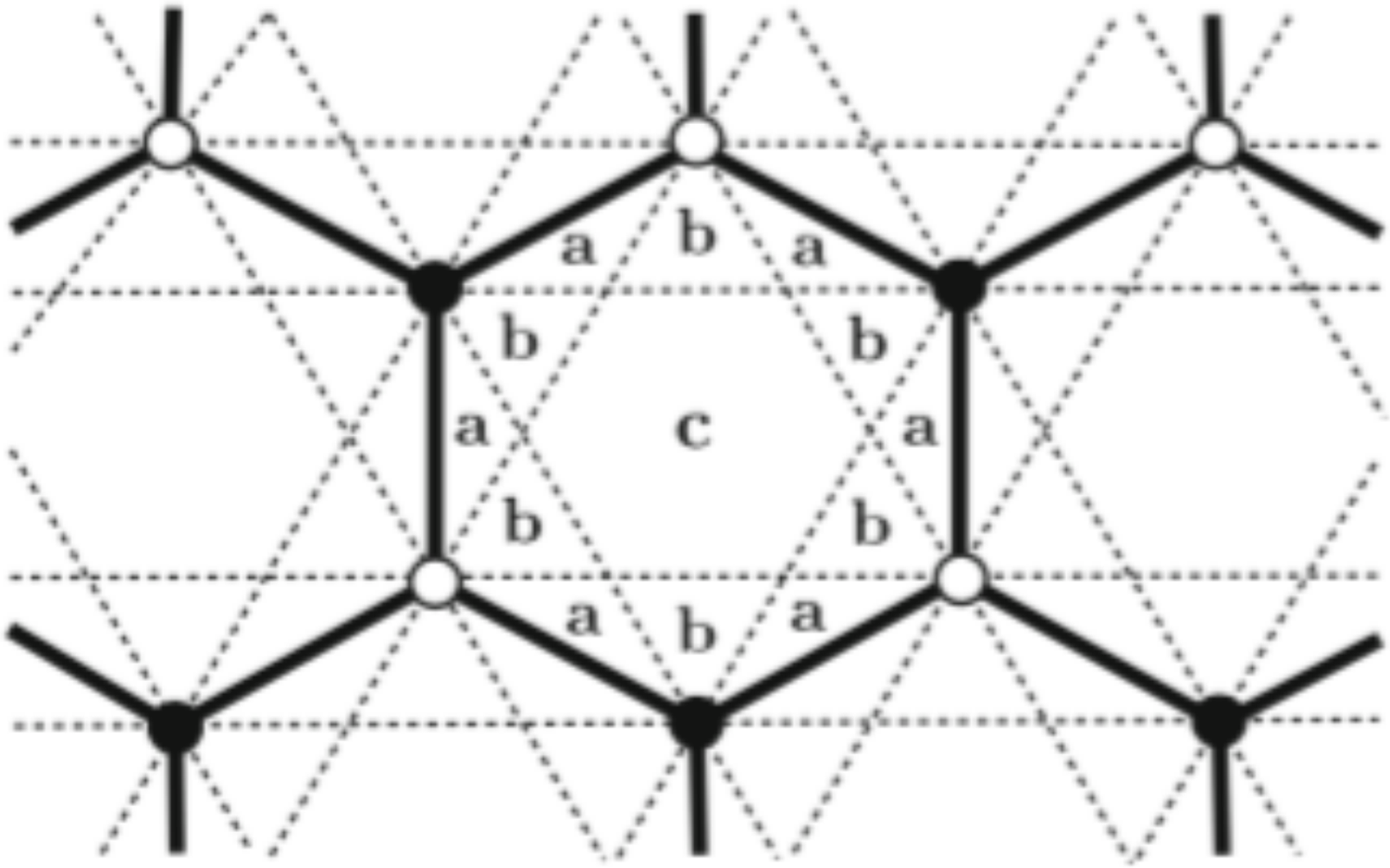
Spinless Fermions

$$H = \epsilon(\vec{k})\ 1_{2\times 2} + \vec{d}(\vec{k}) \cdot \vec{\sigma}$$

Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.1

Haldane Honeycomb Model

Simple example



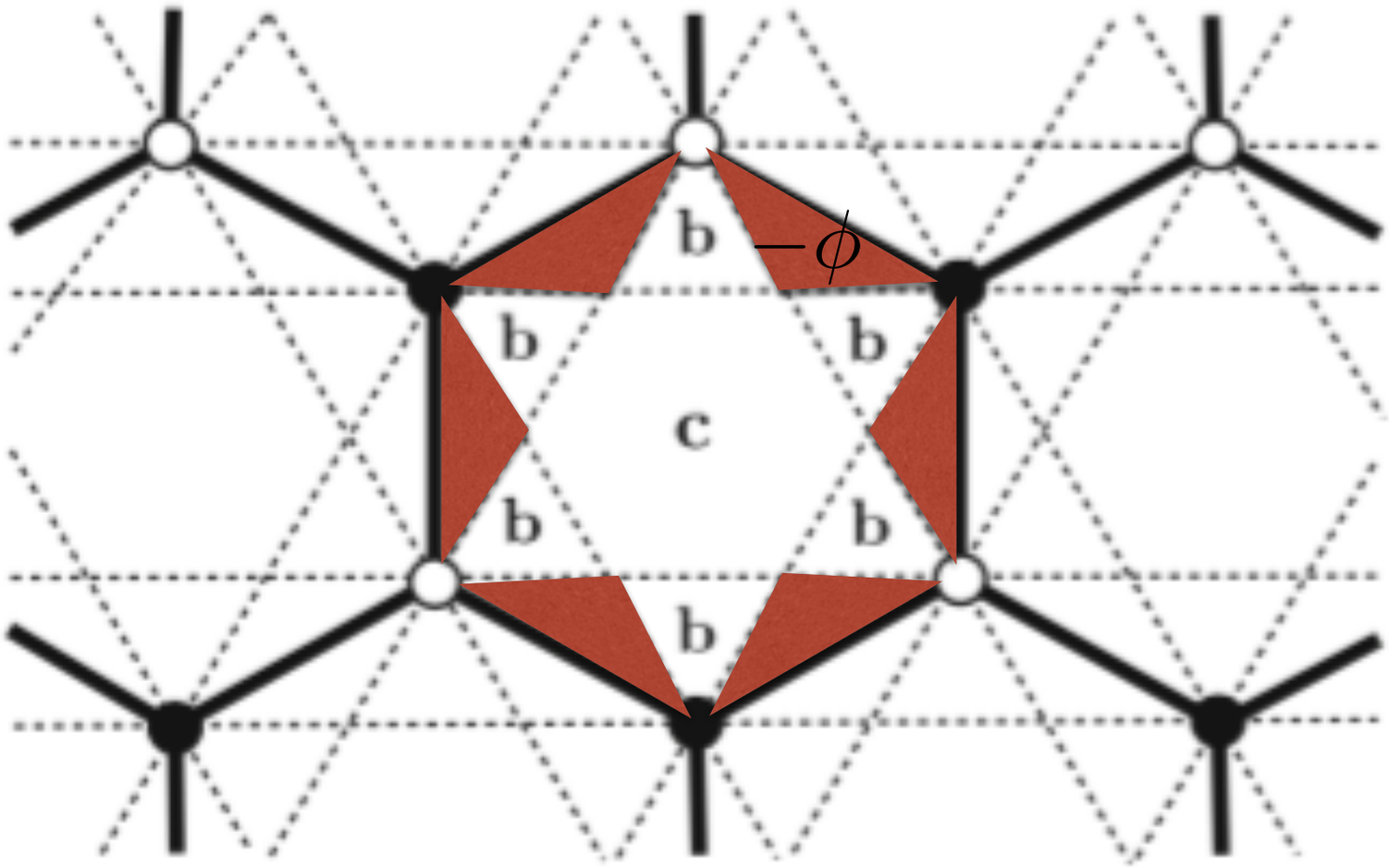
Spinless Fermions

$$H = \epsilon(\vec{k})\ 1_{2\times 2} + \vec{d}(\vec{k}) \cdot \vec{\sigma}$$

Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.1

Haldane Honeycomb Model

Simple example



Spinless Fermions

$$H = \epsilon(\vec{k})\ 1_{2\times 2} + \vec{d}(\vec{k}) \cdot \vec{\sigma}$$

Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.1

Haldane Honeycomb Model

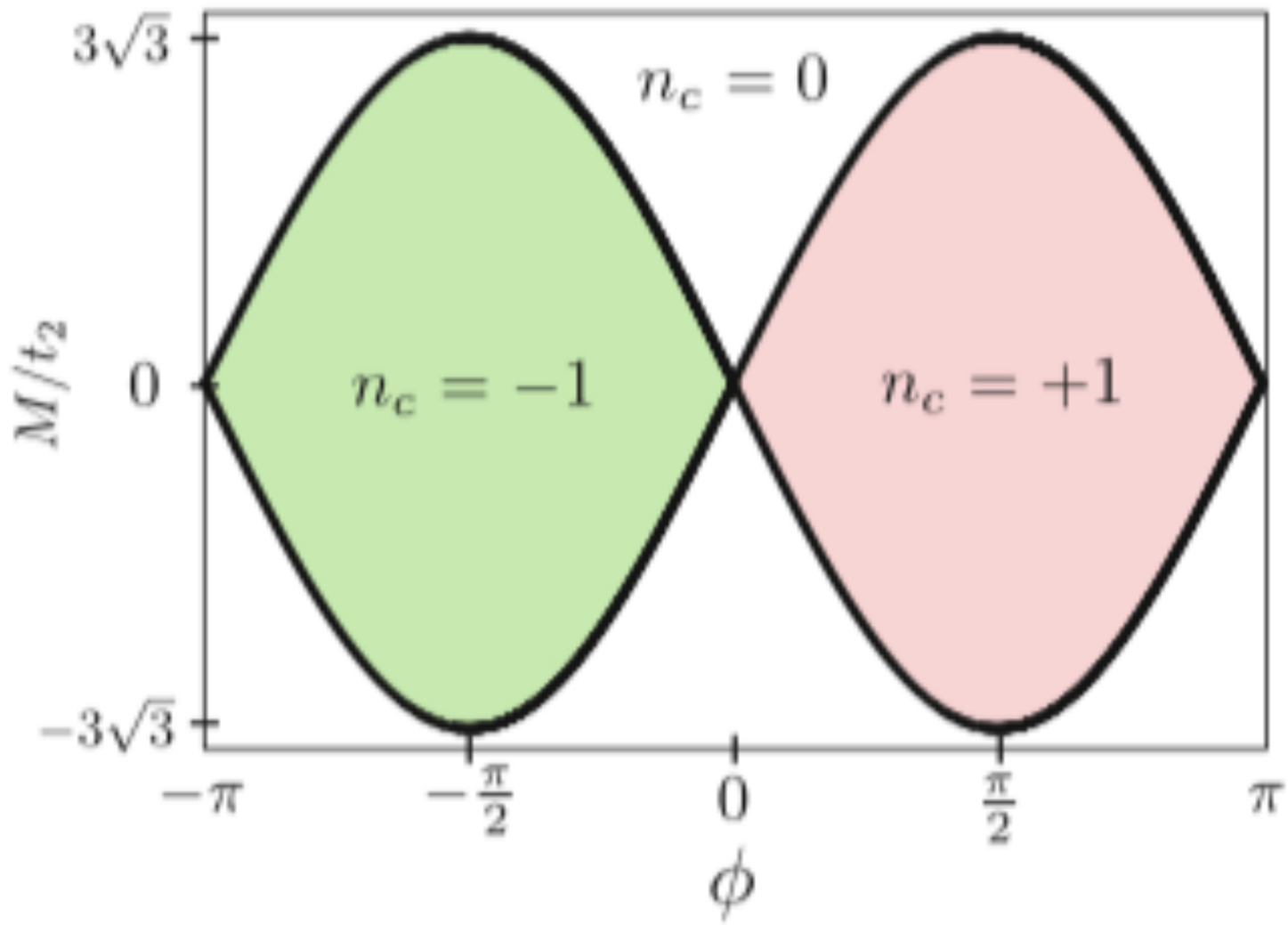


Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.2
Image :: RevModPhys.82.3045, Hasan, Kane

Haldane Honeycomb Model

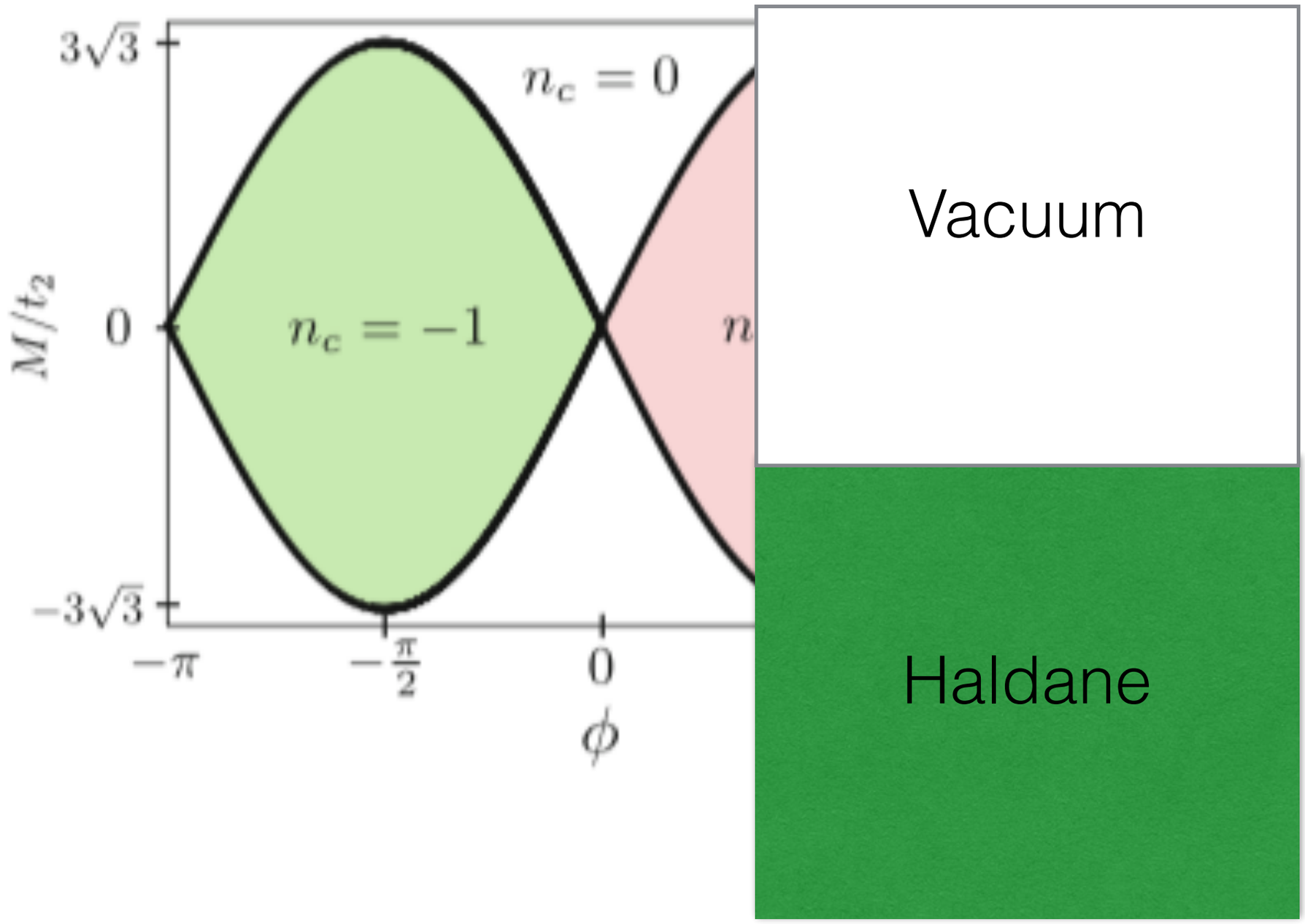


Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.2
Image :: RevModPhys.82.3045, Hasan, Kane

Haldane Honeycomb Model

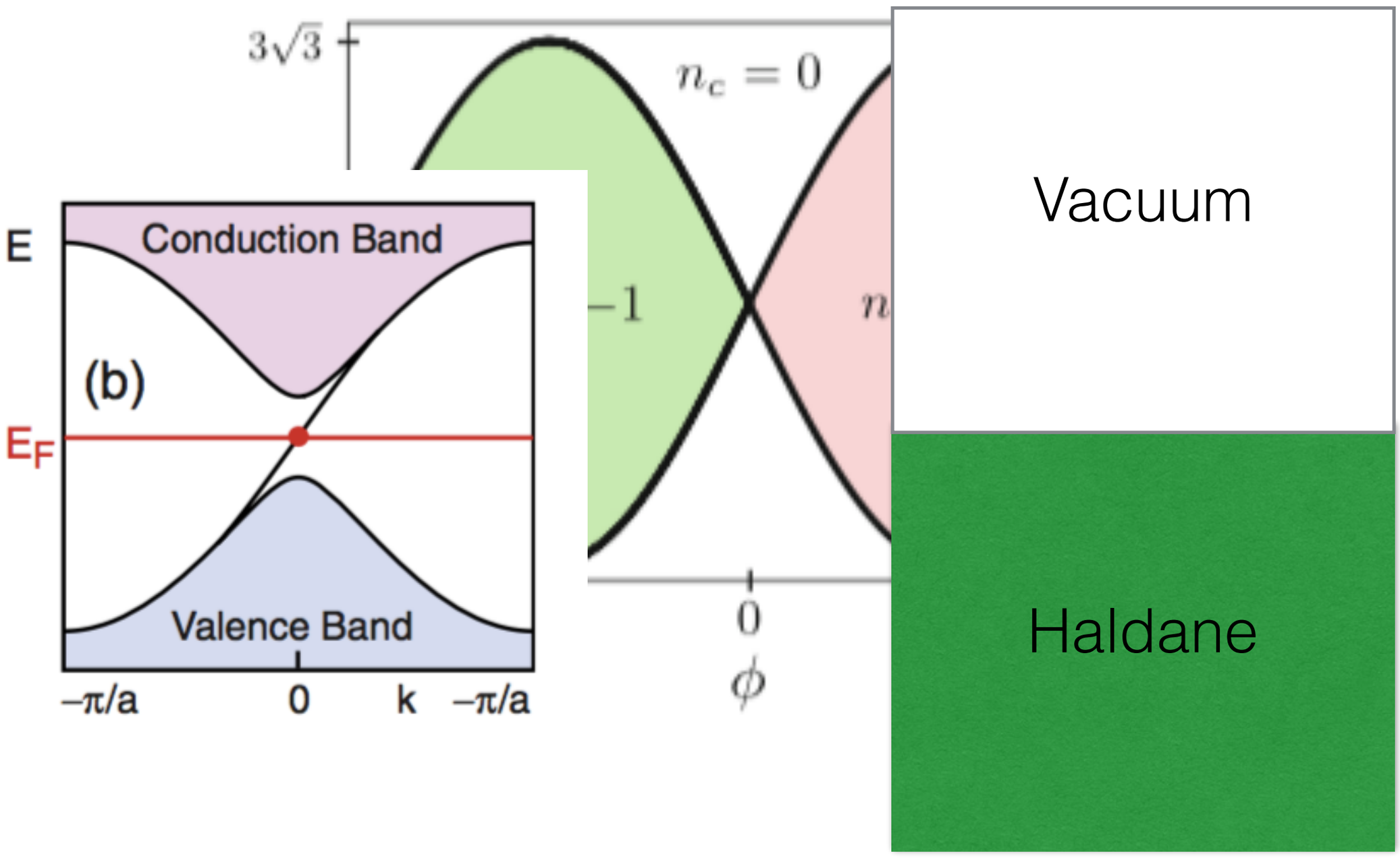


Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.2
Image :: RevModPhys.82.3045, Hasan, Kane

Haldane Honeycomb Model

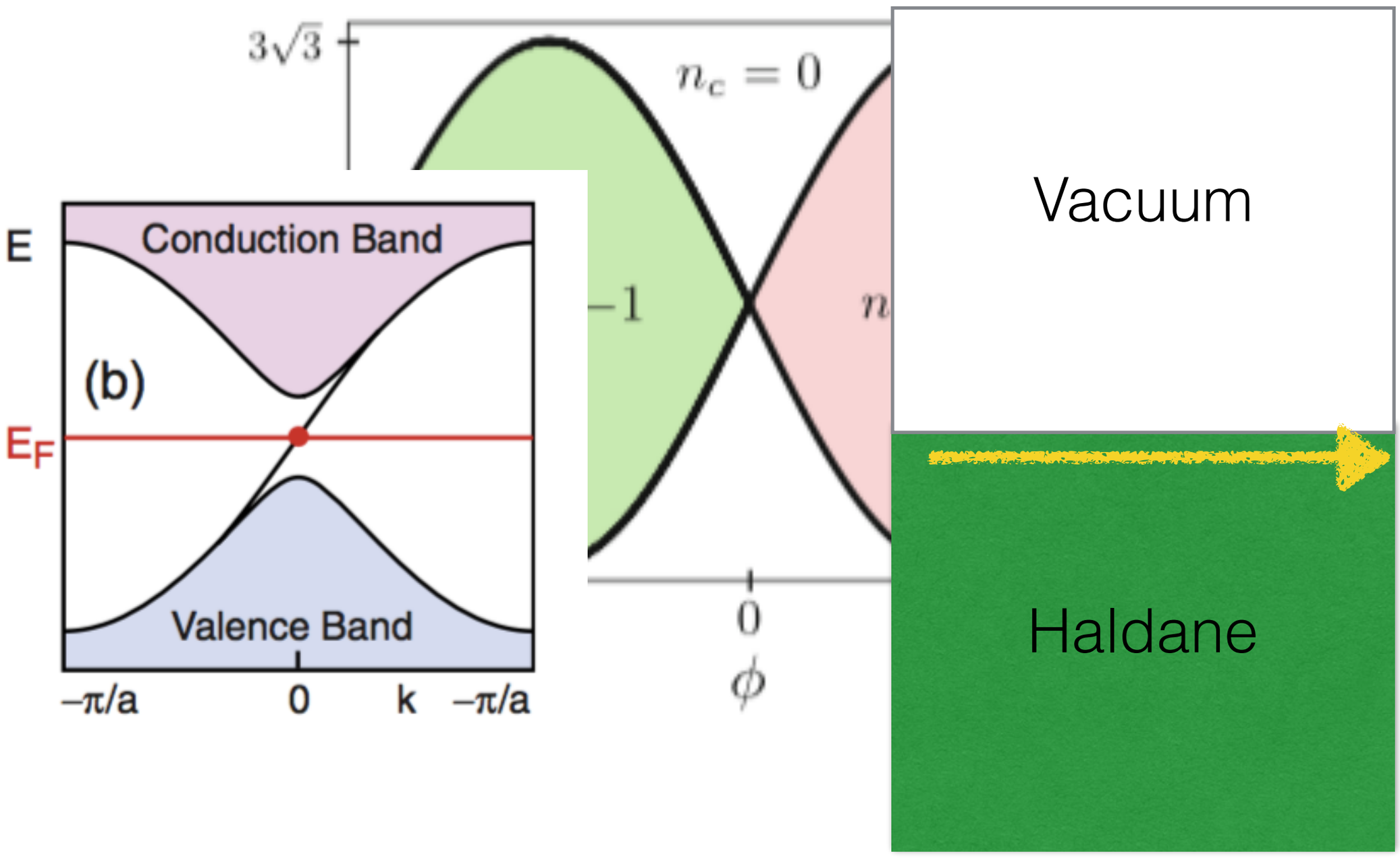


Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.2
Image :: RevModPhys.82.3045, Hasan, Kane

Haldane Honeycomb Model

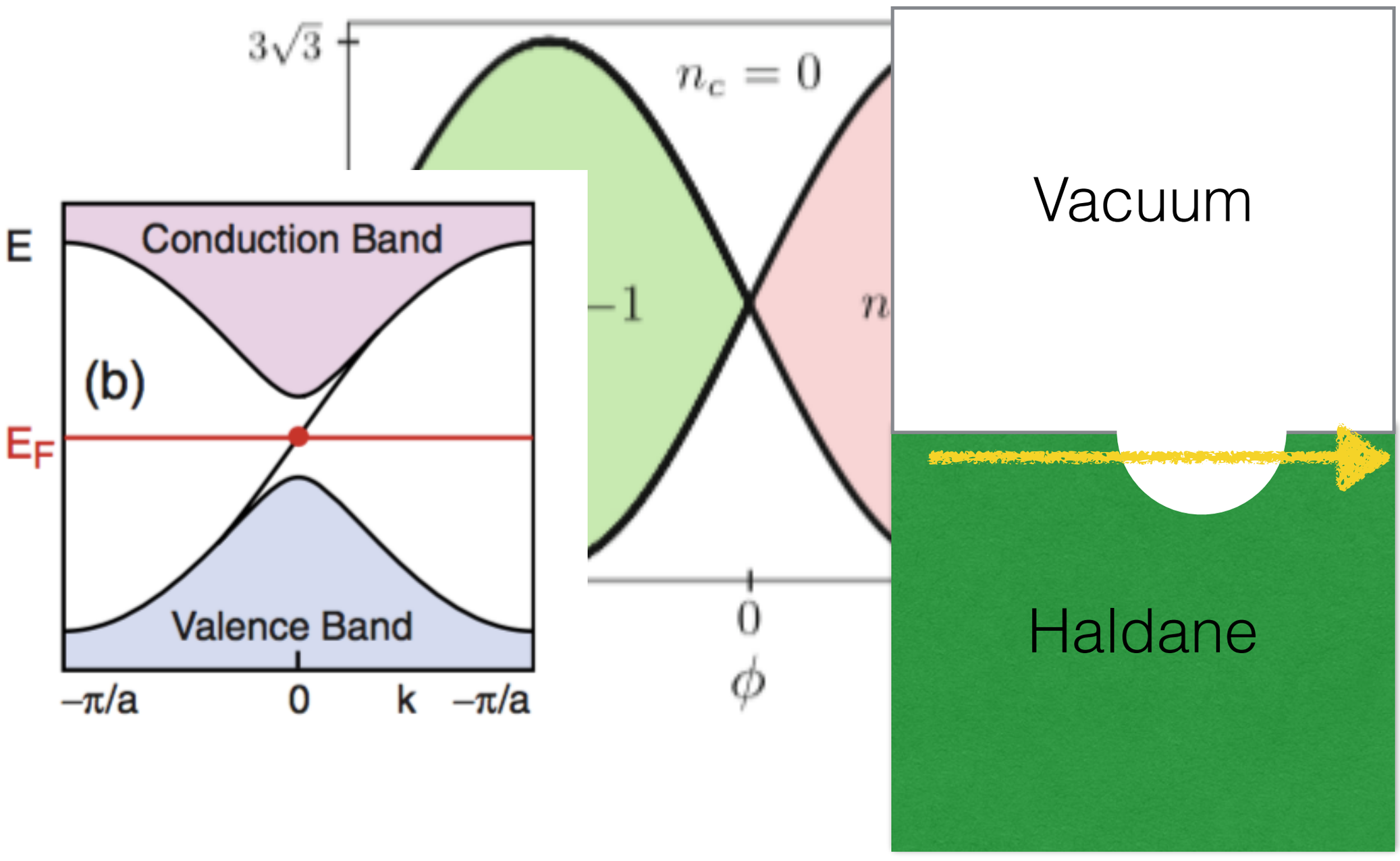


Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.2
Image :: RevModPhys.82.3045, Hasan, Kane

Haldane Honeycomb Model

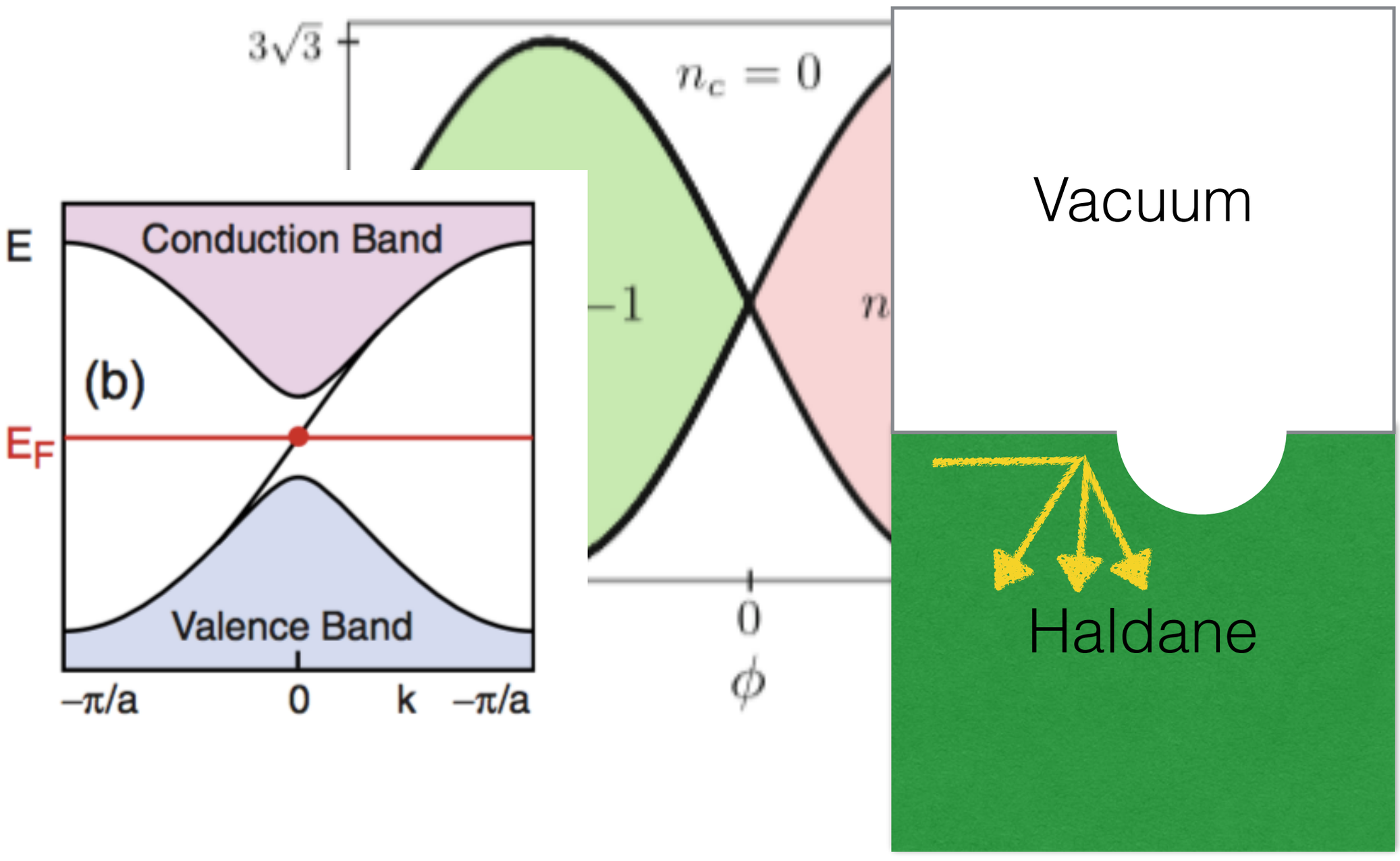


Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.2
Image :: RevModPhys.82.3045, Hasan, Kane

Haldane Honeycomb Model

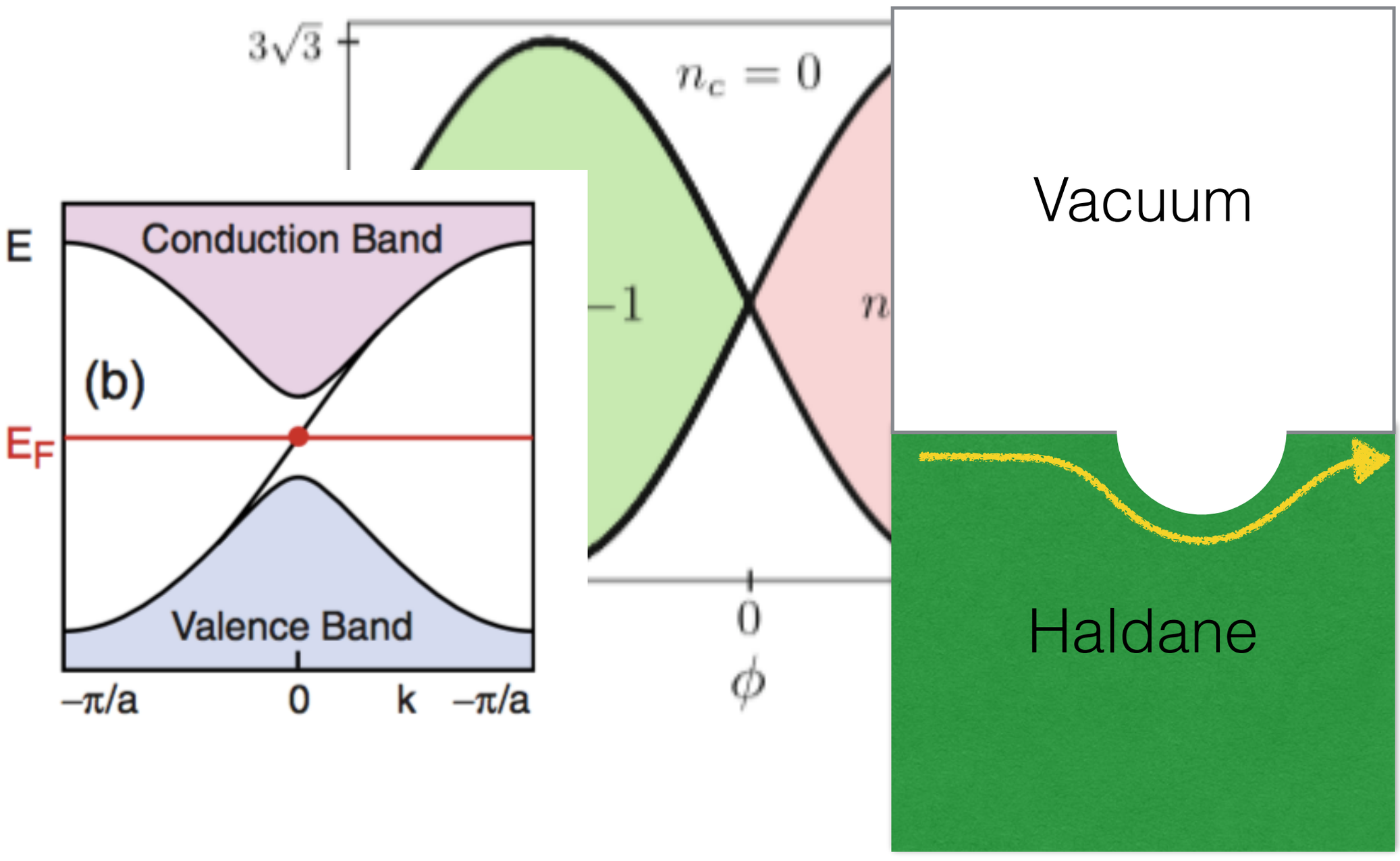
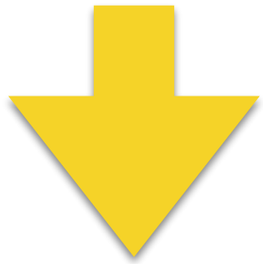


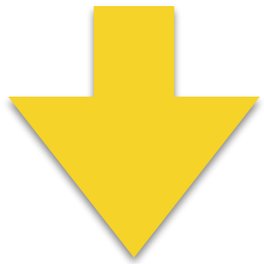
Image :: Topological Insulators : Dirac Equation in Condensed matters, Shen, Fig. 6.2
Image :: RevModPhys.82.3045, Hasan, Kane

Why the magnetic field is important ?

Time-Reversal Symmetry (TR)



Zero Chern number



Trivial ?

Quantum Spin Hall Effect

Kane and Mele Model
Spin 1/2 Fermions

Two copies of Haldane model :
Without external magnetic field
With Spin-Orbit coupling

Respect Time-reversal symmetry

Quantum Spin Hall Effect

Two phases exist

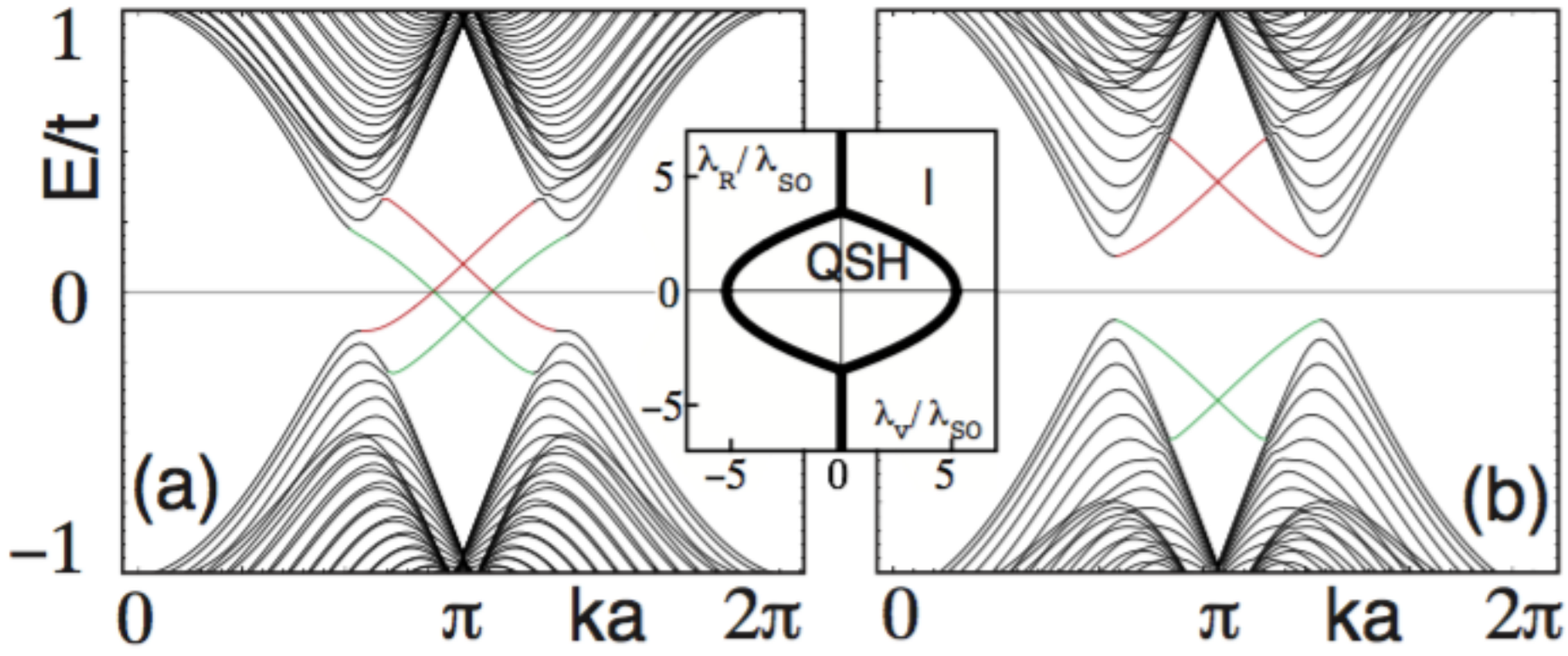


Image :: RevModPhys.82.3045, Hasan, Kane
Image :: PRL 2005 vol.95 146802, Z2 Topological order and the quantum spin Hall effect

Quantum Spin Hall Effect

Two phases exist

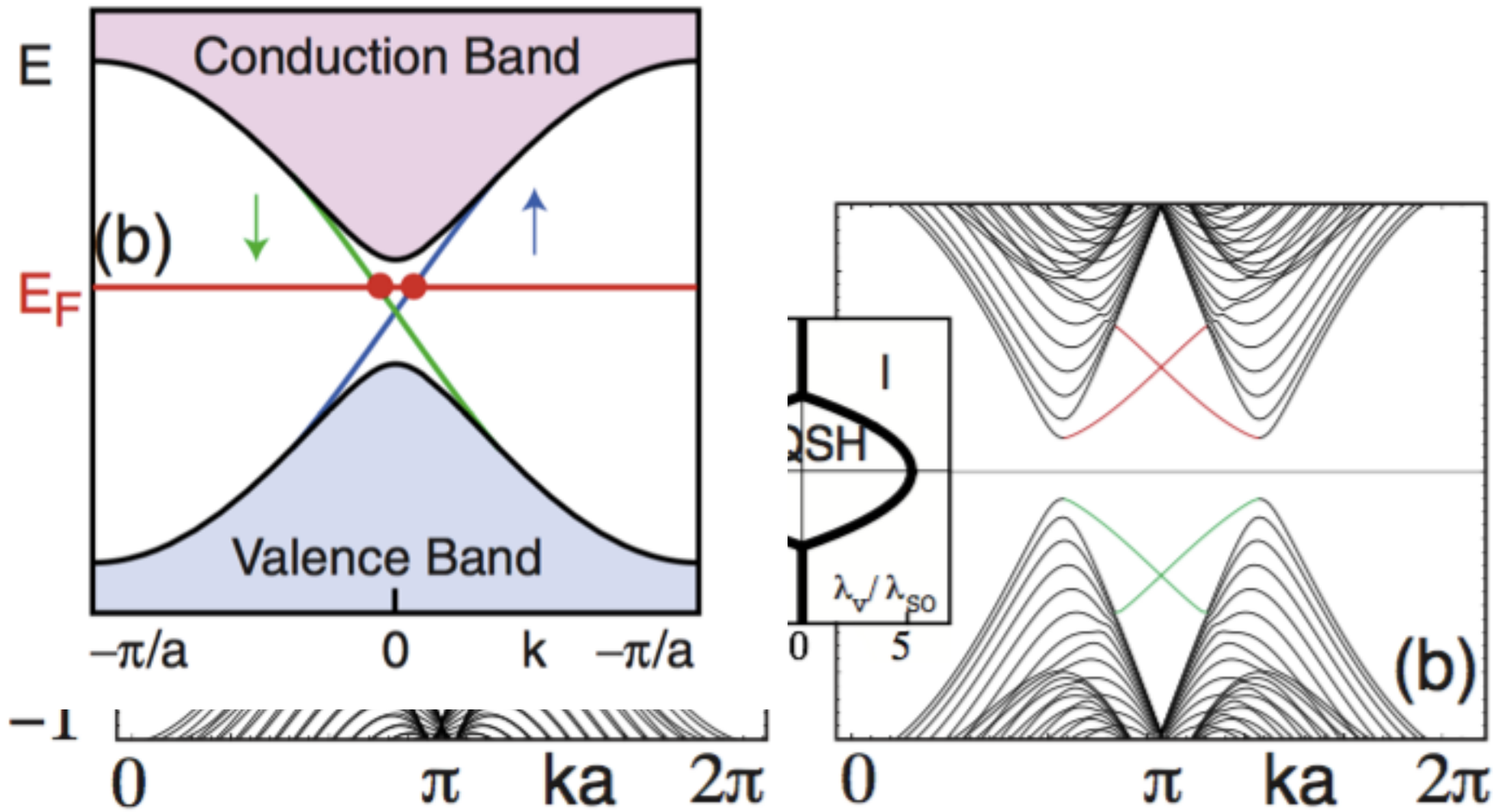


Image :: RevModPhys.82.3045, Hasan, Kane
Image :: PRL 2005 vol.95 146802, Z2 Topological order and the quantum spin Hall effect

Quantum Spin Hall Effect

Two phases exist

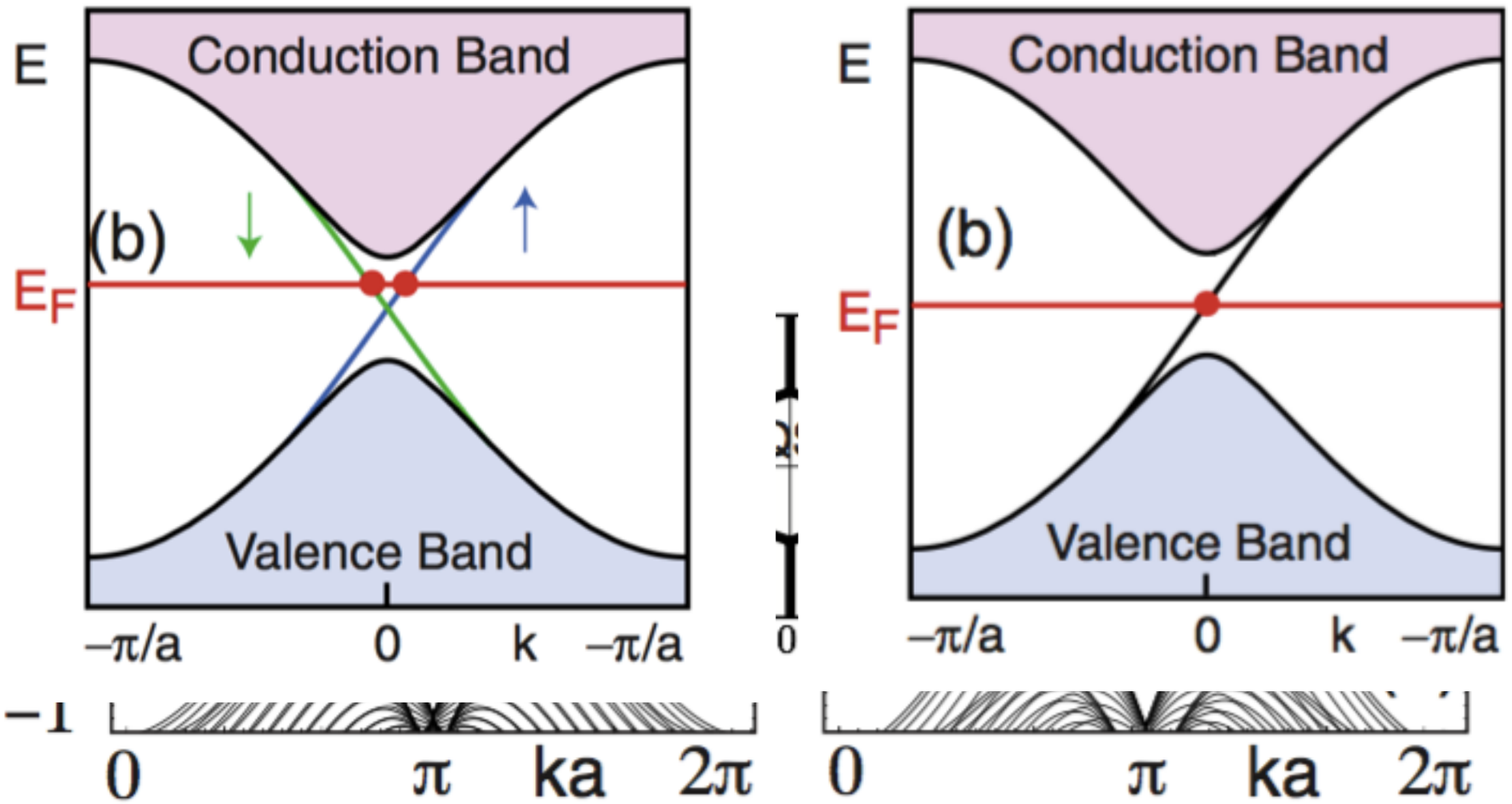
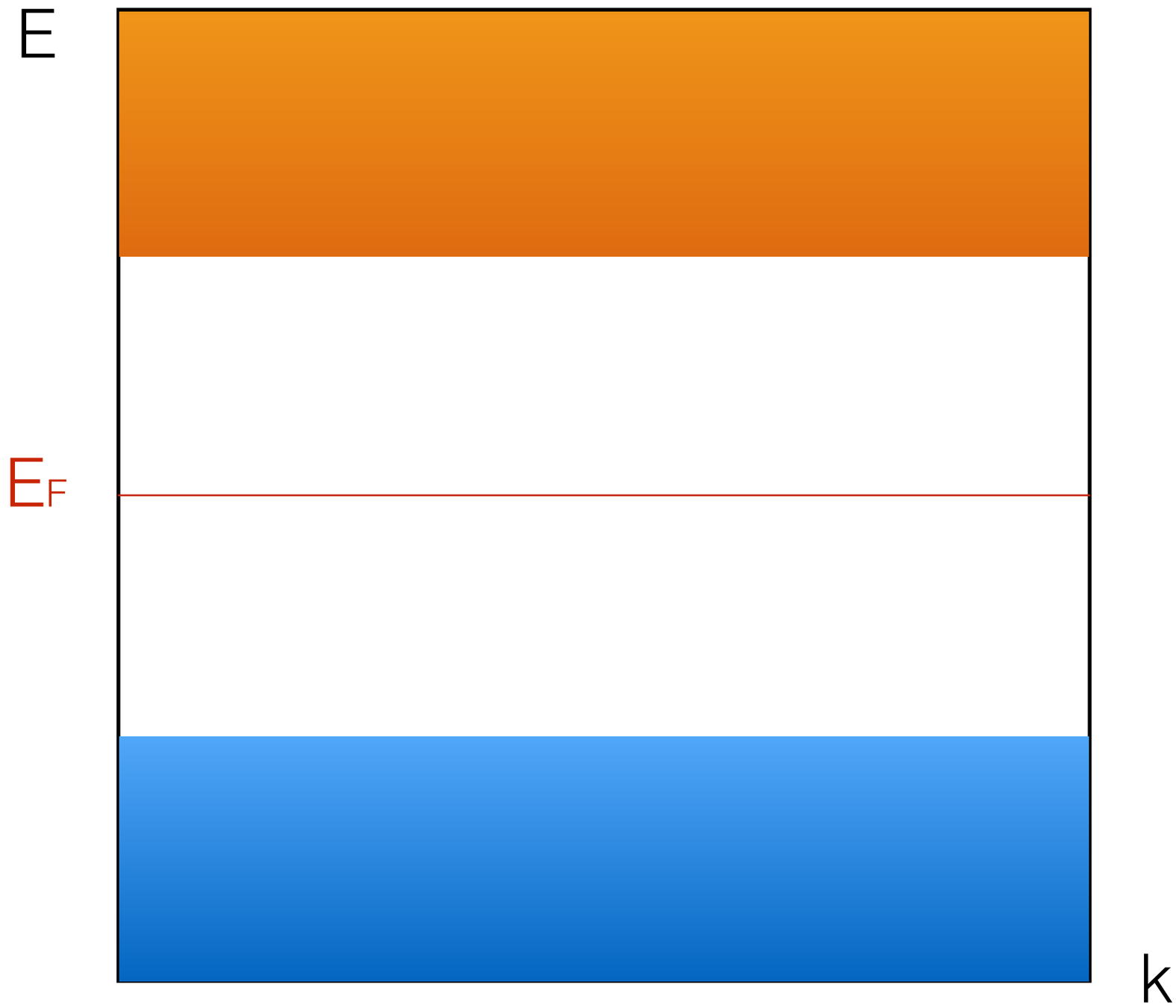


Image :: RevModPhys.82.3045, Hasan, Kane
Image :: PRL 2005 vol.95 146802, Z2 Topological order and the quantum spin Hall effect

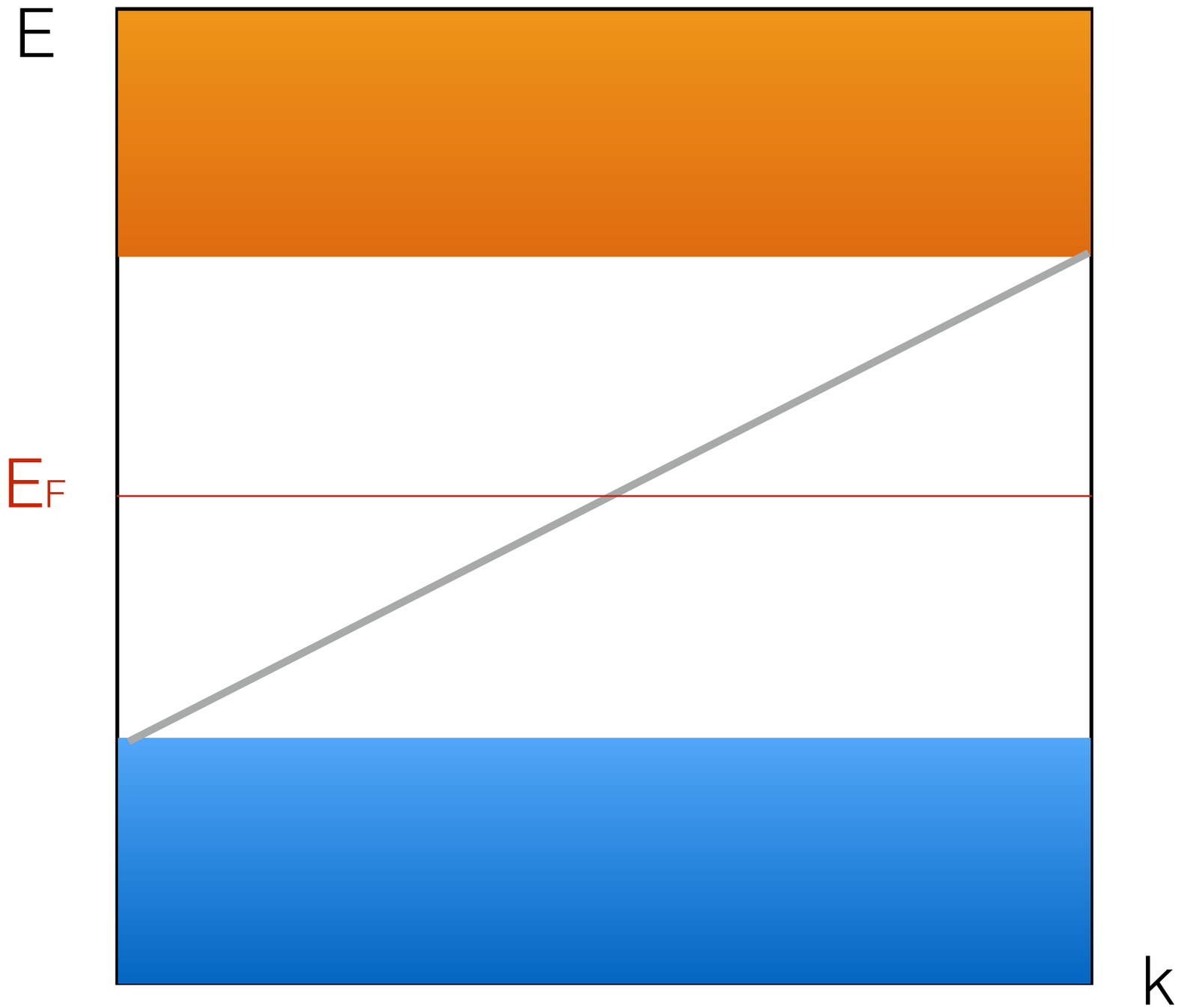
Quantum Spin Hall Effect

Two copies of the Haldane model



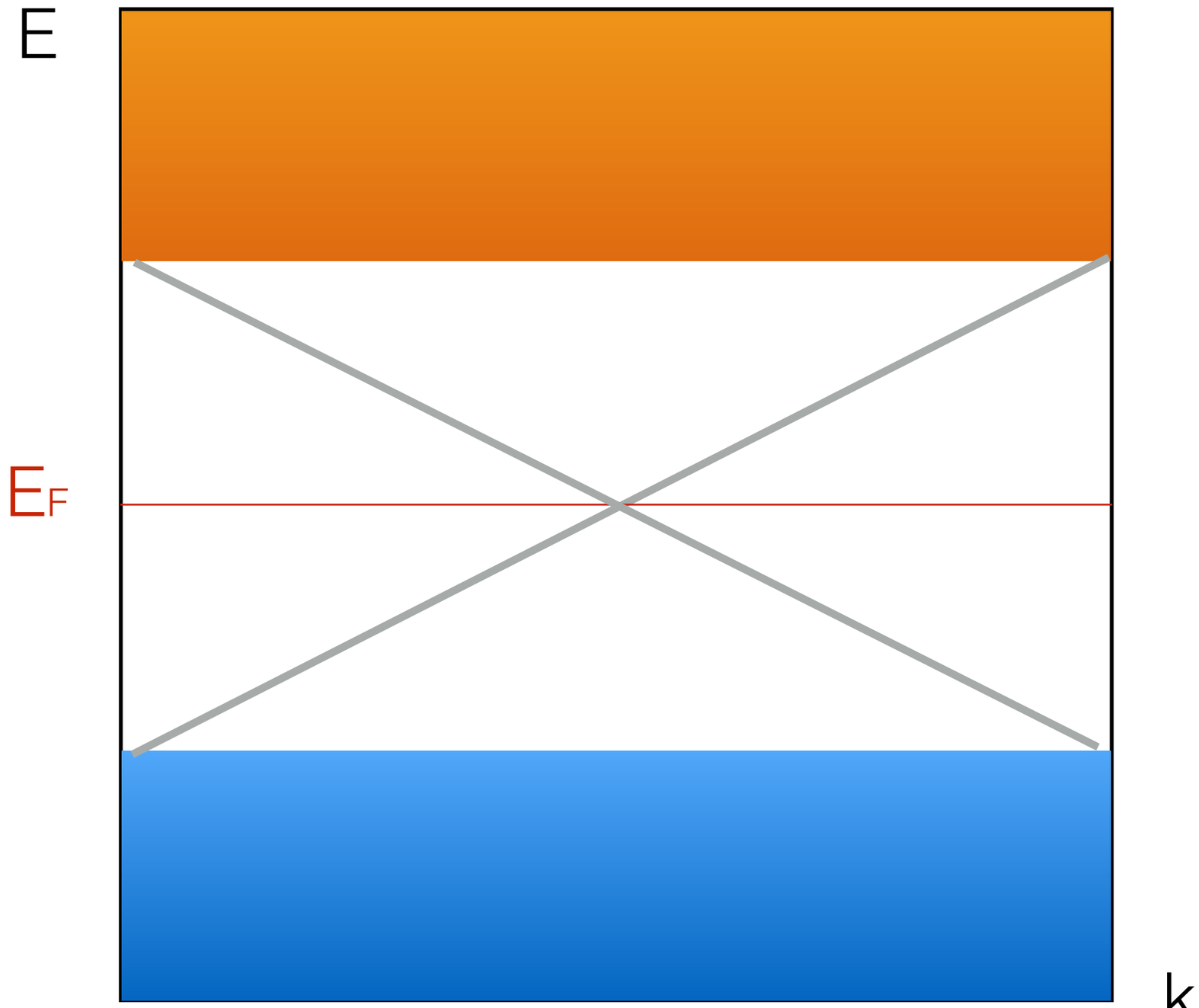
Quantum Spin Hall Effect

Two copies of the Haldane model



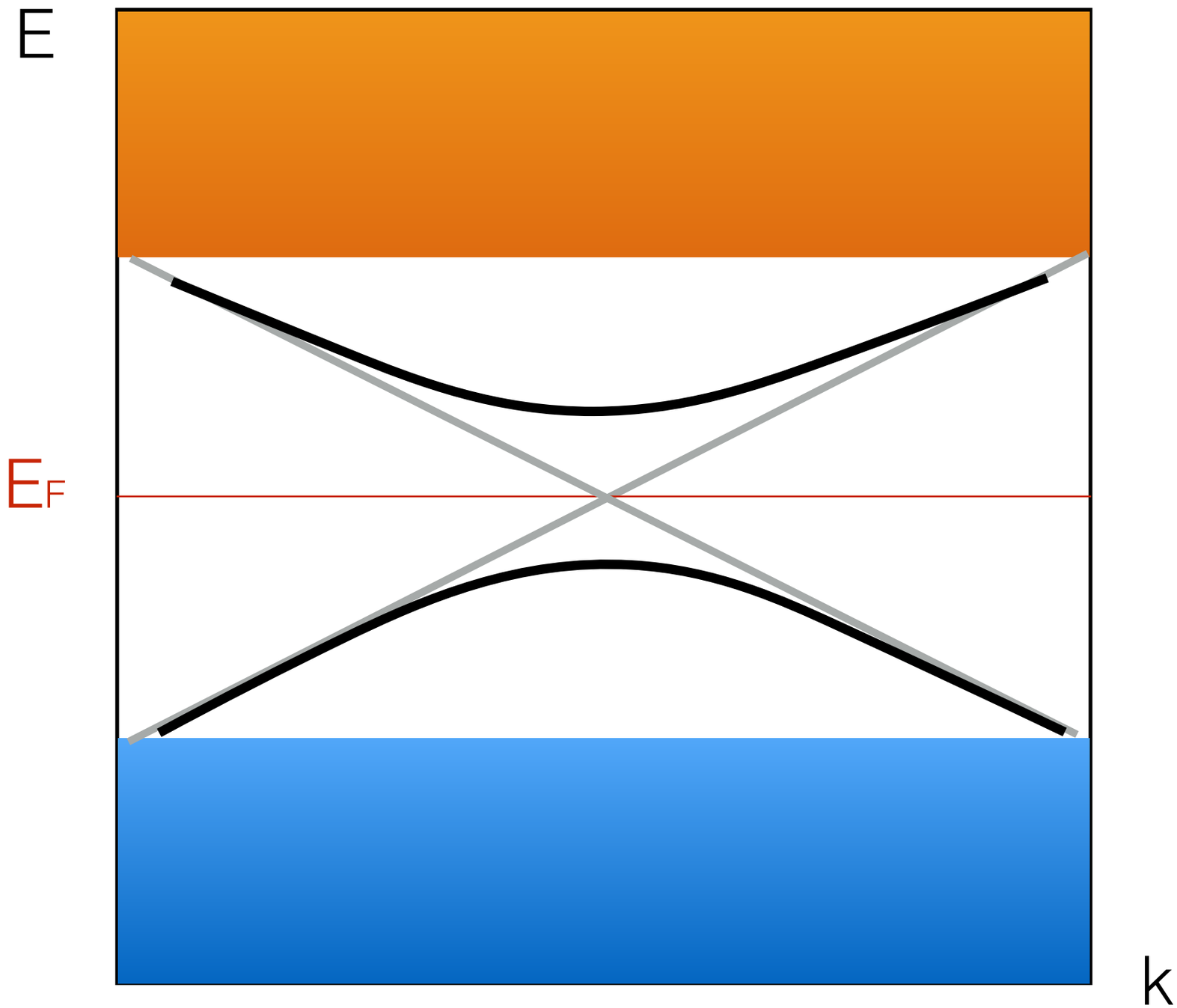
Quantum Spin Hall Effect

Two copies of the Haldane model



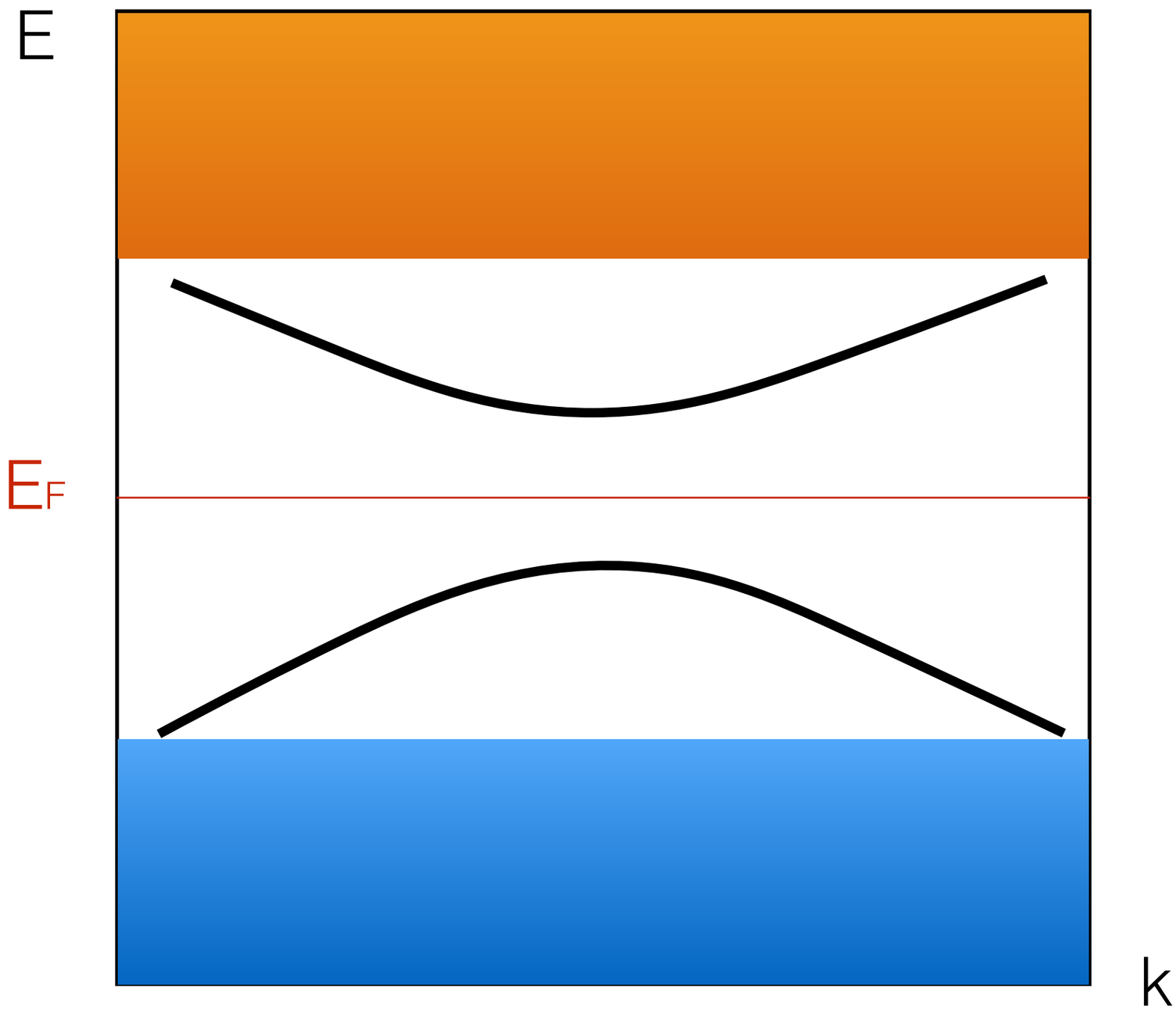
Quantum Spin Hall Effect

Two copies of the Haldane model



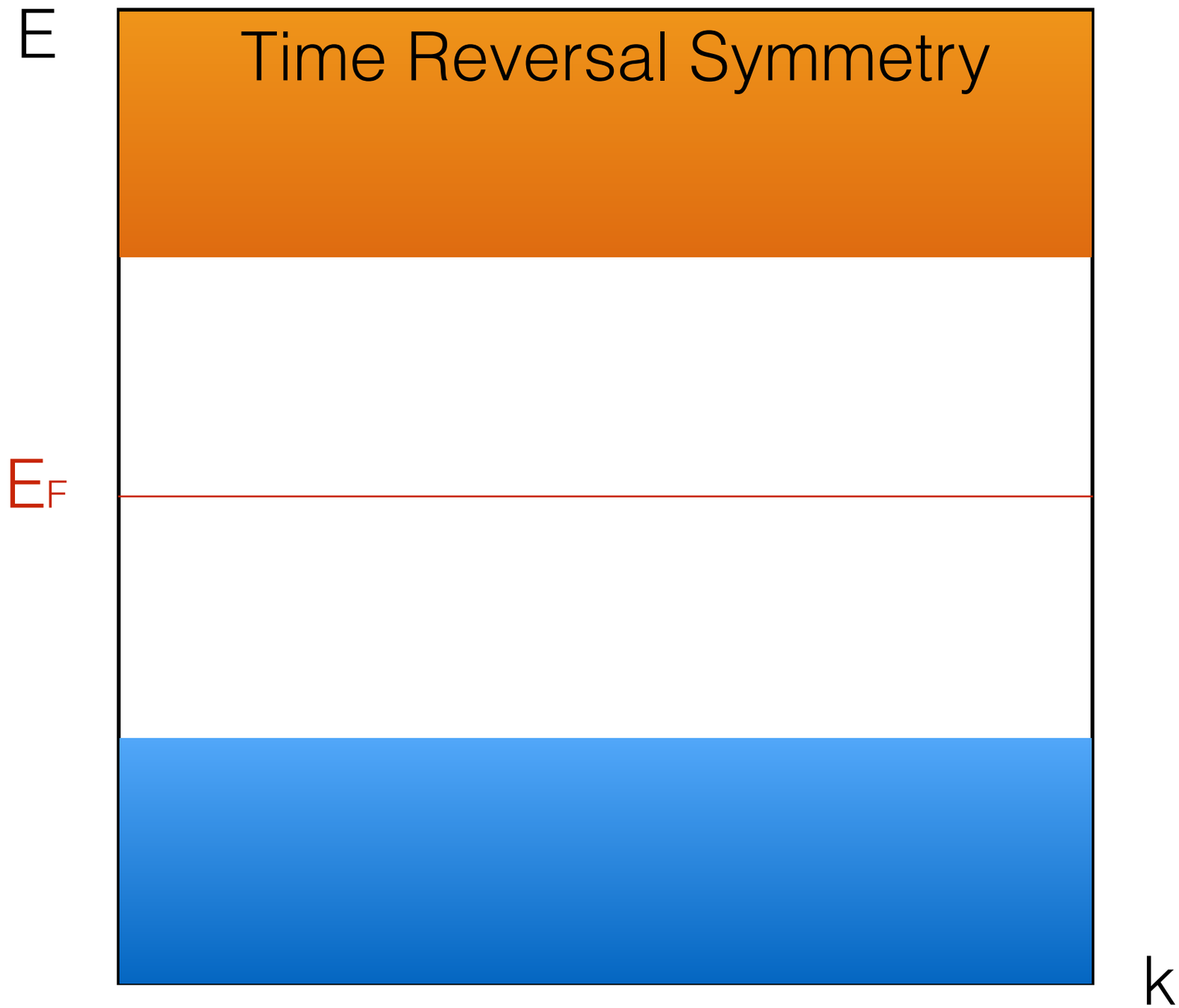
Quantum Spin Hall Effect

Two copies of the Haldane model



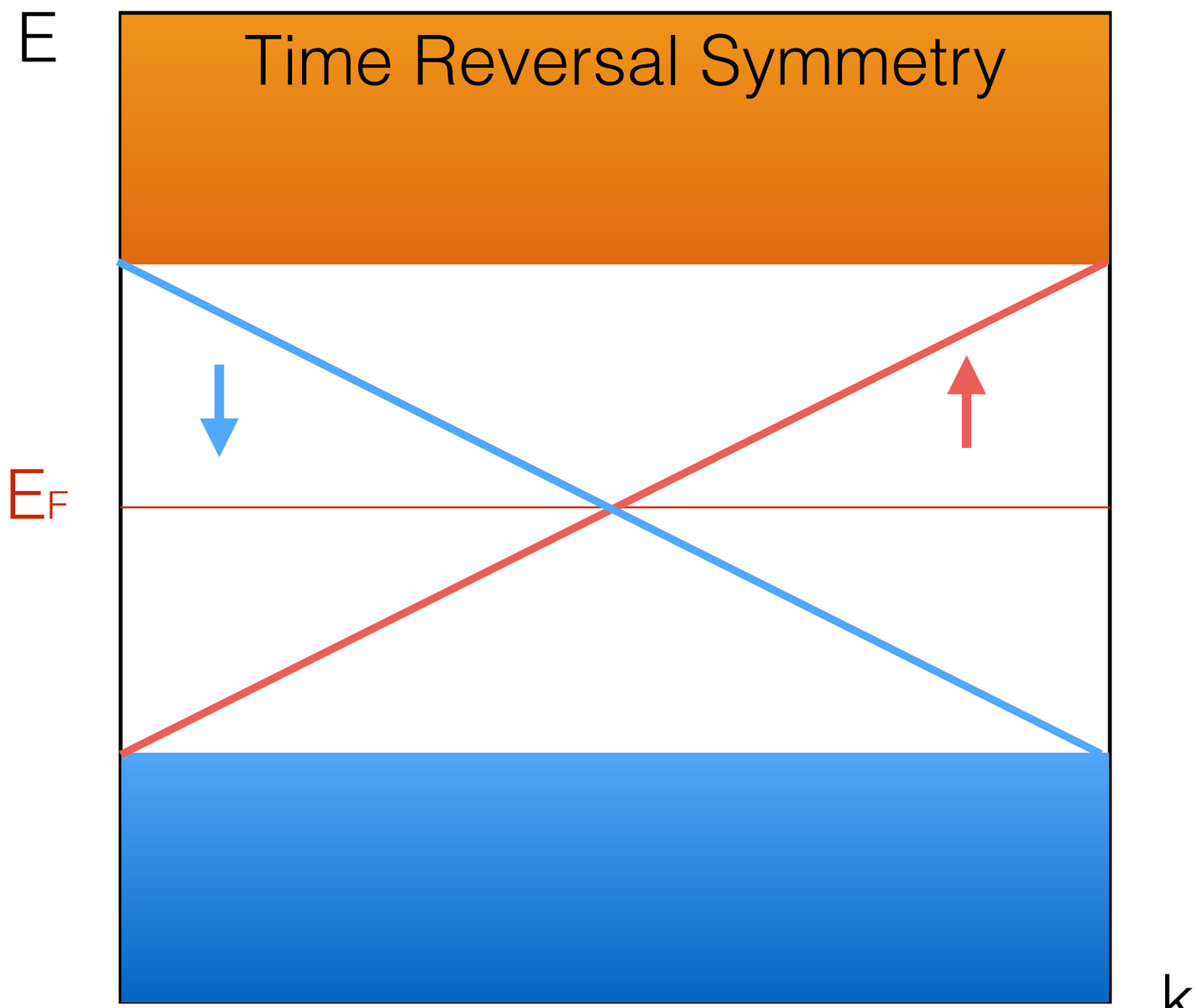
Spin Quantum Hall Effect

Kane and Mele model



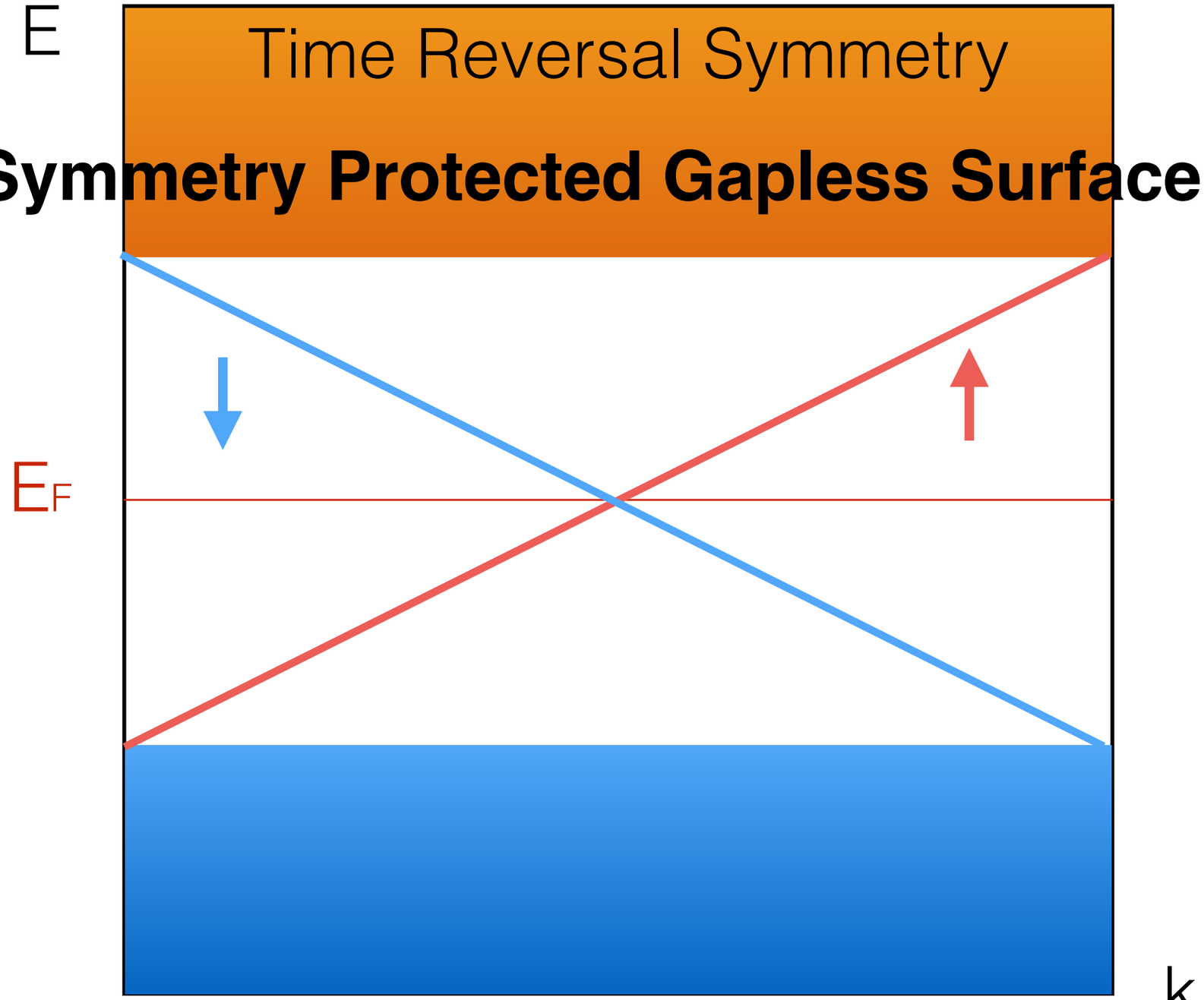
Spin Quantum Hall Effect

Kane and Mele model



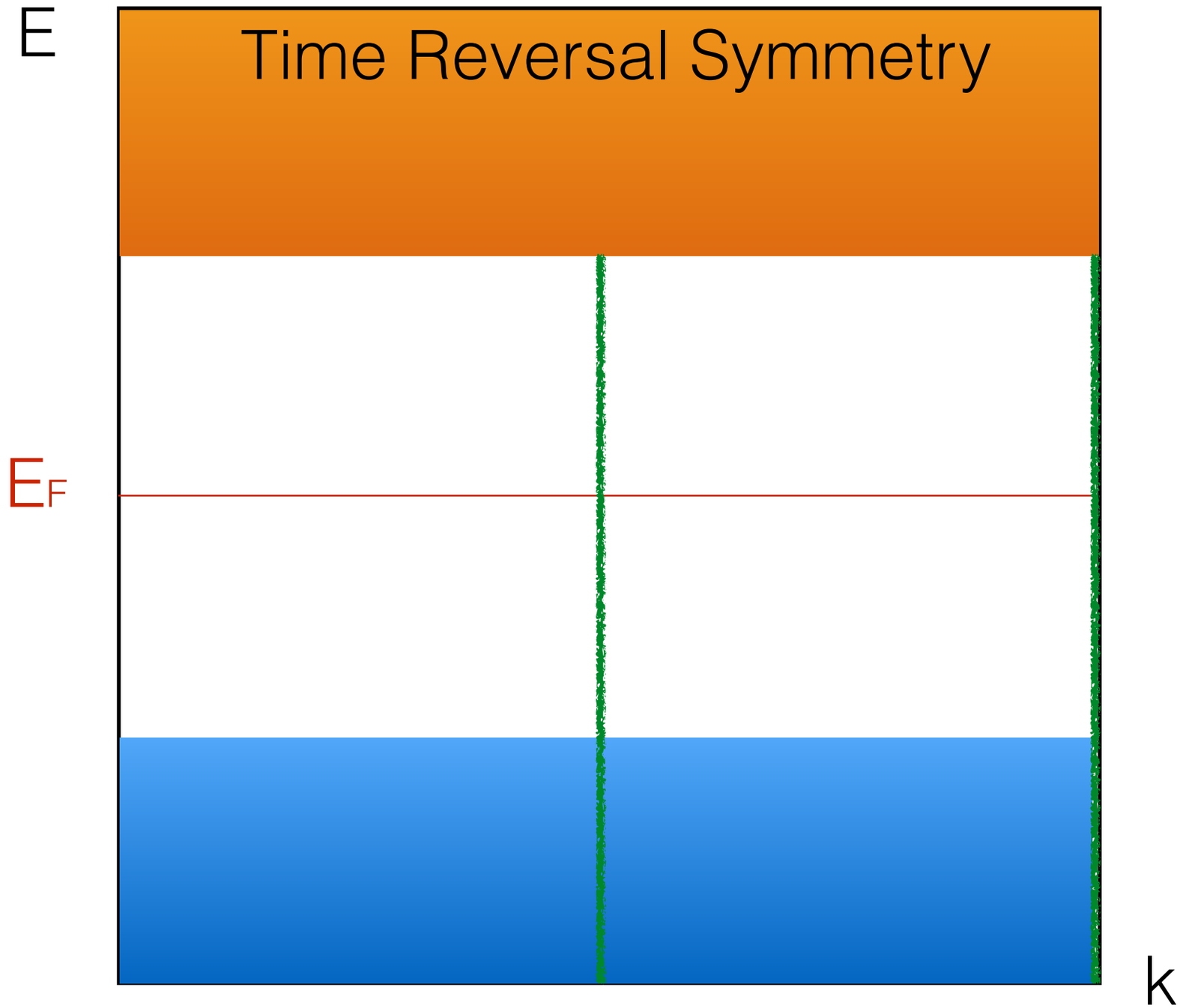
Spin Quantum Hall Effect

Kane and Mele model



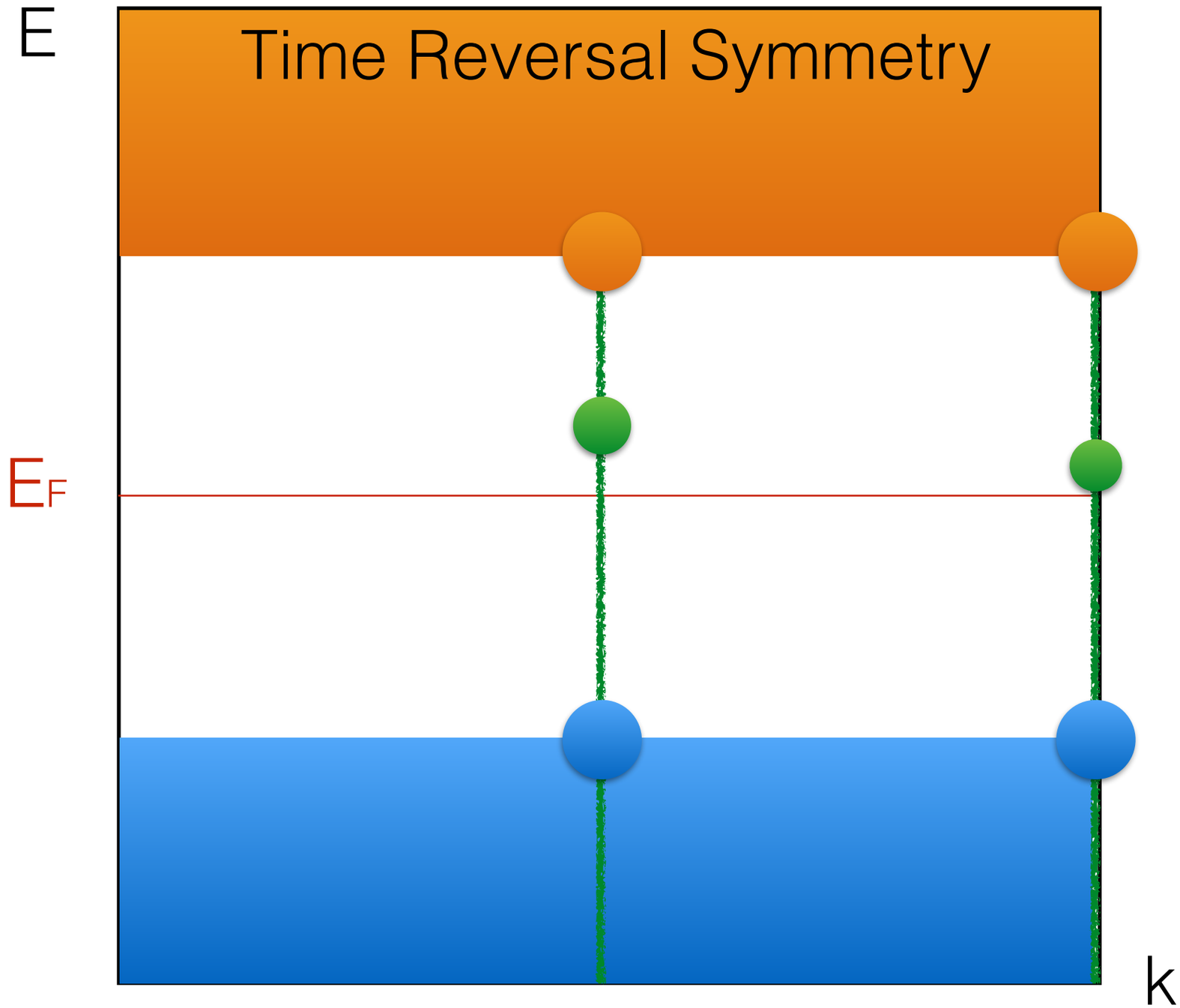
Kramers Degeneracy

At TRIM, there should be at least two fold degeneracy



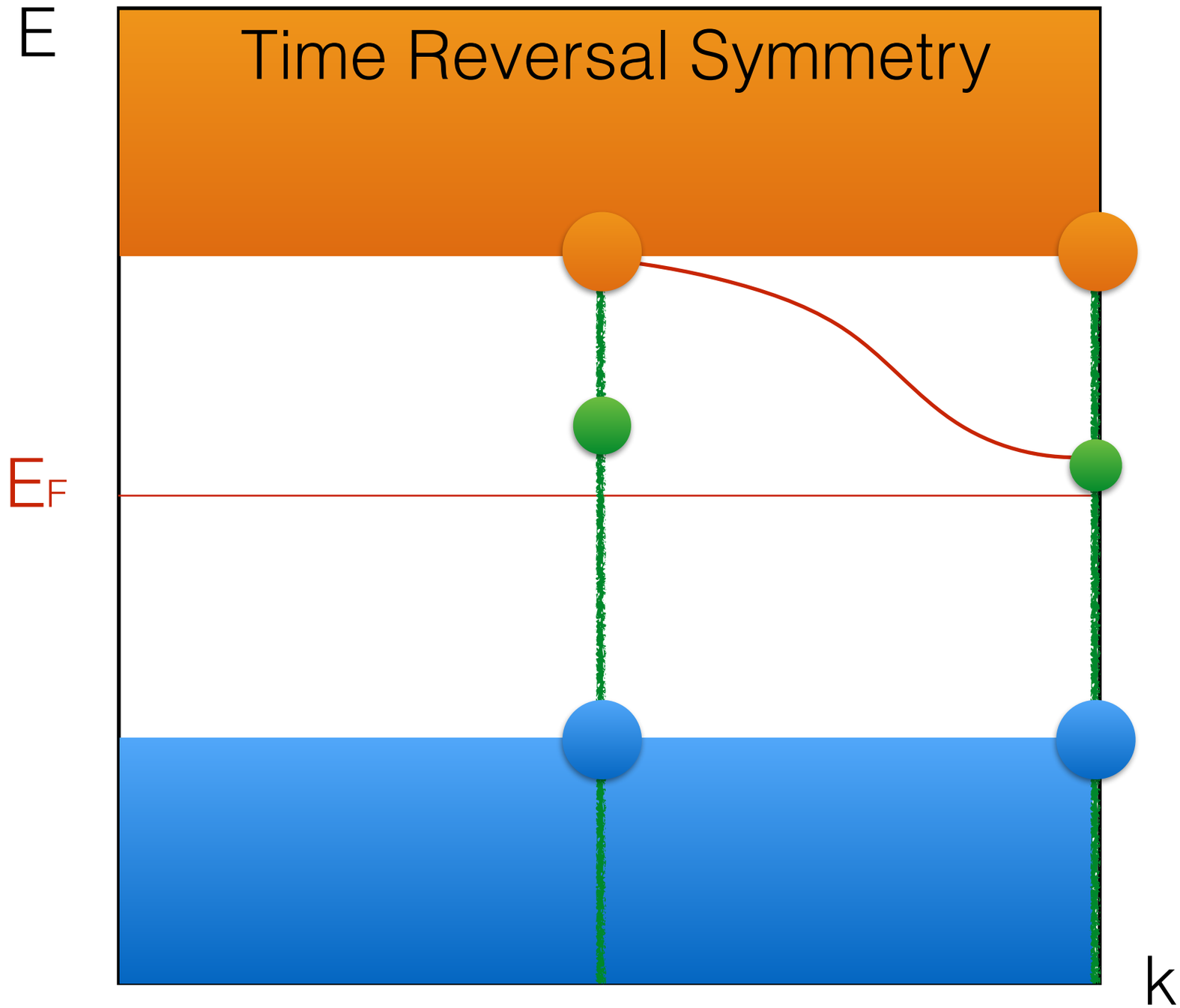
Kramers Degeneracy

At TRIM, there should be at least two fold degeneracy



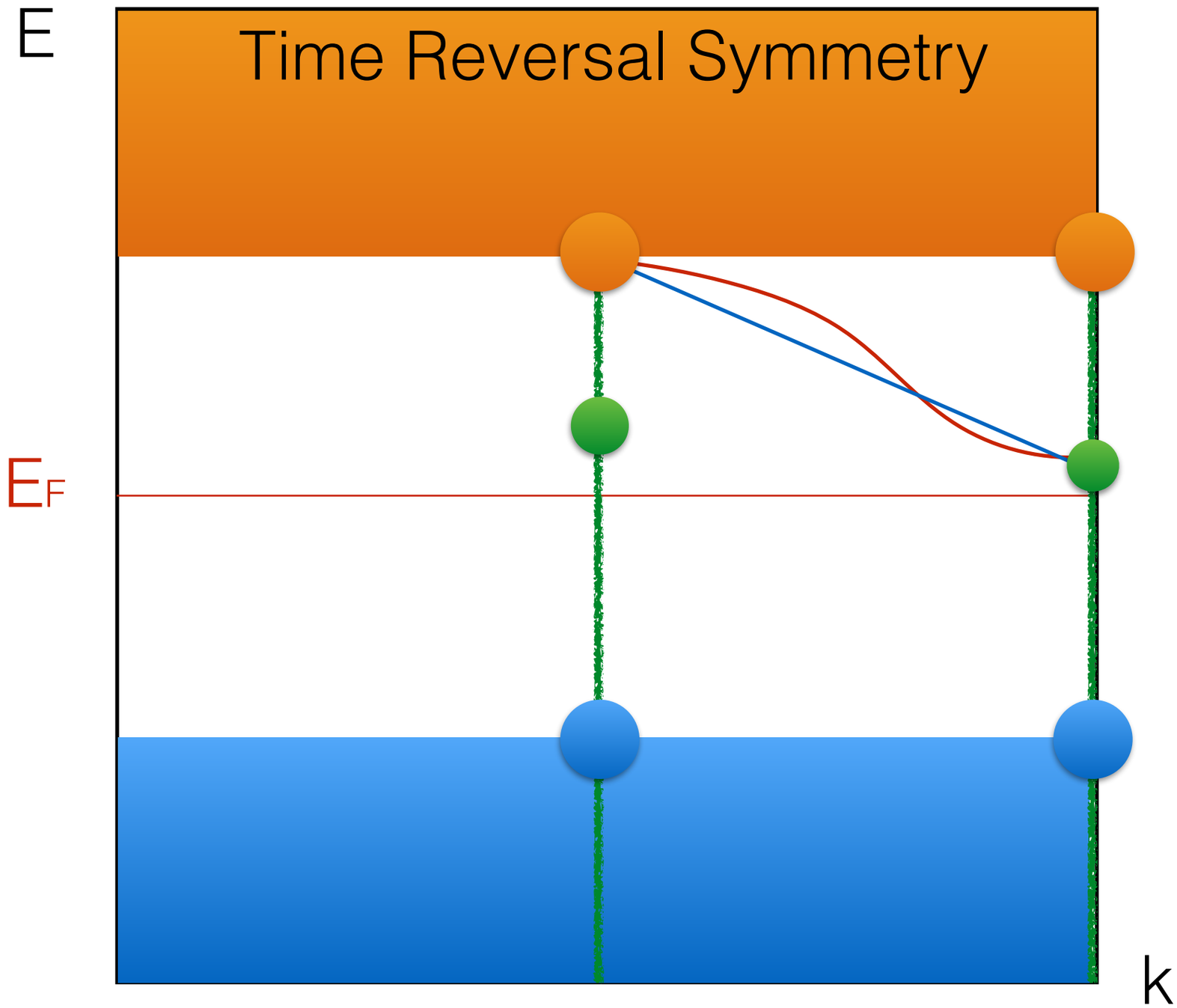
Kramers Degeneracy

At TRIM, there should be at least two fold degeneracy



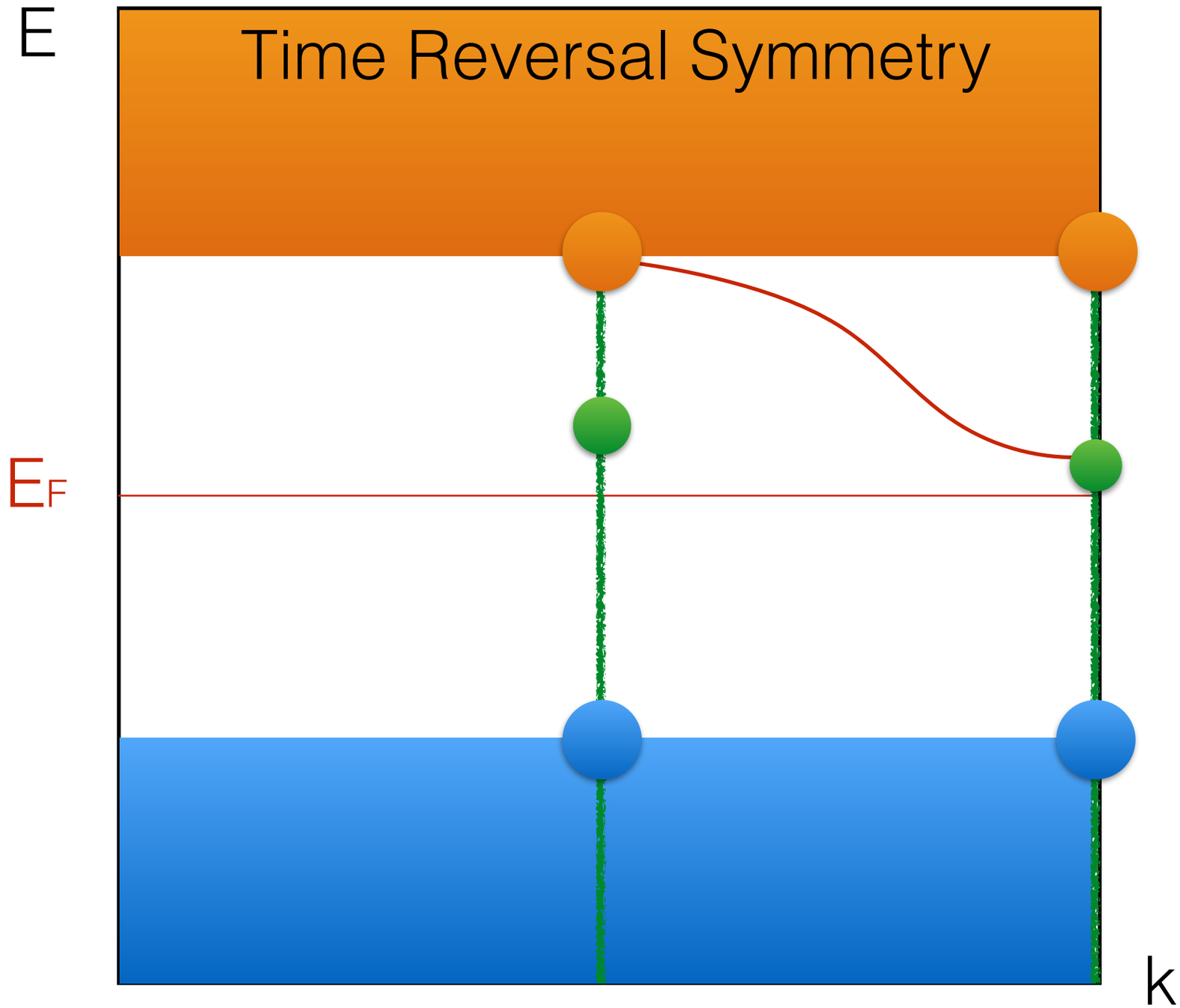
Kramers Degeneracy

At TRIM, there should be at least two fold degeneracy



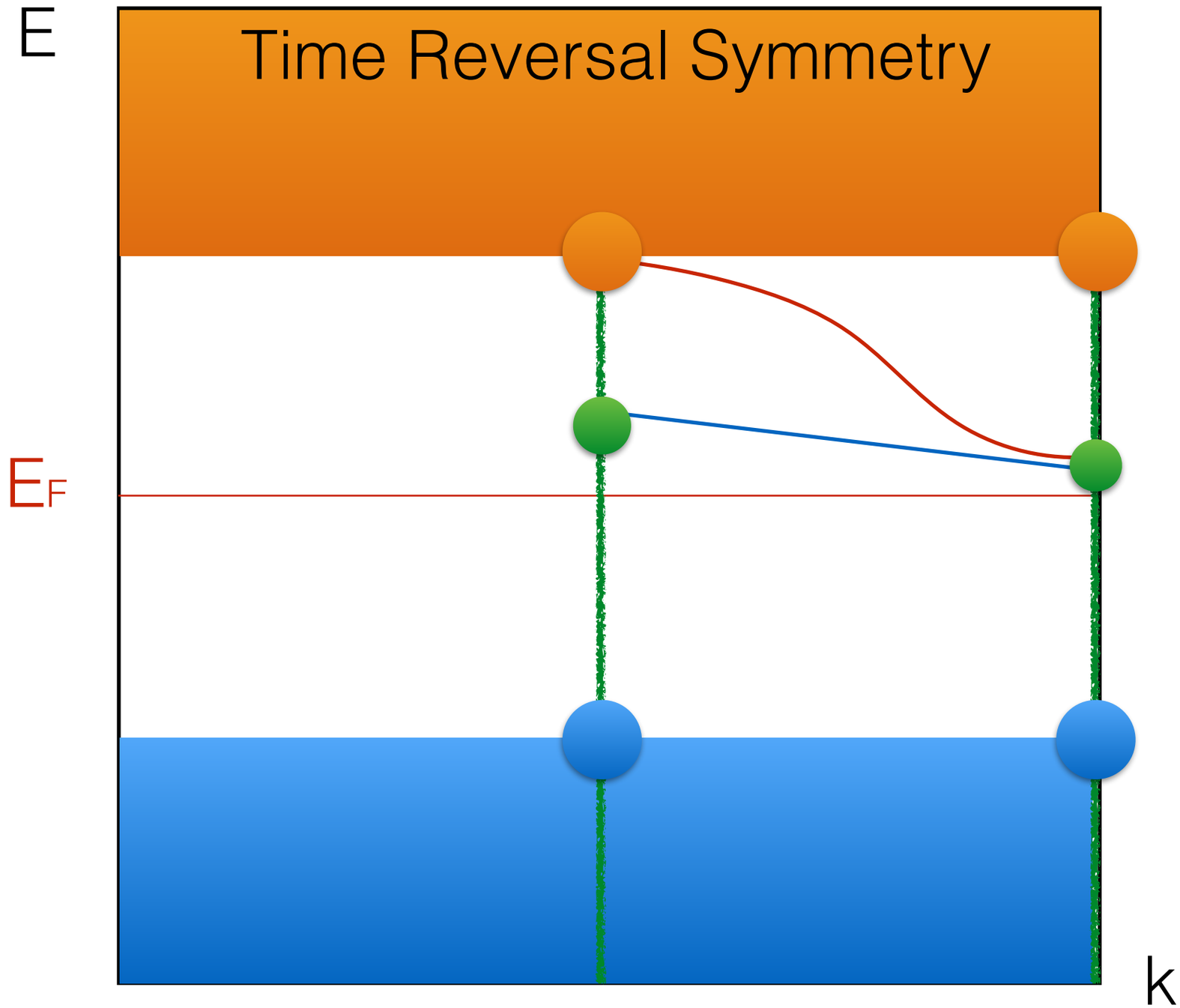
Kramers Degeneracy

At TRIM, there should be at least two fold degeneracy



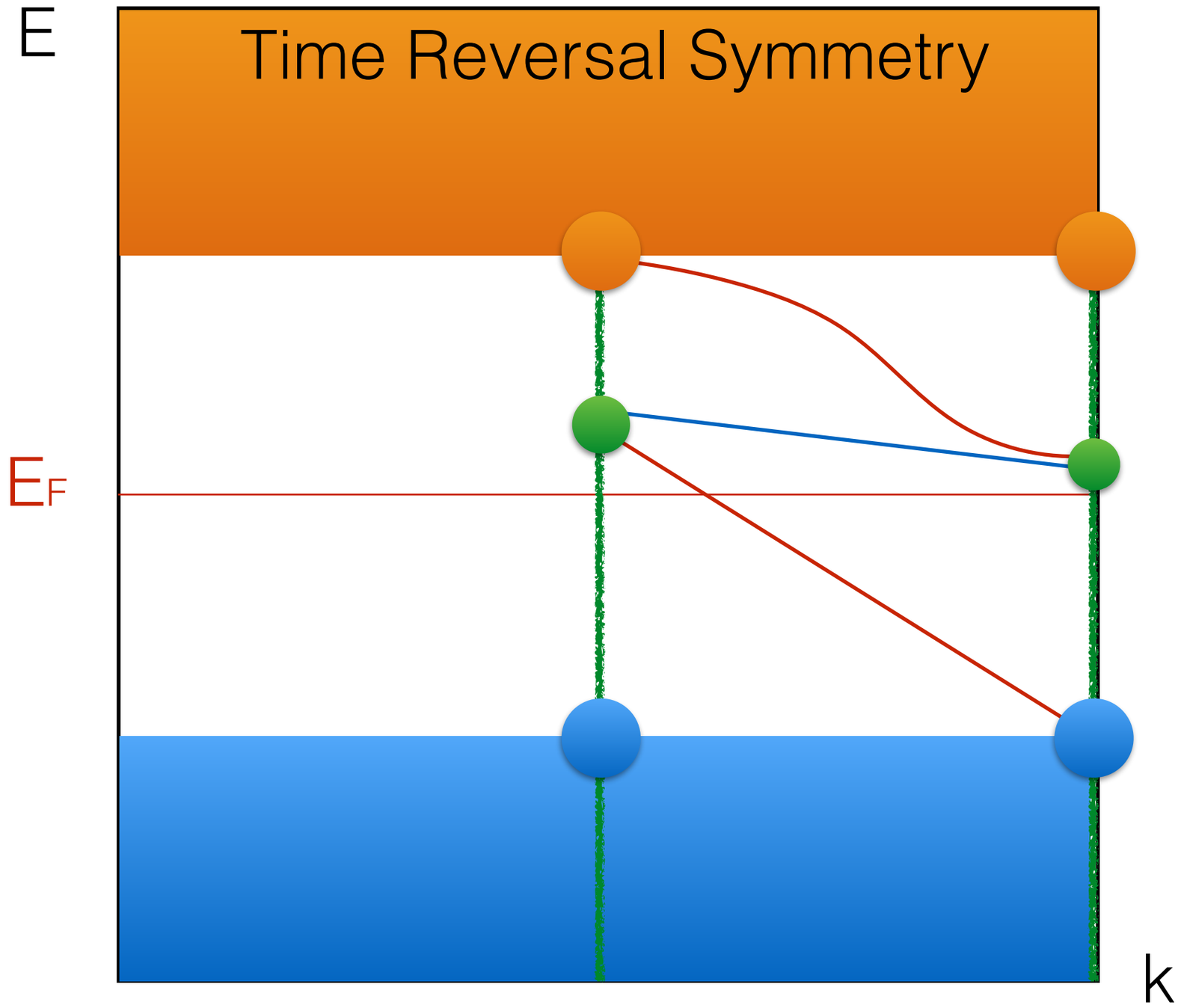
Kramers Degeneracy

At TRIM, there should be at least two fold degeneracy



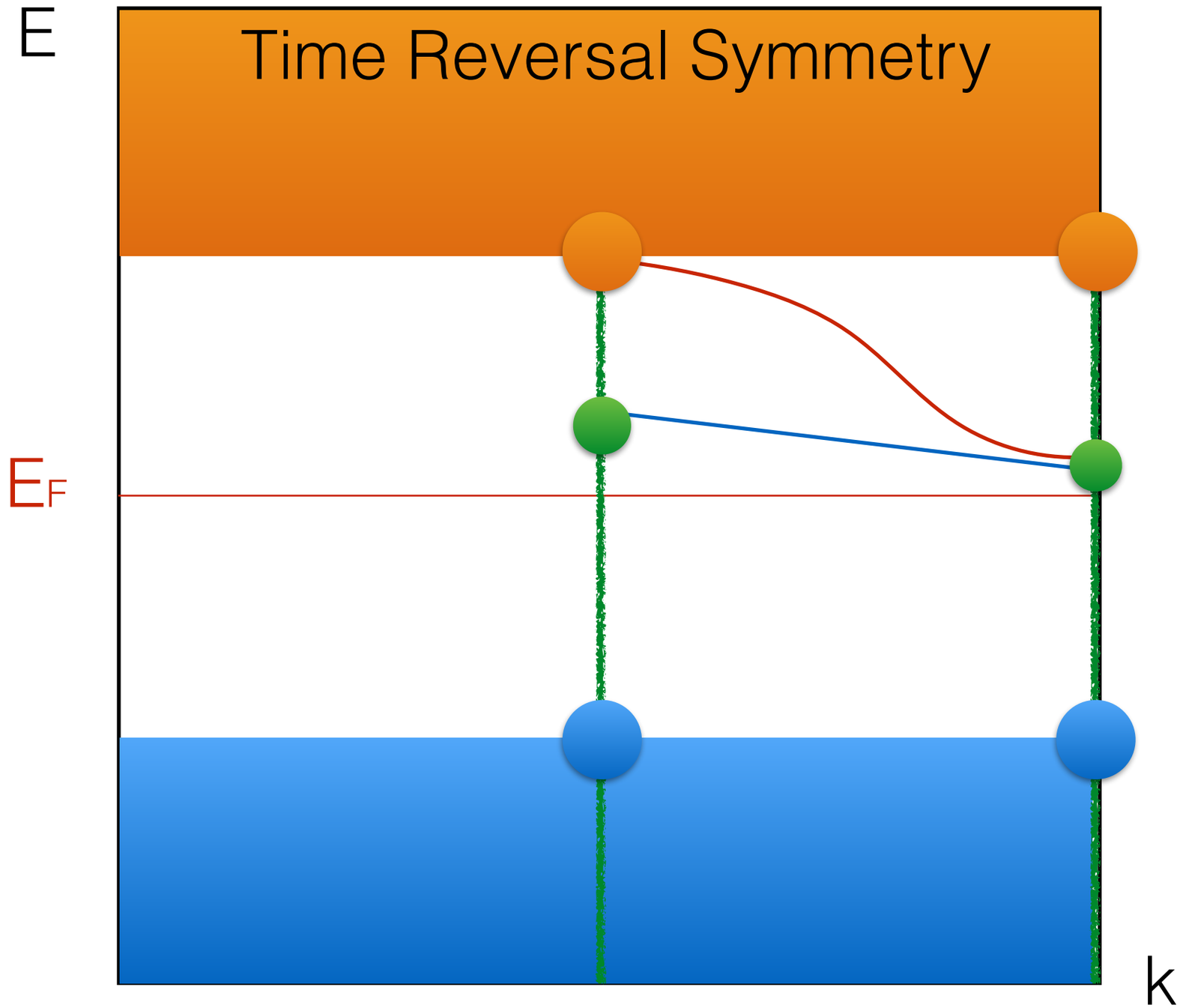
Kramers Degeneracy

At TRIM, there should be at least two fold degeneracy



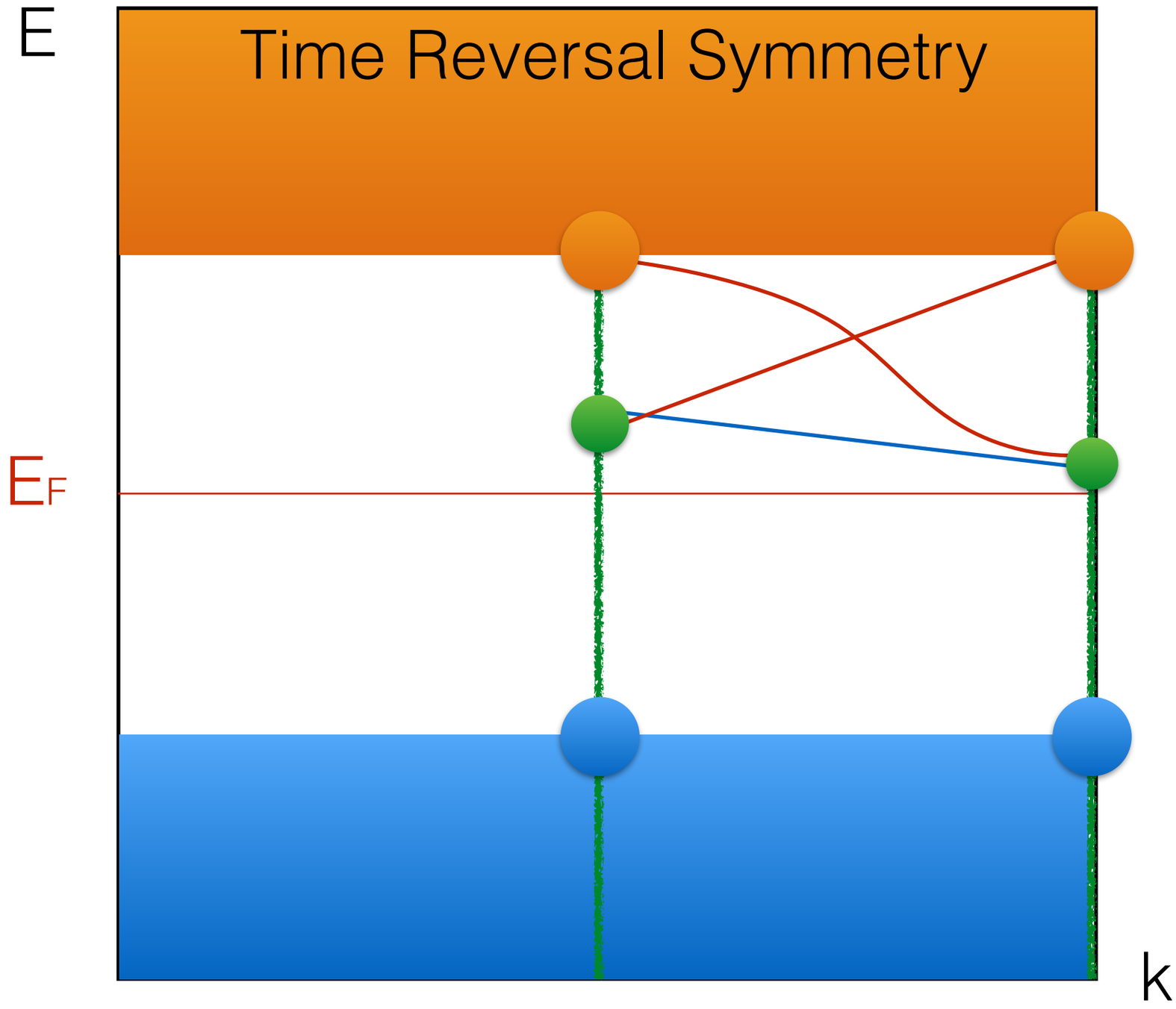
Kramers Degeneracy

At TRIM, there should be at least two fold degeneracy



Kramers Degeneracy

At TRIM, there should be at least two fold degeneracy



Summary

Quantum Hall Effect : Topological point of view

Simple model : Haldane Model

Time reversal symmetry

Kane and Mele Model

Z2 Topological Invariant