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# Quantum phase transitions in capacitively coupled two-dimensional Josephson-junction arrays

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**Abstract.** Quantum phase transitions in two layers of ultrasmall Josephson junctions, coupled capacitively with each other, are investigated. As the inter-layer capacitance is increased, the system at zero temperature is found to exhibit an insulator-to-superconductor transition. It is shown that, unlike in the case for one-dimensional arrays with a similar coupling configuration, the transition cannot be accounted for exclusively by particle–hole pairs.

Capacitively coupled systems of charges have attracted significant attention in recent years, raising the possibility of current drag effects: the current fed through either of the systems, owing to Coulomb interaction, induces a secondary current in the other system. Such a drag effect depends strongly on the dimensionality and the structure of the system as regards its mechanism and behaviour. The current drag in two capacitively coupled two-dimensional (2D) electron gases [1] was attributed to a momentum-transfer mechanism due to Coulomb scattering [2] and is fairly small in magnitude. By contrast, recent theoretical predictions [3] and experimental demonstrations [4, 5] with two capacitively coupled one-dimensional (1D) arrays of submicron metallic tunnel junctions have shown that the primary and the secondary currents are comparable in magnitude but opposite in direction in a certain region of applied voltage. In such tunnel junction systems, the current drag is attributed to the transport of electron–hole pairs, which are bound by the electrostatic energy of the coupling capacitance. Lately, it has been suggested that the momentum-transfer mechanism can also lead to absolute current drag in 1D electron channels coupled electrostatically with each other [6]. The current drag effects in capacitively coupled 2D arrays of tunnel junctions have not been studied and will be examined in this work.

More interestingly, when the tunnelling junctions are composed of ultrasmall superconducting grains, the counterpart of the electron–hole pair becomes the pair of excess and deficit Cooper pair, which will simply be called the particle–hole pair. Furthermore, in such ultrasmall Josephson-junction systems the competition between the charging energy and the Josephson coupling energy is well known to lead to novel effects of quantum fluctuations [7–10]. Combined with these quantum fluctuation effects, the pair transport phenomena in coupled 1D Josephson-junction arrays (JJAs) have recently been proposed to drive an insulator-to-superconductor transition [11].

In this paper, two 2D arrays of ultrasmall Josephson junctions, coupled capacitively with each other, are considered. Quantum phase transitions are examined at zero temperature,

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focusing on the roles of the particle–hole pairs. The system is transformed into two three-dimensional (3D) systems of classical vortex loops, which are *topologically* coupled but otherwise independent of each other. The resulting model reveals that as the coupling capacitance increases, in appropriate regions of parameters, the system exhibits an insulator-to-superconductor transition. Contrary to the 1D counterpart with a similar coupling scheme [11], the transition cannot be ascribed exclusively to the condensation of the particle–hole pairs. Accordingly, it is also remarked briefly that the accompanying drag of supercurrents in the superconducting phase is not absolute in general. In the vicinity of the transition, however, the particle–hole pairs still play major roles, and therefore current drag can be large.

As a matter of fact, capacitively coupled 2D JJAs have been studied by several authors, but in a different context and for different regions of parameter space [12]. Besides, the capacitive coupling should be distinguished from the Josephson coupling such as that considered in multi-layered systems [13]. The capacitively coupled JJAs can presumably be realized in experiment by current techniques, which have already made it possible to fabricate submicron metallic junction arrays with large inter-array capacitances [4, 5] as well as large arrays of ultrasmall Josephson junctions [14].

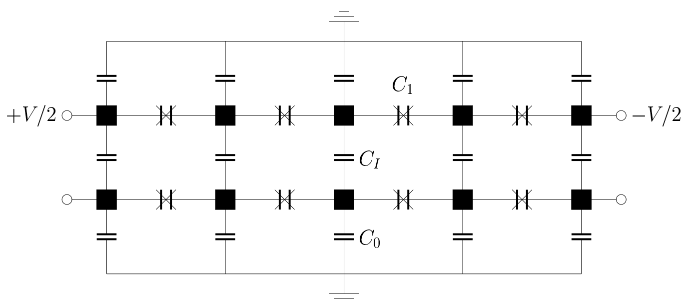


**Figure 1.** A schematic side view of the system. Each chain in the figure represents a 2D array.

Each of the two arrays ($\ell = 1, 2$) of Josephson junctions considered here is characterized by the Josephson coupling energy $E_J$ and the charging energies $E_0 \equiv e^2/2C_0$ and $E_1 \equiv e^2/2C_1$, associated with the self-capacitance $C_0$ and the junction capacitance $C_1$, respectively (see figure 1). The two arrays are coupled with each other by the capacitance $C_I$, with which the electrostatic energy $E_I \equiv e^2/2C_I$ is associated, while there is no Cooper-pair tunnelling between the arrays. The intra-array capacitances are assumed to be so small ($E_0, E_1 \gg E_J$) that, without the coupling, the two arrays would each be separately in the insulating phase [8]. It is also assumed that the coupling capacitance is sufficiently large compared with the intra-array capacitances; $C_I \gg C_0, C_1$. In that case, the electrostatic energy of the particle–hole pair ($\sim E_I$) is much smaller than that of an unpaired charge ($\sim E_0, E_1$); the particle–hole pair, bound by the binding energy of order of $E_0 - E_I$ or $E_1 - E_I$, is thus much more favourable than the unpaired charges. For the most part, this work is devoted to the case of identical arrays, but non-identical arrays will also be briefly discussed.

The system can be well described by the Hamiltonian

$$H = \frac{1}{4K}\sum_{\ell,\ell'}\sum_{\boldsymbol{r},\boldsymbol{r}'} n(\ell;\boldsymbol{r})\mathbb{C}^{-1}(\ell,\ell';\boldsymbol{r},\boldsymbol{r}')n(\ell';\boldsymbol{r}') - 2K\sum_{\ell}\sum_{\langle\boldsymbol{r}\boldsymbol{r}'\rangle}\cos\left[\phi(\ell;\boldsymbol{r}) - \phi(\ell;\boldsymbol{r}')\right] \tag{1}$$

where $\boldsymbol{r} \equiv (x, y)$ denotes the 2D lattice vector in units of the lattice constant, the coupling constant has been defined by

$$2K \equiv \sqrt{E_J/4E_I}$$

and the energy has been rescaled in units of the Josephson plasma frequency

$$\hbar\omega_p \equiv \sqrt{4E_I E_J}.$$

The number $n(\ell; \boldsymbol{r})$ of excess Cooper pairs and the phase $\phi(\ell; \boldsymbol{r})$ of the superconducting order parameter on the grain at $\boldsymbol{r}$ in array $\ell$ are quantum mechanically conjugate variables:

$$[n(\ell; \boldsymbol{r}), \phi(\ell'; \boldsymbol{r}')] = \mathrm{i}\delta_{\boldsymbol{r}\boldsymbol{r}'}\delta_{\ell\ell'}.$$

The capacitance matrix in equation (1) takes the form

$$\mathbb{C} = \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{2}$$

where the submatrices $C(\boldsymbol{r}, \boldsymbol{r}')$ are defined by the Fourier transform

$$\widetilde{C}(\boldsymbol{q}) = C_0 + C_1\Delta(\boldsymbol{q})$$

with $\Delta(\boldsymbol{q}) \equiv \Delta(q_x) + \Delta(q_y)$; $\Delta(k) \equiv 2(1 - \cos k)$. Here all of the capacitances have been rescaled with respect to the relevant capacitance scale $2C_I$: $C_0/2C_I \rightarrow C_0$ and $C_1/2C_I \rightarrow C_1$.

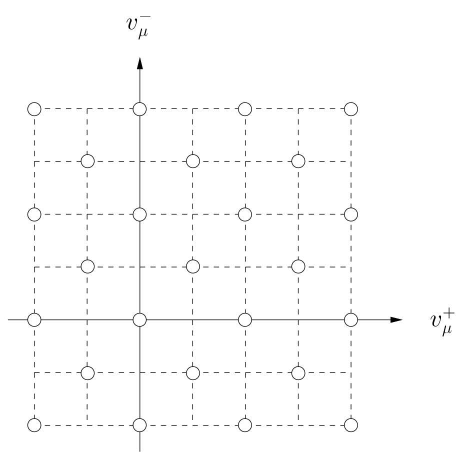


**Figure 2.** Topological coupling of $v_\mu^+$ and $v_\mu^-$. At each space-time position $\vec{r}$, $(v_\mu^+, v_\mu^-)$ can take only half of the elements in $\mathbb{Z} \times \mathbb{Z}$ as depicted with open circles in the figure.

It is convenient to write the partition function of the system in the imaginary-time path integral representation

$$Z = \prod_{\ell,\boldsymbol{r},\tau} \sum_{n(\ell;\boldsymbol{r},\tau)} \int_0^{2\pi} \mathrm{d}\phi(\ell; \boldsymbol{r}, \tau)\ \exp[-\mathcal{S}] \tag{3}$$

with the Euclidean action

$$\mathcal{S} = \frac{1}{4K} \sum_{\ell,\ell'} \sum_{\boldsymbol{r},\boldsymbol{r}',\tau} n(\ell; \boldsymbol{r}, \tau)\mathbb{C}^{-1}(\ell, \ell'; \boldsymbol{r}, \boldsymbol{r}')n(\ell'; \boldsymbol{r}', \tau)$$
$$- 2K \sum_{\ell} \sum_{\boldsymbol{r},\tau} \sum_{j} \cos \nabla_j\phi(\ell; \boldsymbol{r}, \tau) + \mathrm{i}\sum_{\ell}\sum_{\boldsymbol{r}} n(\ell; \boldsymbol{r}, \tau)\nabla_\tau\phi(\ell; \boldsymbol{r}, \tau) \tag{4}$$

where $\nabla_j$ ($j = x, y$) and $\nabla_\tau$ denote the difference operators in the spatial and the imaginary-time directions, respectively, and the (imaginary-) time slice $\delta\tau$ has been chosen to be unity (in units of $\omega_p^{-1}$) [15]. The highly symmetric form of equation (4) with respect to space and time makes it useful to introduce the space-time 3-vector notation $\vec{\boldsymbol{r}} \equiv (\boldsymbol{r}, \tau)$, and analogous notation for all other vector variables. We then apply the Villain approximation [16] to rewrite the cosine term as a summation over integer fields

$$\{m_x(\ell; \vec{\boldsymbol{r}}), m_y(\ell; \vec{\boldsymbol{r}})\} \equiv \{\boldsymbol{m}(\ell; \vec{\boldsymbol{r}})\}.$$

Further, with the aid of the Poisson resummation formula [16] and Gaussian integration, we rewrite the charging energy term as a summation over another integer field $\{m_\tau(\ell; \vec{\boldsymbol{r}})\}$, to obtain the partition function

$$Z \sim \prod_{\ell;\vec{\boldsymbol{r}}} \sum_{\vec{\boldsymbol{m}}(\ell;\vec{\boldsymbol{r}})} \int_{-\infty}^{\infty} \mathrm{d}\phi(\ell; \vec{\boldsymbol{r}}) \;\; \exp\{-\mathcal{S}\} \tag{5}$$

with

$$\mathcal{S} = K \sum_{\ell,\ell';\vec{\boldsymbol{r}},\vec{\boldsymbol{r}}'} \mathbb{C}(\ell, \ell'; \boldsymbol{r}, \boldsymbol{r}')\delta_{\tau\tau'} \left[\nabla_\tau \phi(\ell; \vec{\boldsymbol{r}}) - 2\pi m_\tau(\ell; \vec{\boldsymbol{r}})\right]\left[\nabla_\tau \phi(\ell'; \vec{\boldsymbol{r}}') - 2\pi m_\tau(\ell'; \vec{\boldsymbol{r}}')\right] + K \sum_{\ell;\vec{\boldsymbol{r}}} \left[\nabla\phi(\ell; \vec{\boldsymbol{r}}) - 2\pi \boldsymbol{m}(\ell; \vec{\boldsymbol{r}})\right]^2. \tag{6}$$

The variables $\phi(\ell; \vec{\boldsymbol{r}})$ and $\vec{\boldsymbol{m}}(\ell; \vec{\boldsymbol{r}})$ can be usefully replaced by $\phi^\pm(\vec{\boldsymbol{r}}) \equiv \phi(1; \vec{\boldsymbol{r}}) \pm \phi(2; \vec{\boldsymbol{r}})$ and $\vec{\boldsymbol{m}}^\pm(\vec{\boldsymbol{r}}) \equiv \vec{\boldsymbol{m}}(1; \vec{\boldsymbol{r}}) \pm \vec{\boldsymbol{m}}(2; \vec{\boldsymbol{r}})$, respectively. In this way, one decomposes the Euclidean action in equation (6) into the sum $\mathcal{S} = \mathcal{S}^+ + \mathcal{S}^-$ with $\mathcal{S}^\pm$ defined by

$$\mathcal{S}^\pm = +\frac{1}{2} K \sum_{\vec{\boldsymbol{r}},\vec{\boldsymbol{r}}'} C^\pm(\boldsymbol{r}, \boldsymbol{r}')\delta(\tau, \tau') \left[\nabla_\tau \phi^\pm(\vec{\boldsymbol{r}}) - 2\pi m_\tau^\pm(\vec{\boldsymbol{r}})\right]\left[\nabla_\tau \phi^\pm(\vec{\boldsymbol{r}}') - 2\pi m_\tau^\pm(\vec{\boldsymbol{r}}')\right] + \frac{1}{2} K \sum_{\vec{\boldsymbol{r}}} \left[\nabla\phi^\pm(\vec{\boldsymbol{r}}) - 2\pi \boldsymbol{m}^\pm(\vec{\boldsymbol{r}})\right]^2. \tag{7}$$

Here the new capacitance matrices $C^\pm(\boldsymbol{r}, \boldsymbol{r}')$ are defined via the Fourier transforms $\widetilde{C}^+(\boldsymbol{q}) = \widetilde{C}(\boldsymbol{q})$ and $\widetilde{C}^-(\boldsymbol{q}) = 1 + \widetilde{C}(\boldsymbol{q})$, respectively. Now one follows the standard procedures [17, 8] to integrate out $\{\phi^\pm(\vec{\boldsymbol{r}})\}$. Then, apart from the irrelevant spin-wave part, one can finally obtain the 3D system of classical vortex lines, which is also decomposed into two subsystems $H_V = H_V^+ + H_V^-$ with

$$H_V^\pm = 2\pi^2 K \sum_{\vec{\boldsymbol{r}},\vec{\boldsymbol{r}}'} \sum_\mu v_\mu^\pm(\vec{\boldsymbol{r}}) U_\mu^\pm(\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}') v_\mu^\pm(\vec{\boldsymbol{r}}') \tag{8}$$

where the interactions between vortex line segments are defined via their Fourier transforms

$$\widetilde{U}_\parallel^\pm(\vec{\boldsymbol{q}}) = \frac{\widetilde{C}^\pm(\boldsymbol{q})}{\Delta(\boldsymbol{q}) + \widetilde{C}^\pm(\boldsymbol{q})\Delta(\omega)} \tag{9}$$

$$\widetilde{U}_\tau^\pm(\vec{\boldsymbol{q}}) = \frac{1}{\Delta(\boldsymbol{q}) + \widetilde{C}^\pm(\boldsymbol{q})\Delta(\omega)}. \tag{10}$$

Here the vortex lines $v_\mu^-$ are manifestations of the particle–hole pairs, whereas the $v_\mu^+$ stand for single-particle processes [11]. Note that, in equation (8), the fields $\{v_\mu^\pm(\vec{\boldsymbol{r}})\}$ are subject to the constraint $\vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{v}}^\pm(\vec{\boldsymbol{r}}) = 0$; i.e., all vortex lines either form closed loops or go to infinity. More importantly, it should also be noticed that the two fields $v_\mu^+$ and $v_\mu^-$ cannot be independent of each other, since $m_\mu(1; \vec{\boldsymbol{r}})$ and $m_\mu(2; \vec{\boldsymbol{r}})$ in equation (7), and hence $v_\mu(1; \vec{\boldsymbol{r}}) = [v_\mu^+(\vec{\boldsymbol{r}}) + v_\mu^-(\vec{\boldsymbol{r}})]/2$ and $v_\mu(2; \vec{\boldsymbol{r}}) = [v_\mu^+(\vec{\boldsymbol{r}}) - v_\mu^-(\vec{\boldsymbol{r}})]/2$, can take only integer values. As depicted with open circles in figure 2, $(v_\mu^+, v_\mu^-)$ at each $\vec{r}$ can take only half of the elements in the product set of integers $\mathbb{Z} \times \mathbb{Z}$; $v_\mu^+$ and $v_\mu^-$ are *topologically coupled* with each other. Contrary to the case for the capacitively coupled 1D JJAs [11], this topological coupling plays crucial roles in the present case, which will be discussed in more detail below.

It is not difficult to understand the physics described by each of the Hamiltonians $H_V^\pm$. Unless $C_0 = 0$, the length-scale dependence of the anisotropy factor $\widetilde{C}^+(\boldsymbol{q}) = \widetilde{C}(\boldsymbol{q}) \ll 1$ is screened out at length scales larger than $\sqrt{C_1/C_0}$, and thereby $\widetilde{U}_\mu^+(\vec{q})$ is simply reduced to the highly anisotropic current-like interaction:

$$\widetilde{U}_\parallel^+(\vec{q}) \simeq C_0/[\Delta(\boldsymbol{q}) + C_0\Delta(\omega)] \qquad \widetilde{U}_\tau^+(\vec{q}) \simeq 1/[\Delta(\boldsymbol{q}) + C_0\Delta(\omega)].$$

Such an anisotropic model has been studied in reference [17], and is known to exhibit an anisotropic 3D transition which is associated with the disruption of the vortex loops, at $K = K_c^+$ close to the 2D Berezinskii–Kosterlitz–Thouless (BKT) transition point [18]: $K_c^+ \sim 2/\pi$. In the case of $C_0 = 0$, the vortex lines $v_\mu^+$ even form 2D pancake vortices residing on decoupled 2D layers with

$$\widetilde{U}_\parallel^+(\vec{q}) \simeq C_1/[1 + C_1\Delta(\omega)] \qquad \widetilde{U}_\tau^+(\vec{q}) \simeq 1/\Delta(\boldsymbol{q})[1 + C_1\Delta(\omega)]$$

and the phase transition is precisely BKT-type. In any case, the system of vortex lines $v_\mu^+$ exhibits a phase transition at $K = K_c^+ \sim 2/\pi$. On the other hand, neglecting the anisotropy at short-length scales, $\widetilde{U}_\mu^-(\vec{r})$ are isotropic in space-time:

$$\widetilde{U}_\parallel^-(\vec{q}) \simeq \widetilde{U}_\tau^-(\vec{q}) \simeq 1/[\Delta(\boldsymbol{q}) + \Delta(\omega)].$$

In consequence, it follows that the system of vortex lines $v_\mu^-$ exhibits the isotropic 3D $XY$-type phase transition at $K = K_c^- \sim 1/2\sqrt{2}$. At this point, one might be tempted to conclude that, as $K$ is increased, the total system $H_V$ might go through two successive transitions, one at $K_c^-$ and the other at $K_c^+$, the first of which would be ascribed to condensation of particle–hole pairs [11]. This scenario of successive transitions, however, should be tested against the topological coupling discussed above between $v_\mu^+$ and $v_\mu^-$.

For this goal, it is convenient to consider the subsystem $\{v_\mu^*\}$ of $\{v_\mu^+\}$ satisfying $v_\mu^- = 0$. In this subsystem, $v_\mu^*(\vec{r})$ can take only even values, and hence the phase transition could take place at $K = K_* = K_c^+/4 \sim 1/2\pi$, which is substantially lower than $K_c^-$. This means that vortices $v_\mu^*$ could be tightly bound even before the vortices $v_\mu^-$ get bound, contradicting the assumption that $v_\mu^- = 0$. Consequently, it follows that the actual phase transition should take place at $K_c$ between $K_c^-$ and $K_c^+$, and cannot be accounted for exclusively by $v^-$, i.e., by particle–hole pairs. This is distinctively different from the case of the capacitively coupled 1D chains [11], where the vortices $v^*$ in analogous subsystems always form a plasma of free vortices regardless of $K$ in the presumed configuration $C_0, C_1 \ll 1$ and the topological coupling is thus irrelevant; the free vortices $v^*$ completely screen out the interaction among the vortices $v_\mu^+$.

Nevertheless, it is evident that any correction of the vortex lines $v_\mu^+$ in the vicinity of $K_c$ is exponentially small in the creation energy $\mu_c^+$ of the smallest vortex loops (or nearby pancake vortex–antivortex pairs when $C_0 = 0$): $\mu_c^+ \sim K_c\pi$ [18]. In particular, the shift of the transition point $K_c$ with respect to $K_c^-$ can be estimated by

$$(K_c - K_c^-)/K_c^- \sim \mathrm{e}^{-2\mu_c^+} \sim 0.1 \tag{11}$$

where the factor 2 in the exponent is due to the topological coupling.

Now I examine briefly and qualitatively the current drag effects in the superconducting phase, by means of the linear response $\sigma_{\ell\ell'}(\omega)$ of the current in the array $\ell$ to the voltage applied across the array $\ell'$ (see figure 1):

$$\sigma_{\ell\ell'}(\omega) = \frac{1}{\mathrm{i}\omega} \lim_{q\to 0} \widetilde{\mathcal{G}}_{\ell\ell'}(\boldsymbol{q}, \mathrm{i}\omega' \to \omega + \mathrm{i}0^+) \tag{12}$$

where $\widetilde{\mathcal{G}}_{\ell\ell'}$ is the Fourier transform of the imaginary-time Green's function

$$\mathcal{G}_{\ell\ell'}(\boldsymbol{r}, \tau) = \left\langle T_\tau[I(\ell; \boldsymbol{r}, \tau) I(\ell'; \boldsymbol{0}, 0)]\right\rangle$$

with the time-ordered product $T_\tau$ and the current operators $I(\ell; \boldsymbol{r}) \equiv \sin \nabla_x \phi(\ell; \boldsymbol{r})$ (since the system is isotropic in the $x$- and $y$-directions, only the current in the $x$-direction is considered here). Due to the symmetry between the two arrays, it follows that

$$\sigma_{11}(\omega) = \left[\sigma_+(\omega) + \sigma_-(\omega)\right]/2 \qquad \sigma_{21}(\omega) = \left[\sigma_+(\omega) - \sigma_-(\omega)\right]/2$$

where the $\sigma_\pm$ are defined in a manner analogous to equation (12) with

$$I^\pm(x) \equiv I(1; x) \pm I(2; x).$$

According to the discussion above on the phase transition, at $K > K_c$, both $\sigma_+$ and $\sigma_-$ show superconducting behaviour: $\sigma_\pm(\omega) = \sigma_\pm^0 \delta(\omega)$ $(\omega \ll 1)$, which means that the drag of supercurrents along the two arrays is not perfect in general. However, in the vicinity of the phase transition, where $\sigma_+^0 \ll \sigma_-^0$, the currents in the two arrays can be comparable in magnitude. This suggests the following: the particle–hole pair is not so tight as in the 1D case, distributing over a few lattice constants. Yet it is still energetically favourable enough to play significant (if not exclusively crucial) roles in the phase transition and the transport.

Before concluding, I remark briefly on non-identical arrays. The difference in the intra-array capacitances leads to additional coupling between the vortices $v_\mu^+$ and $v_\mu^-$ with the coupling strength proportional to the difference. The arguments on identical arrays therefore remain valid qualitatively as long as

$$\left|\widetilde{C}(1; \boldsymbol{q}) - \widetilde{C}(2; \boldsymbol{q})\right| \ll \left|\widetilde{C}(1; \boldsymbol{q}) + \widetilde{C}(2; \boldsymbol{q})\right|.$$

The difference in Josephson coupling energy, on the other hand, can be effectively incorporated in the capacitance difference by renormalizing the parameters, since all of the effects considered in this work depend only on the relative strength of the Josephson coupling energy and the charging energies.

In conclusion, quantum phase transitions in two capacitively coupled 2D JJAs have been investigated. In particular, it has been found that as the coupling capacitance increases, in appropriate parameter ranges ($E_J/E_0, E_J/E_1 \ll 1$; $E_J/E_0, E_J/E_1 \ll E_J/E_I < \infty$), the system exhibits an insulator-to-superconductor transition. Contrary to the case for the capacitively coupled 1D chains, the transition cannot be accounted for exclusively by the condensation of particle–hole pairs. Accordingly, the drag of supercurrents along the two arrays is not absolute in general.

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