Comment on "Josephson Current through a Nanoscale Magnetic Quantum Dot"

In Ref. [1], Siano and Egger (SE) studied the Josephson current through a quantum dot in the Kondo regime using the quantum Monte Carlo (QMC) method. Several of their results were unusual, and inconsistent with those from the numerical renormalization group (NRG) studies [2,3] among others. Those results in Ref. [1] are not reliable as (i) the definition of the Kondo temperature was wrong and (ii) there were substantial finite-temperature effects.

We first clarify point (i). The *normal-state* Kondo temperature [4,5] *in the absence of superconductivity* provides one of the most significant energy scales of the system. SE defined the Kondo temperature as

$$T_K^{\rm SE} = \exp[\pi\epsilon_0(\epsilon_0 + U)/\Gamma_{\rm SE}U]\sqrt{\Gamma_{\rm SE}U}/2 \qquad (1)$$

with $\Gamma_{\rm SE} = 2\pi\rho_0|t|^2$, where $|t|^2$ denotes the coupling to *one lead* and ρ_0 the density of states (DOS) at the Fermi level. In Ref. [2] we defined it as

$$T_K = \exp[\pi\epsilon_0(\epsilon_0 + U)/2\Gamma U]\sqrt{\Gamma U/2} \qquad (2)$$

with $\Gamma = 2\pi N_0|V|^2$, where $|V|^2$ denotes the coupling to *one lead* and N_0 the DOS at the Fermi level *per spin* (the factor 2 in the coupling comes from the two leads). It is important to clarify the difference between the two definitions since different definitions of T_K result in significantly different scaling behaviors of physical quantities. We note that both forms, Eqs. (1) and (2), appear in the literature. However, in Eq. (1) $\Gamma_{\rm SE}$ should be the *full* width at half maximum of the single particle level of the noninteracting dot [6], whereas in Eq. (2), Γ should be the *half* width at half maximum (HWHM) of the single particle level. To see the precise meaning of $\Gamma_{\rm SE}$, let us take the limit $\Delta = 0$ and $U = 0$ in the local Green's function (GF) in Eq. (6) in Ref. [1], which yields the spectral function $A(E) = \Gamma_{\rm SE}/\pi(E^2 + \Gamma_{\rm SE}^2)$. Therefore, $\Gamma_{\rm SE}$ is the *HWHM*; i.e., $\Gamma_{\rm SE} = \Gamma$ in Eqs. (1) and (2). It thus follows that $T_K^{\rm SE} = T_K^2/\sqrt{\Gamma U}$, which implies that the scale $\Delta/T_K^{\rm SE}$ differs from the scale given in Ref. [2]. The unusual definition of the Kondo temperature in Eq. (1) explains the (otherwise) unusual behaviors of $I(\phi)$ with respect to U/Δ in Fig. 2 of SE.

We now move on to point (ii). SE did all calculations at a finite temperature $T = 0.1\Delta$ and note that "this appears to be quite close to the ground-state limit". This is particularly important in the determination of the current-phase relation. To estimate the Josephson energy we note that $E_J(\phi) = \int^\phi d\phi' I_S(\phi') \sim \Delta I_c/I_c^{\rm short}$, where I_c is the effective critical current of the system and $I_c^{\rm short} \equiv e\Delta/\hbar$ the critical current of the open contact. According to the numerical results in Ref. [1], $I_c/I_c^{\rm short} \le 0.1$ for $\Delta/T_K^{\rm SE} \gtrsim 5$ ($\Delta/T_K \gtrsim 1$ in Ref. [2]). We think that in most plots in Ref. [1] the current-phase relation contains significant amounts of thermal activation. To confirm this we have performed NRG calculations at finite temperatures and the results in Fig. 1 demonstrate the strong finite-temperature effects. The sharp transition at zero temperature is washed out and the critical current is reduced by a factor of 5 for $T/\Delta = 0.1$. The discrepancy between the NRG and QMC data in the new Fig. 2 of the Reply [7] may simply reflect the different estimates of critical value Δ_c/T_K (i.e., the NRG and QMC data are in different phases), and may not be an evidence that the NRG is less accurate.

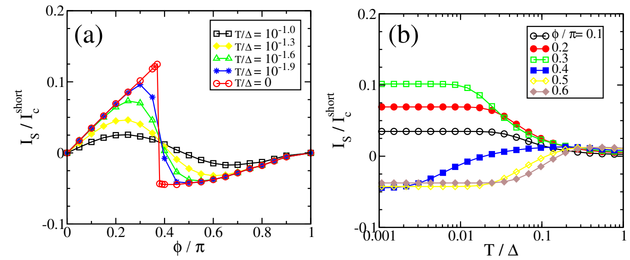

FIG. 1 (color online). (a) Josephson current $I_S(\phi)$ at different temperatures. (b) Josephson current as a function of temperature for different values of ϕ. $\Delta/T_K = 1.6$.

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