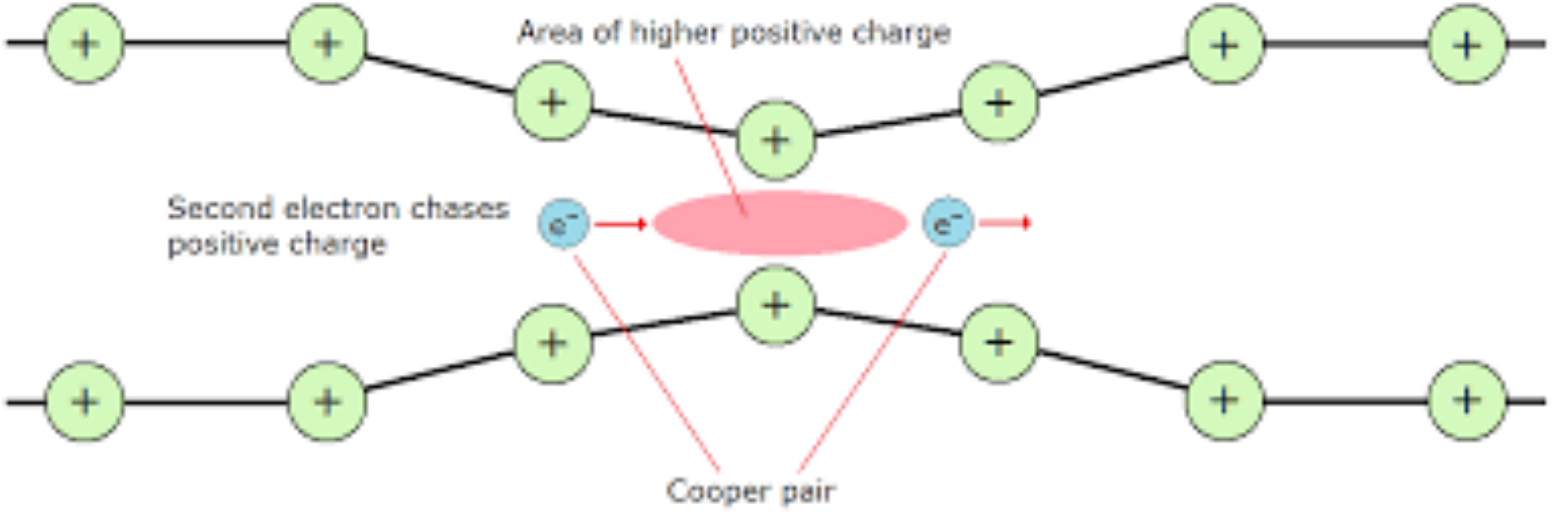
Weak measurement of superconducting qubit in superconducting circuit QED systems

Wednesday, August 14th

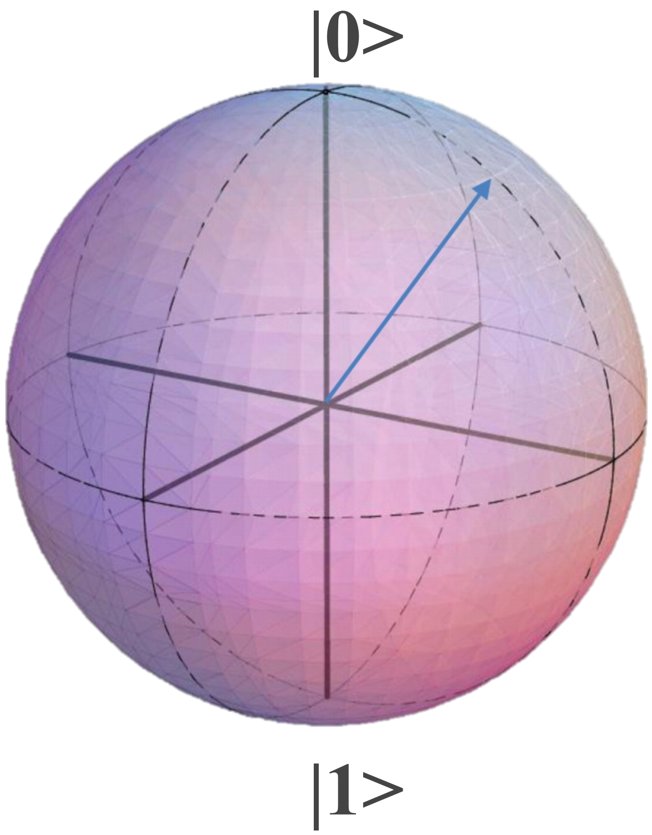
Summary

- Notions
- Hamiltonian
- Qubit read-out

Superconducting wires

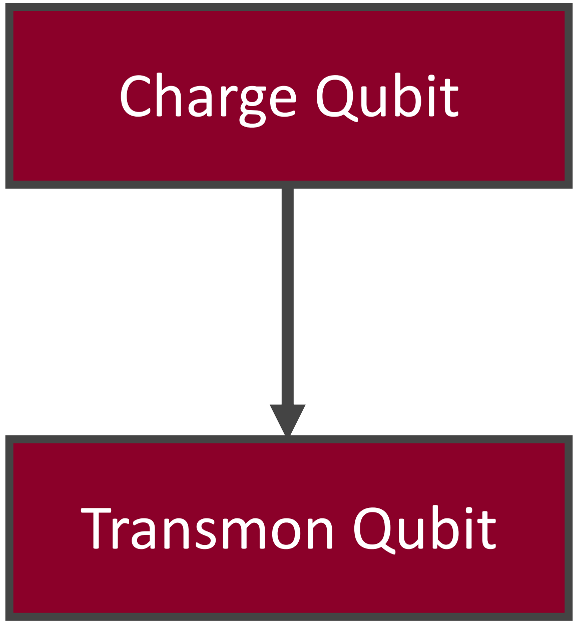


Qubit

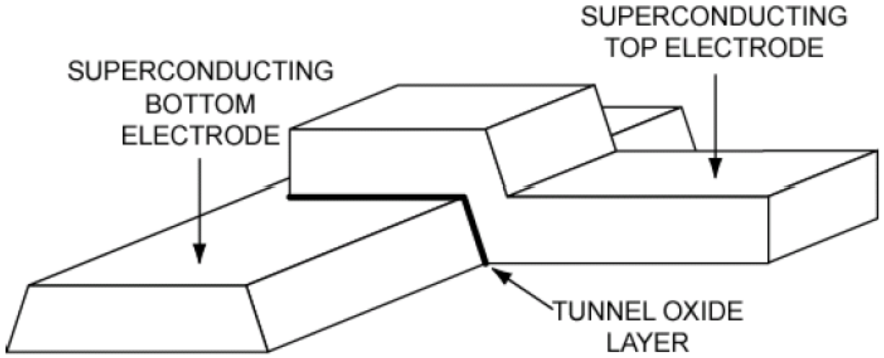


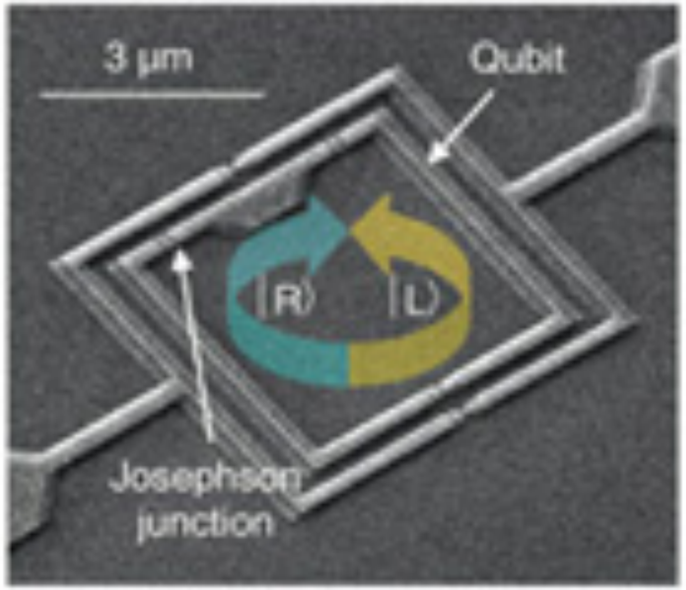
$$|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

Qubit



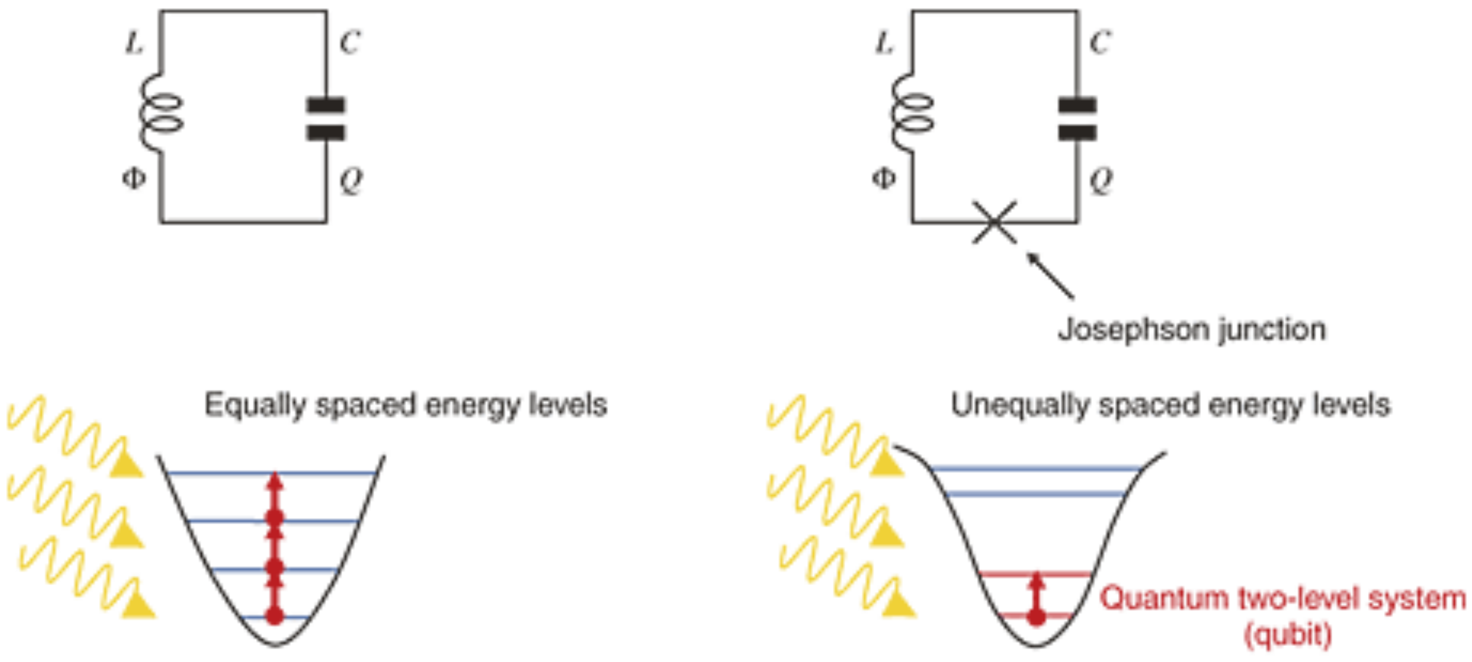
Josephson Junction





=> Tunelling though the insulator due to Josephson effect

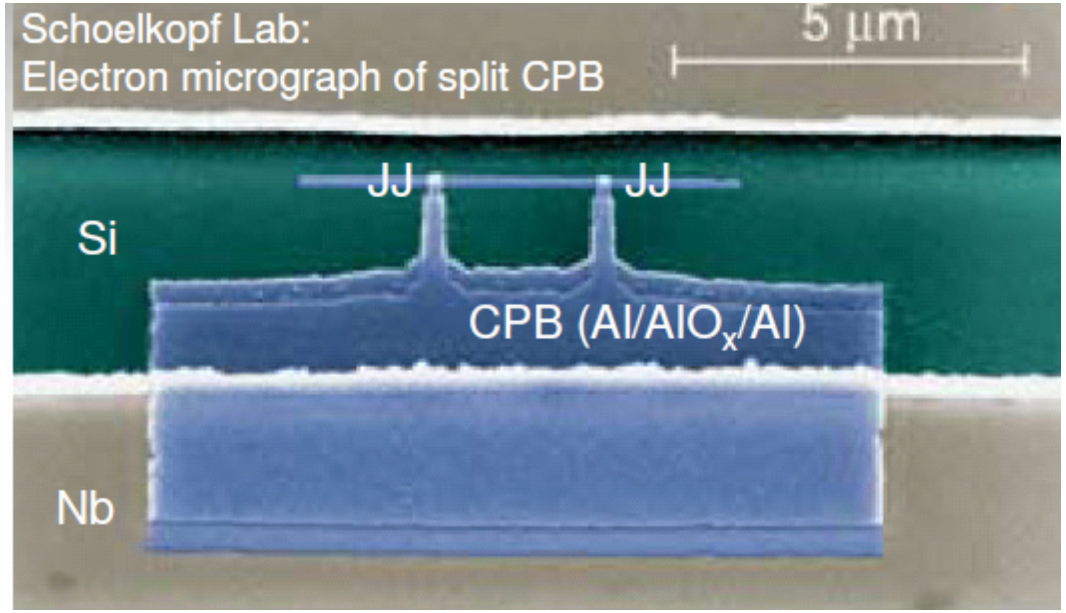
Charge Qubit



(a) LC-circuit without Josephson junction (b) LC-circuit with Josephson junction

N Cooper pairs in the Cooper pair box

Charge Qubit



N Cooper pairs in the Cooper pair box

Energy of the Cooper pair box

$$E = \frac{Q^2}{C}$$

Capacitance equation

E : Energy
Q : Charge
C : Capacity

$$E = \boxed{\frac{(2e)^2}{2C}} \mathrm{N}^2$$

Ec

Energy of the Cooper pair box

|0>

|1>

$$E = Ec\ \mathrm{N}^2$$

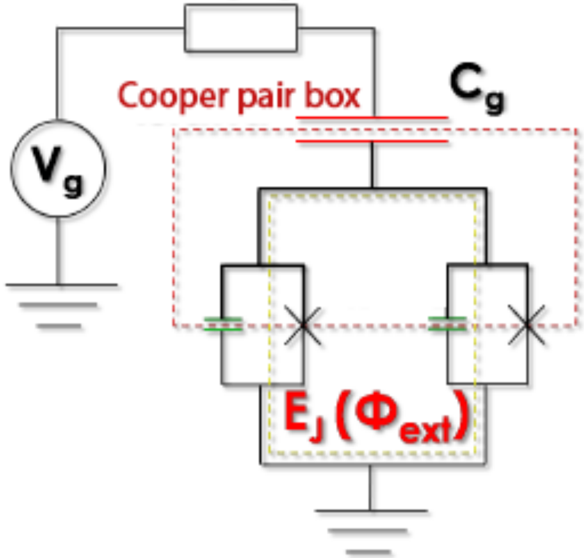
$$E = Ec\ (\mathrm{N}+1)^2$$

=> Necessity of a Josephson Junction

Hamiltonian

$$\hat{H} = Ec.\hat{N} - E_J.\cos(\hat{\phi})$$

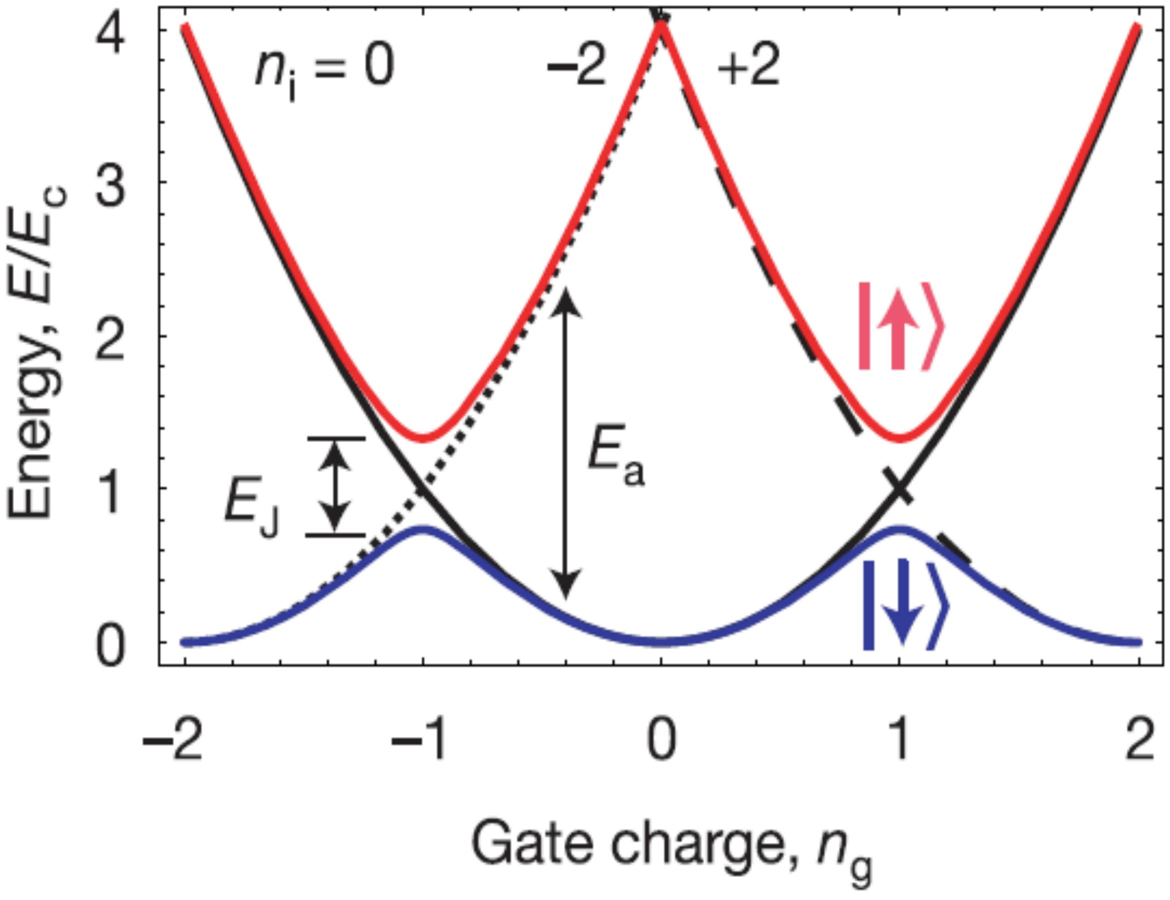
Hamiltonian



$$Cg = \frac{Ng.2e}{Vg}$$

$$\hat{H} = Ec.(\hat{N} - Ng) - E_J.\cos(\hat{\phi})$$

Energy level of the system



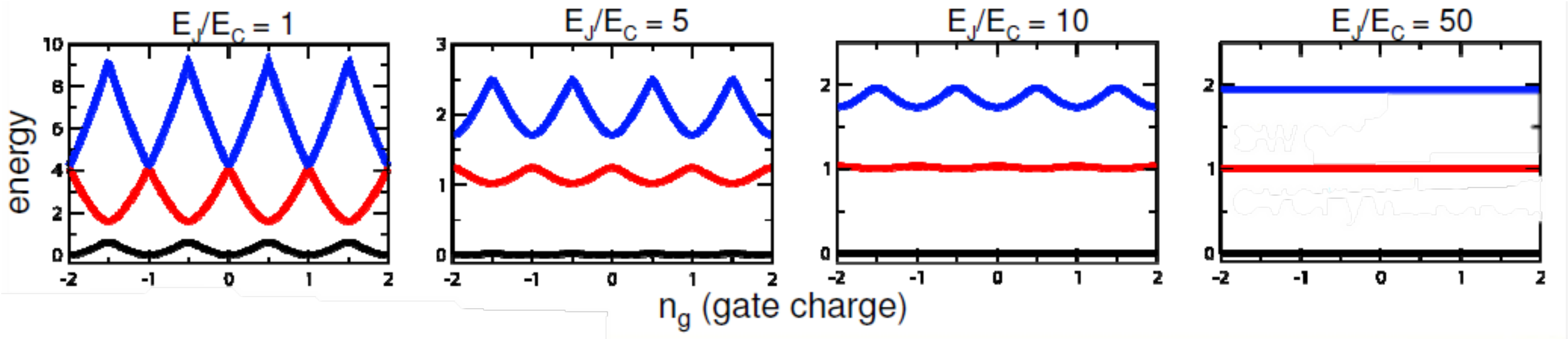
Con about charge qubit

Affected by random charges in the system

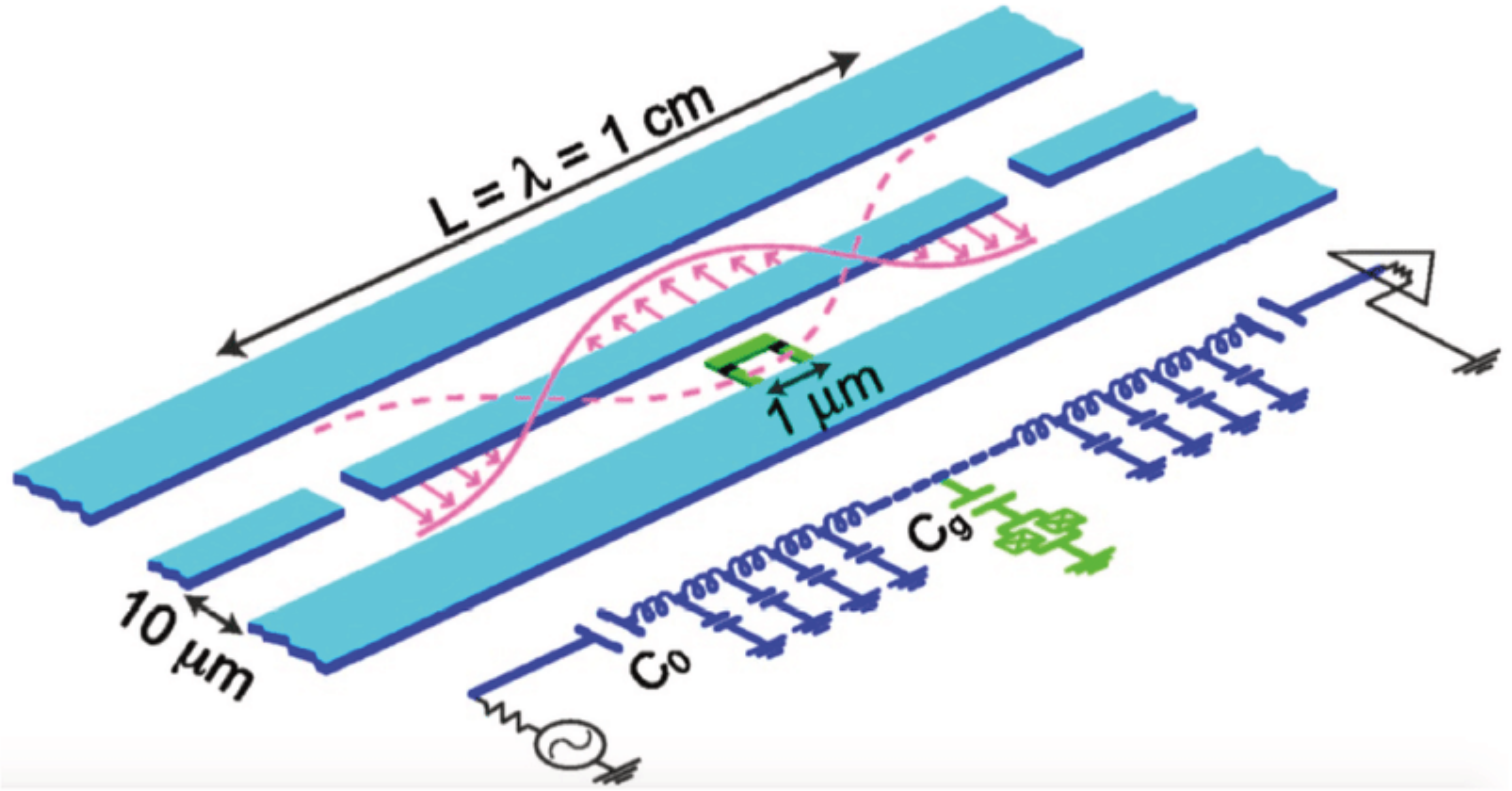
Transmon Qubit

Larger capacitor

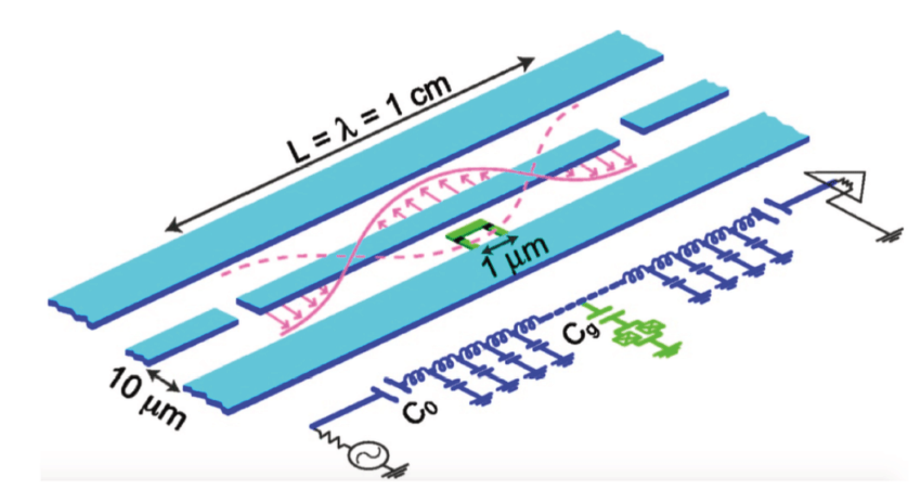
Reduced sensitivity to charge noise

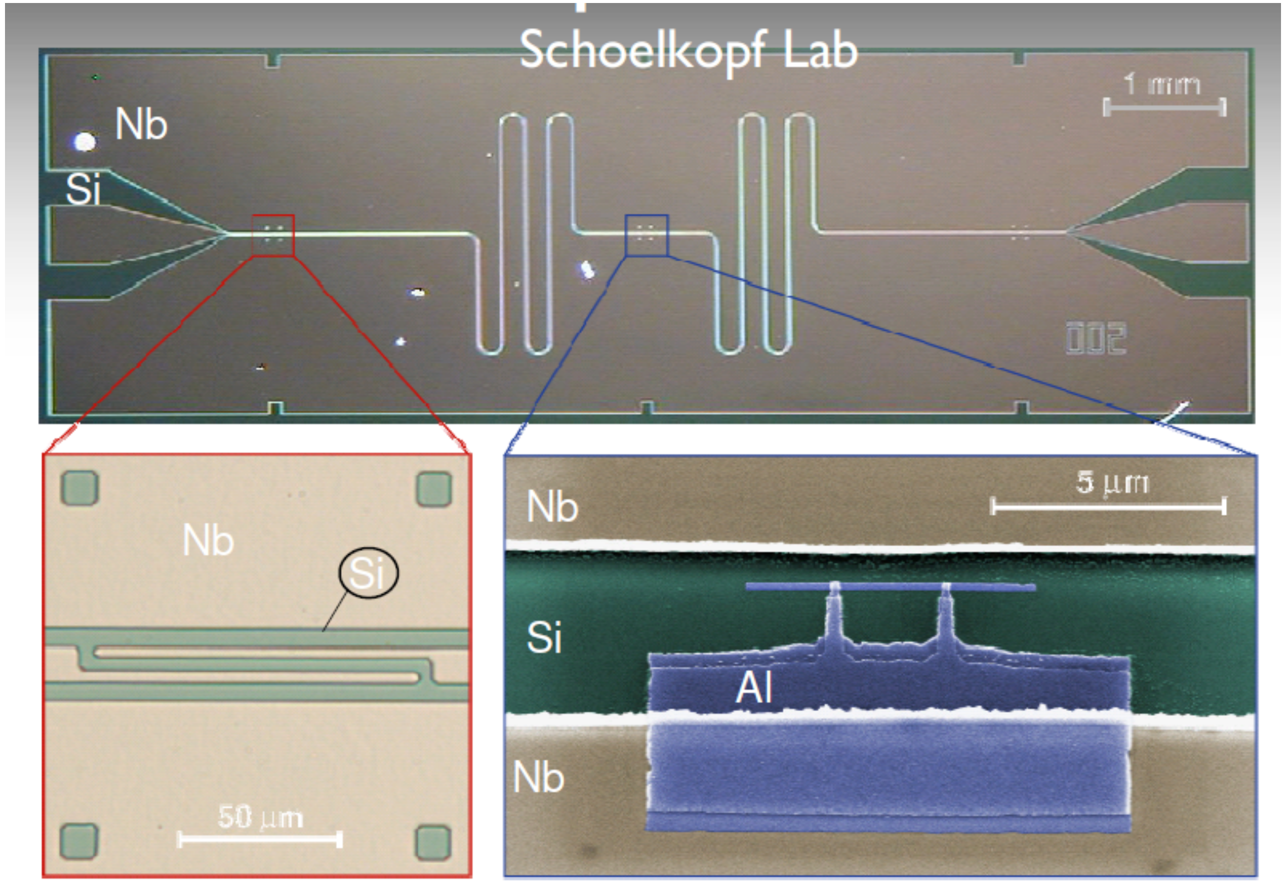


Superconducting circuit



Superconducting circuit





Rabi Hamiltonian

Resonator

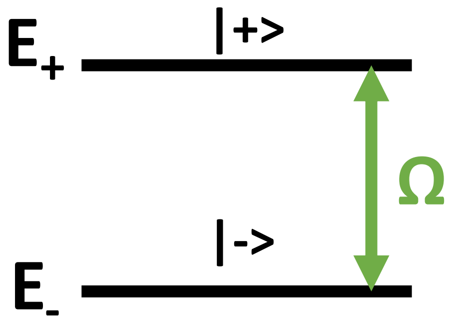
Renonator ⇔ LC Circuit

$$\hbar\omega\left(a^{+}a+\frac{1}{2}\right)\psi(x)=E\psi(x)$$

$$H_{res}=\omega a^{+}a$$

Rabi Hamiltonian

Two level system

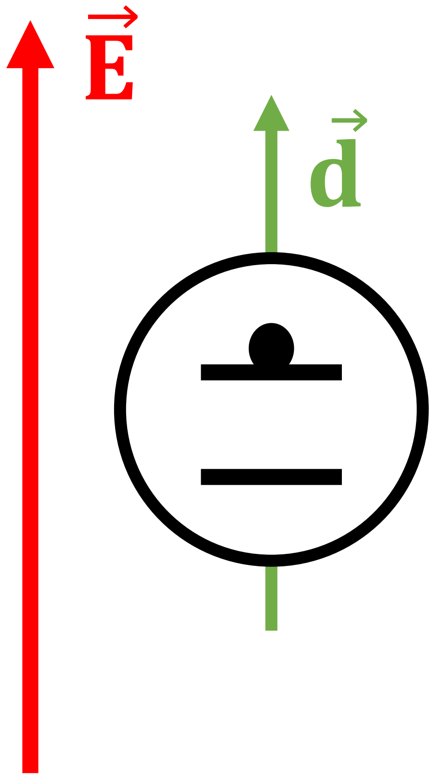


$$H = E_+|+\rangle\langle+| + E_-|-\rangle\langle-|$$

$$H_{TLS} = \frac{1}{2}\Omega\sigma_Z$$

Rabi Hamiltonian

Qubit-Resonator coupling

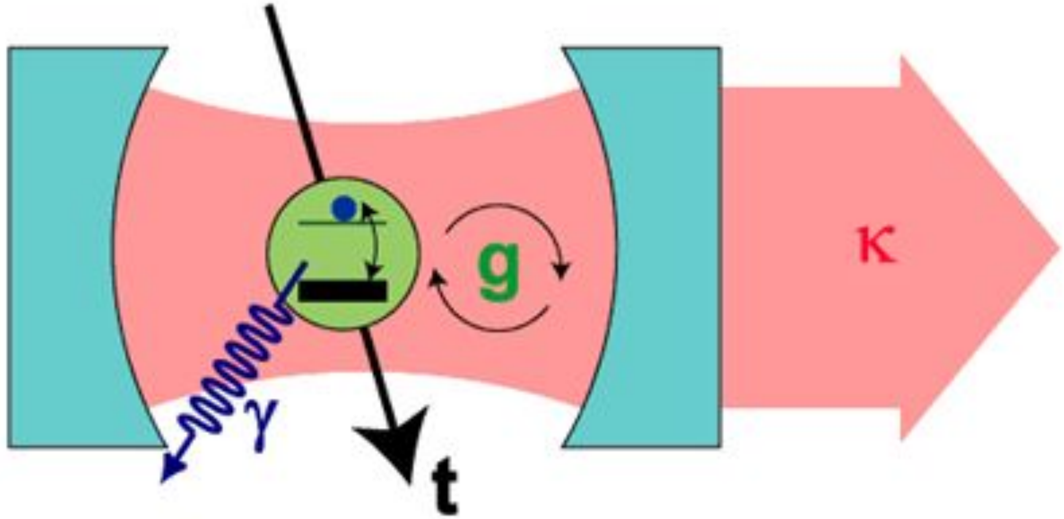


$$H_{int} = -\vec{d}.E(t)$$

$$H_{int} = g(\sigma_+ + \sigma_-)(a^+ + a)$$

Rabi Hamiltonian

Energy loss



Photon loss rate from the resonator κ → Hκ

Photon loss rate from the two level system γ → Hγ

Rabi Hamiltonian

$$H_{Rabi} = \omega(a^{+}a) + \frac{1}{2}\Omega\sigma^{Z} + g(a^{+} + a)(\sigma^{+} + \sigma^{-}) + H_{\kappa} + H_{\gamma}$$

Jaymes-Cummings Hamiltonian

$$H_{Rabi} = \omega(a^{+}a) + \frac{1}{2}\Omega\sigma^{Z} + g(a^{+} + a)(\sigma^{+} + \sigma^{-}) + \cancel{H_{\kappa}} + \cancel{H_{\gamma}}$$

Jaymes-Cummings Hamiltonian

Rotating wave approximation (RWA) :

Near resonance : Δ=Ω-ω

$$H_{int} = g(\sigma_+ + \sigma_-)(a^+ + a)$$

$$H_{int} = g(a^+\sigma_- + a\sigma_+)$$

Jaymes-Cummings Hamiltonian

$$\boldsymbol{H_{RWA} = \omega(a^{+}a) + \frac{1}{2}\Omega\sigma^{Z} + g(a^{+}\sigma_{-} + a\sigma_{+})}$$

Jaymes-Cummings Hamiltonian

$$\boldsymbol{H_{RWA} = \omega(a^{+}a) + \frac{1}{2}\Omega\sigma^{Z} + g(a^{+}\sigma_{-} + a\sigma_{+})}$$

$$\{\ |O,\uparrow\rangle, |O,\downarrow\rangle, |1,\uparrow\rangle, |1,\downarrow\rangle\ \}$$

$$H_{RWA}|O,\uparrow\rangle = \frac{\Omega}{2}|O,\uparrow\rangle + \mathrm{g}\,|1,\downarrow\rangle$$

$$H_{RWA}|O,\downarrow\rangle = -\frac{\Omega}{2}|O,\downarrow\rangle$$

$$H_{RWA}|1,\uparrow\rangle = \frac{\Omega}{2}|1,\uparrow\rangle + \mathrm{g}\sqrt{2}\,|2,\downarrow\rangle$$

$$H_{RWA}|1,\downarrow\rangle = \left(-\frac{\Omega}{2} + \omega\right)|1,\downarrow\rangle + \mathrm{g}\,|O,\uparrow\rangle$$

$$H_{RWA} = \begin{bmatrix} \frac{\Omega}{2} & g \\ g & -\frac{\Omega}{2} + \omega \end{bmatrix}$$

Jaymes-Cummings Hamiltonian

$$\boldsymbol{H_{RWA} = \omega(a^{+}a) + \frac{1}{2}\Omega\sigma^{Z} + g(a^{+}\sigma_{-} + a\sigma_{+})}$$

$$H^{RWA^{(n)}} = \begin{bmatrix} \frac{\Omega}{2} + \omega n & g\sqrt{n+1} \\ g\sqrt{n+1} & -\frac{\Omega}{2} + \omega(n+1) \end{bmatrix}$$

Research of the energy ≡ Research of the eigenstates

$$H^{RWA^{(n)}} = B_x.\sigma^x + B_y.\sigma^y + B_z.\sigma^z$$

$$E_{\pm}^{(n)} = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

Jaymes-Cummings Hamiltonian

$$\boldsymbol{H_{RWA} = \omega(a^{+}a) + \frac{1}{2}\Omega\sigma^{Z} + g(a^{+}\sigma_{-} + a\sigma_{+})}$$

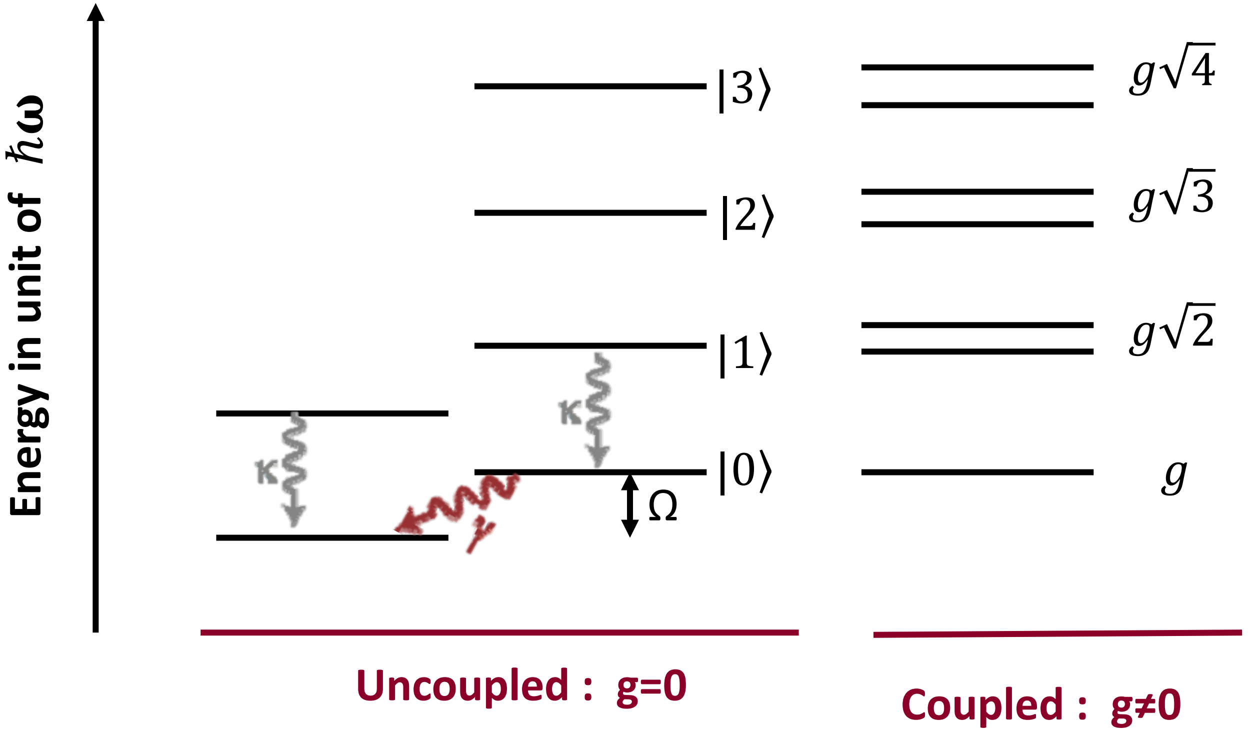
$$E_{\pm}{}^{(n)} = \left(n + \frac{1}{2}\right)\omega \pm \sqrt{g^{2}(n+1) + \left(\frac{\Delta}{2}\right)^{2}}$$

On resonance : $E_{\pm}{}^{(n)} = \left(n + \frac{1}{2}\right)\omega \pm g\sqrt{n+1}$

Jaymes-Cummings Hamiltonian

$$E_{\pm}^{(n)} = \left(n + \frac{1}{2}\right)\omega \pm g\sqrt{n+1}$$

Non degenerate case

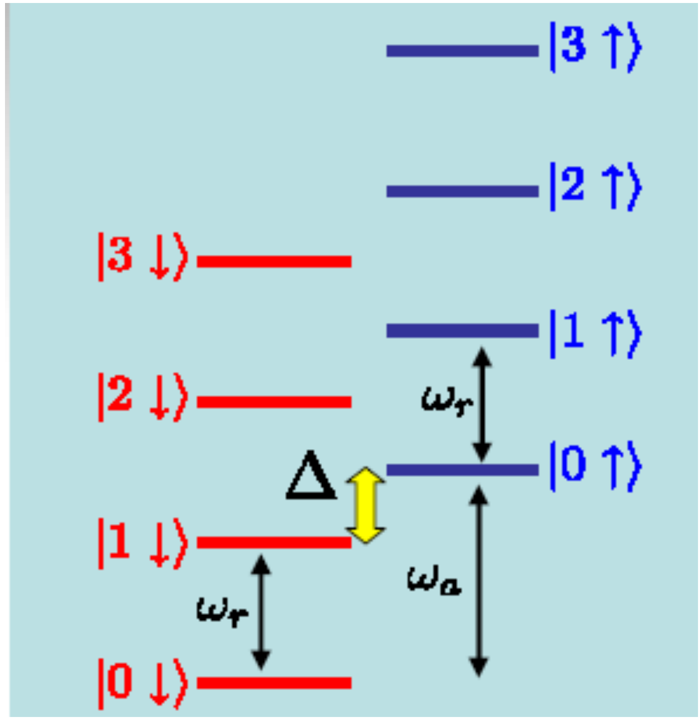


Dispersive Hamiltonian

+ : Lifetime enhancement

Dispersive limit : $g \ll |\Delta|$

Dispersive Hamiltonian



Dispersive Hamiltonian

Unitary Transformation (Baker–Campbell–Hausdorff formula)

$$D^{+}H_{RWA}D = H_{RWA} + \lambda\,[H_{RWA}, X_{-}] + \frac{\lambda^{2}}{2}\left[[H_{RWA}, X_{-}], X_{-}\right] + \cdots + \frac{\lambda^{n}}{n!}\Big[\underbrace{[[H_{RWA}, X_{-}], X_{-}], \ldots, X_{-}}_{\text{n times}}\Big]$$

$\lambda = \frac{g}{\lambda}$ and $D = e^{\lambda X_{-}}$ with $X_{\pm} = a^{+}\sigma_{-} \pm a\sigma_{+}$

Dispersive Hamiltonian

$H_{Disp} = D^{+} H_{RWA} D$ on second order in λ

$$\boldsymbol{H_{Disp} = \frac{1}{2}\Omega\sigma^{Z} + \frac{g^{2}}{2\Delta}\sigma^{Z} + \left(\omega + \frac{g^{2}}{\Delta}\sigma^{Z}\right)a^{+}a}$$

Dispersive Hamiltonian

$H_{Disp} = D^{+} H_{RWA} D$ on second order in λ

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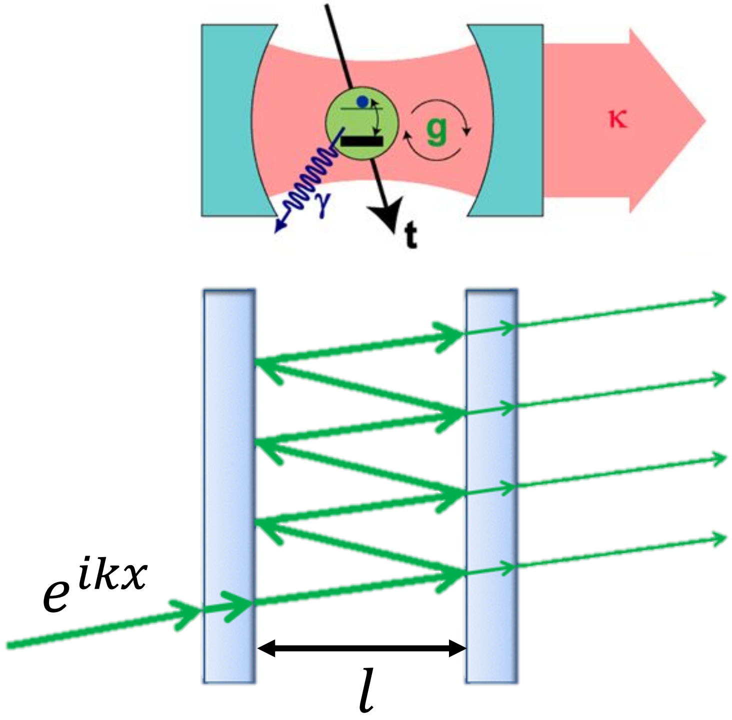
Qubit-Readout
Form of the transmission

Fabry Perot Interferometer



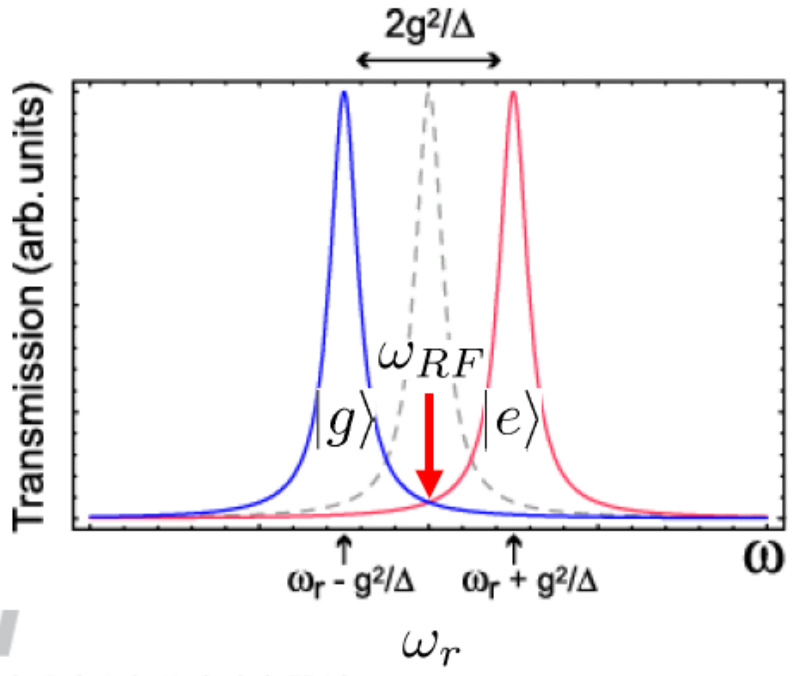
$$Transmission \propto \frac{1}{1 - r^2 (e^{ikl})^2}$$

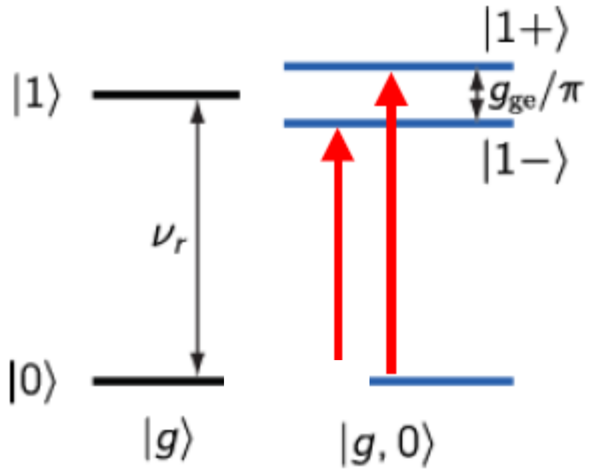
=> Lorentzian

Qubit read-out

$$H_{Disp} = \frac{1}{2}\Omega\sigma^Z + \frac{g^2}{2\Delta}\sigma^Z + \left(\omega + \frac{g^2}{\Delta}\sigma^Z\right)a^+a$$

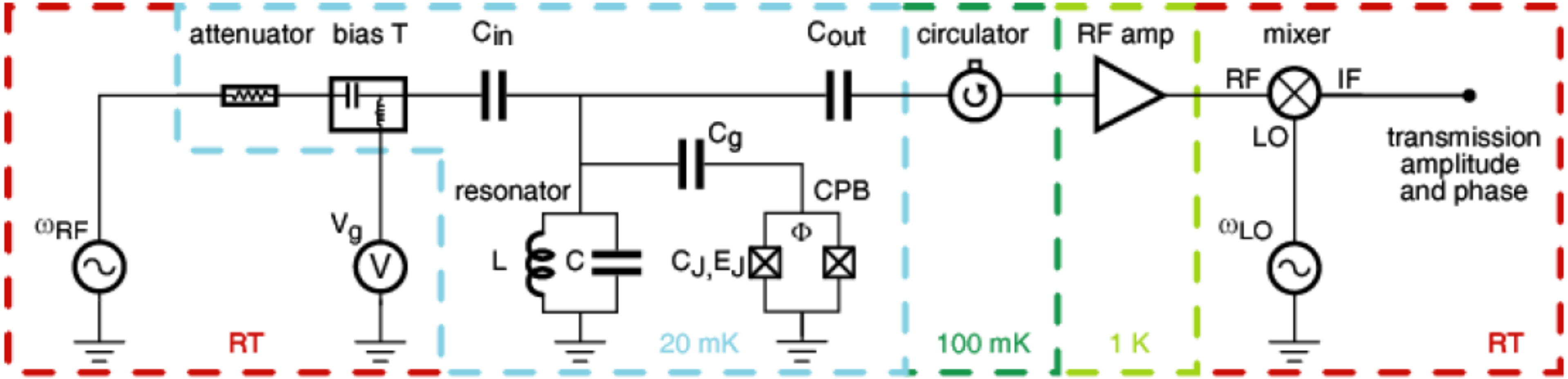
Cavity frequency shift





=> state readout by measurement of transmission

Qubit read-out



With Two Qubits

$$\begin{pmatrix} g_1 g_1 & g_1 g_2 \\ g_2 g_1 & g_2 g_2 \end{pmatrix}$$

One qubit excite the second by virtual photon

$$\begin{pmatrix} \frac{1+2a^\dagger a S_1^z + S_1^z}{\omega-\Omega_1} & \frac{(2\omega-\Omega_1-\Omega_2)\,(S_1^+ S_2^- (2+\Omega_1-\Omega_2) + S_1^- S_2^+ (2-\Omega_1+\Omega_2))}{2\,(\omega-\Omega_1)\,(\omega-\Omega_2)} \\ 0 & \frac{1+2a^\dagger a S_2^z + S_2^z}{\omega-\Omega_2} \end{pmatrix}$$

Perspective and conclusions

What has been achieved

- Understanding QED Systems
- Computing the Hamiltonian
- Using the hamiltonian to understand the qubit read-out
- Interaction between 2 qubits

What remains to be done

- Complete understanding multiple-qubit read-out
- Achieve the n qubit algorithm

Thank you for listening

Do you have any questions ?