Tutorial: Circuit Quantum Electrodynamics

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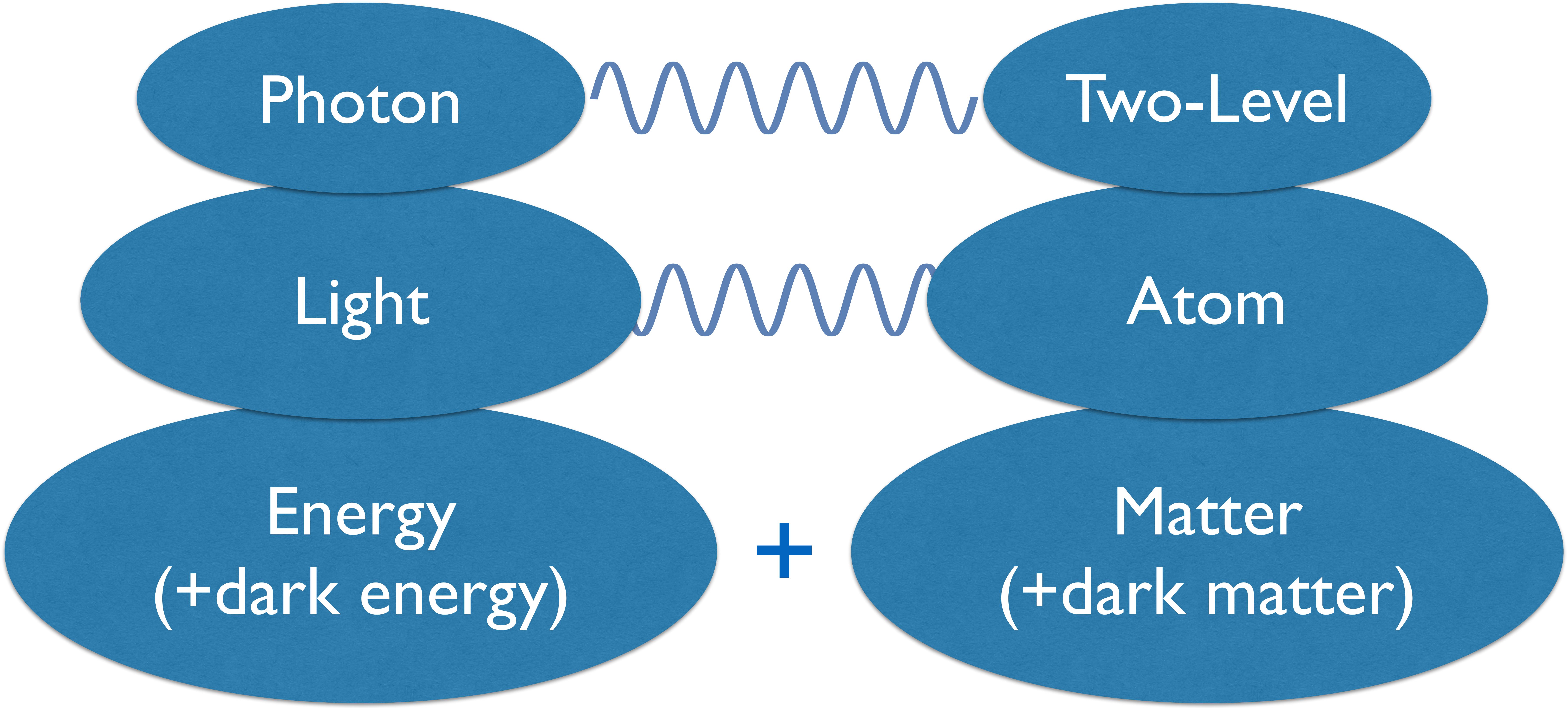
OVERVIEW

1. Introduction
2. Superconducting Resonator
3. Superconducting Circuit QED Systems
4. Exotic Quantum States of Photons
5. Quantum Sensing of Charge Fluctuations
6. Fundamental light-topological matter interaction
7. Summary

General Introduction

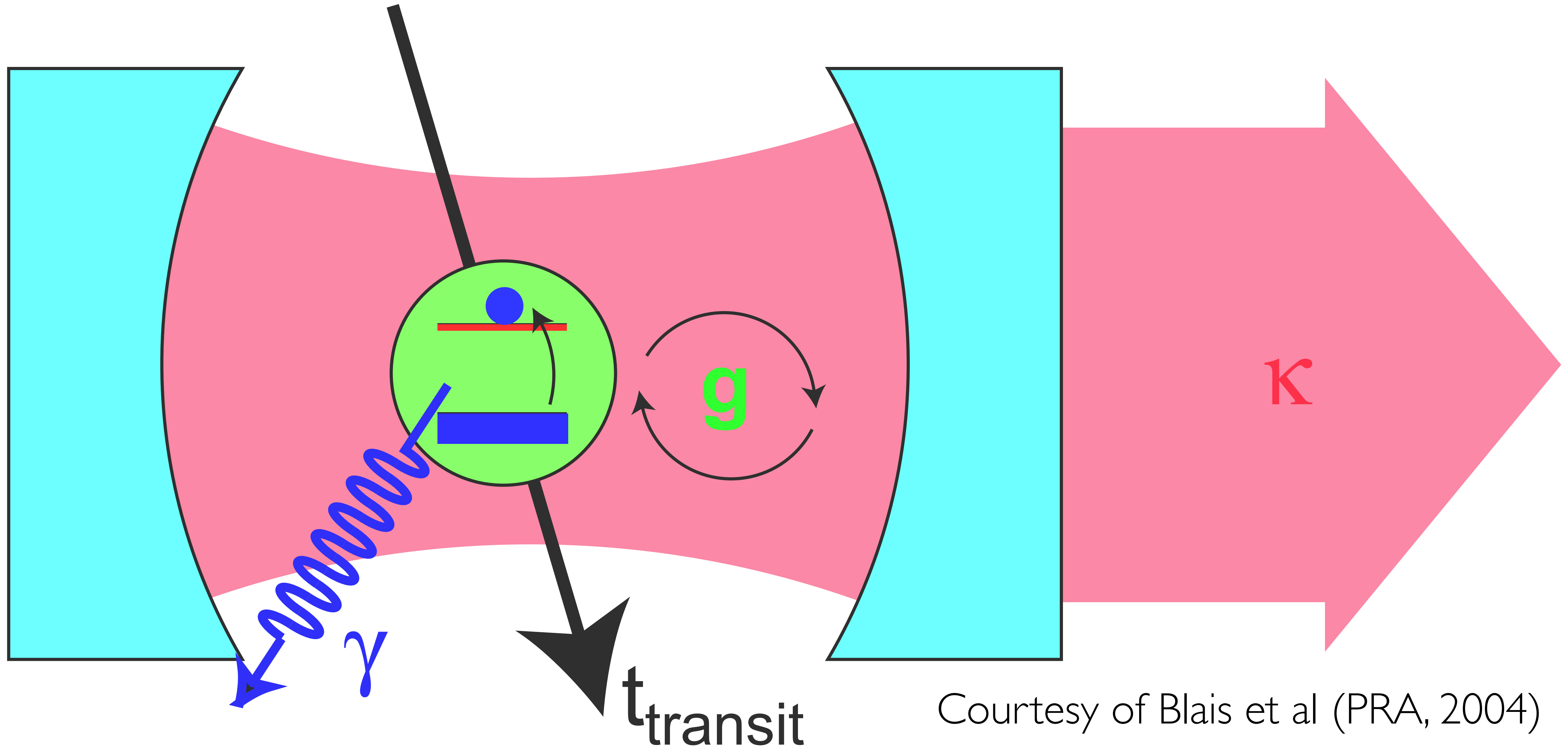
Background & Motivations

Light-Matter Interaction



CAVITY QED

- Simple yet highly non-trivial.
- All essential features of light-matter interaction.



Courtesy of Blais et al (PRA, 2004)

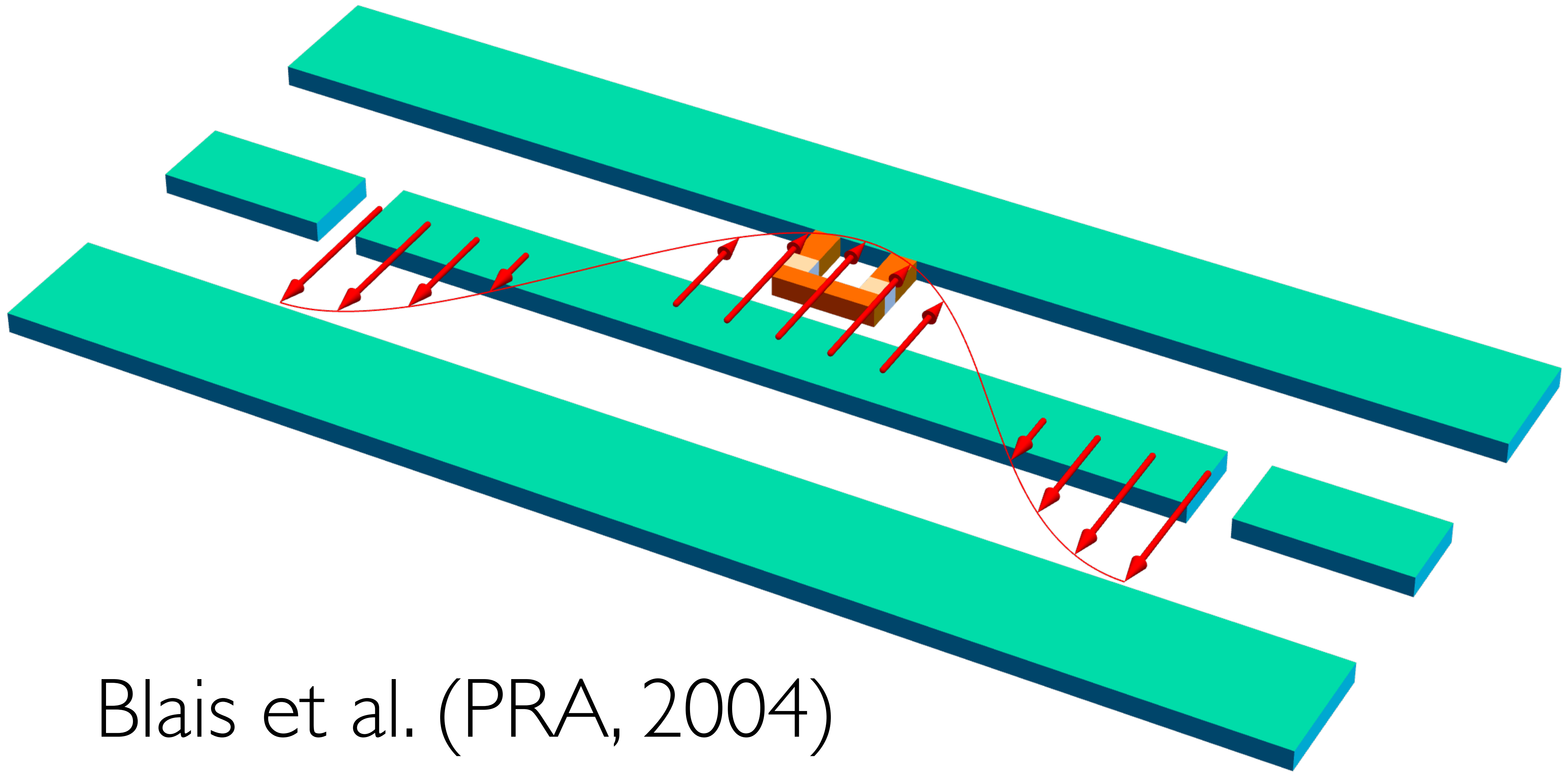
$$H = \omega a^\dagger a + g(a + a^\dagger)\sigma^x + \frac{1}{2}\Omega\sigma^z$$

Two Limitations

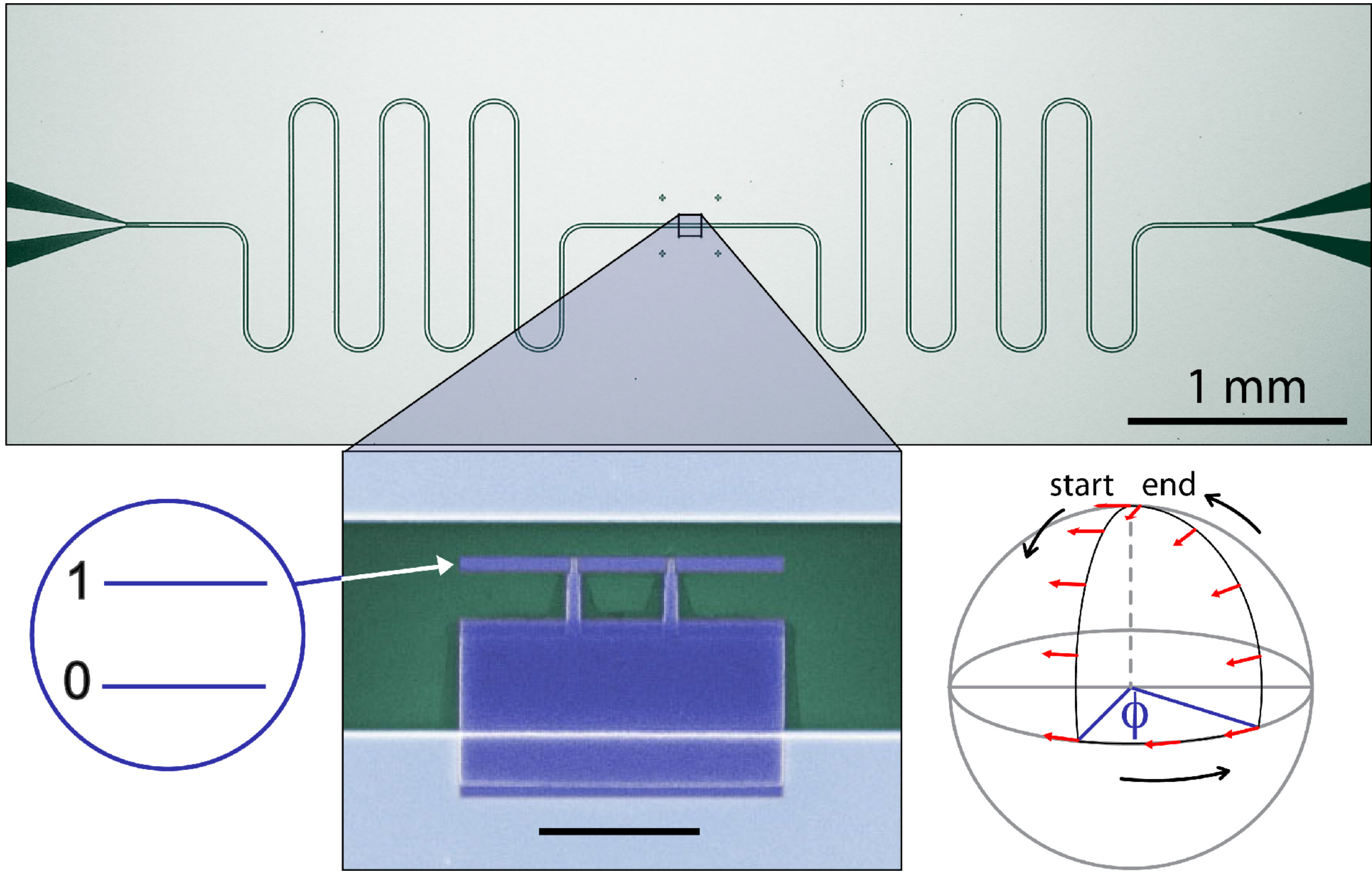
of the conventional cavity QED

1. The coupling is weak.
2. The qubit is not topological.

Circuit QED System



Blais et al. (PRA, 2004)



Wallraff et al. (Nature, 2004)

Light & Topological Matter

What is the smallest unit (if any) of the topological matter?

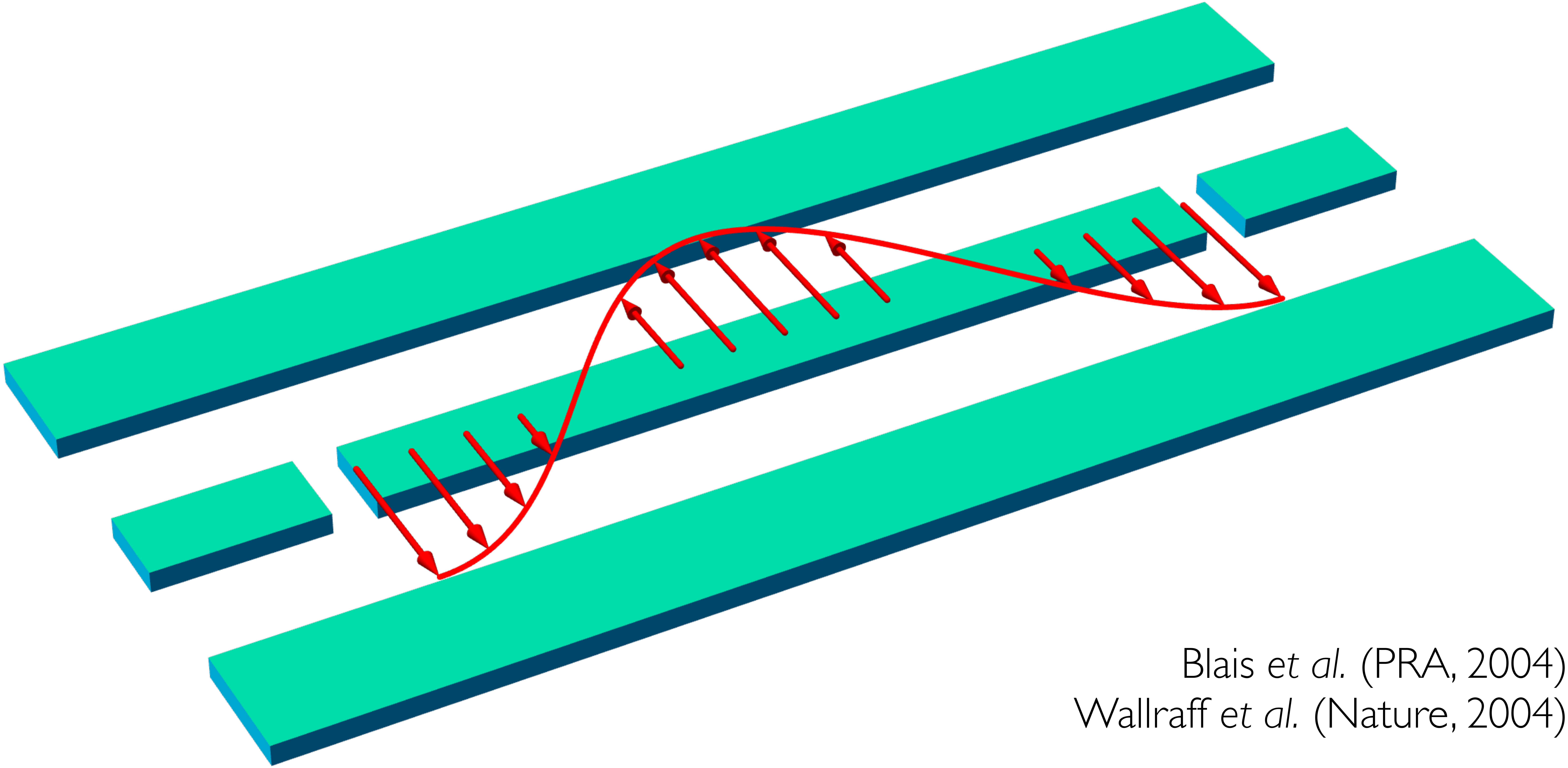
A simple yet quintessence-seizing model of topological matter?

1. To realize *topological qubits* based on Josephson junction arrays.
2. To achieve the *topological QED architecture* (with strong coupling).
3. To explore the *fundamental light-topological matter interaction*.

Superconducting Resonator

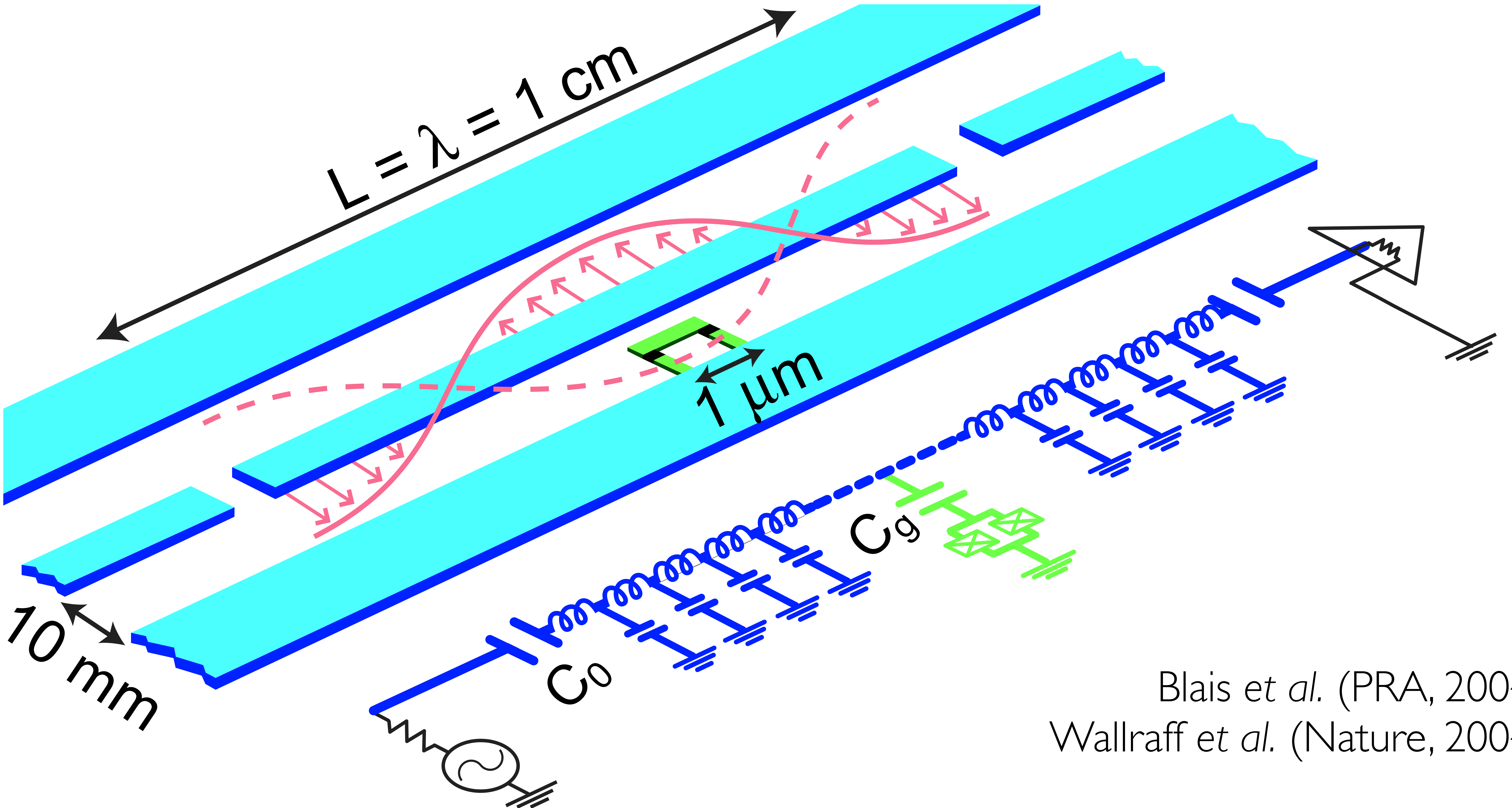
Cavity for Micro-Wave Photons

Superconducting Transmission Line



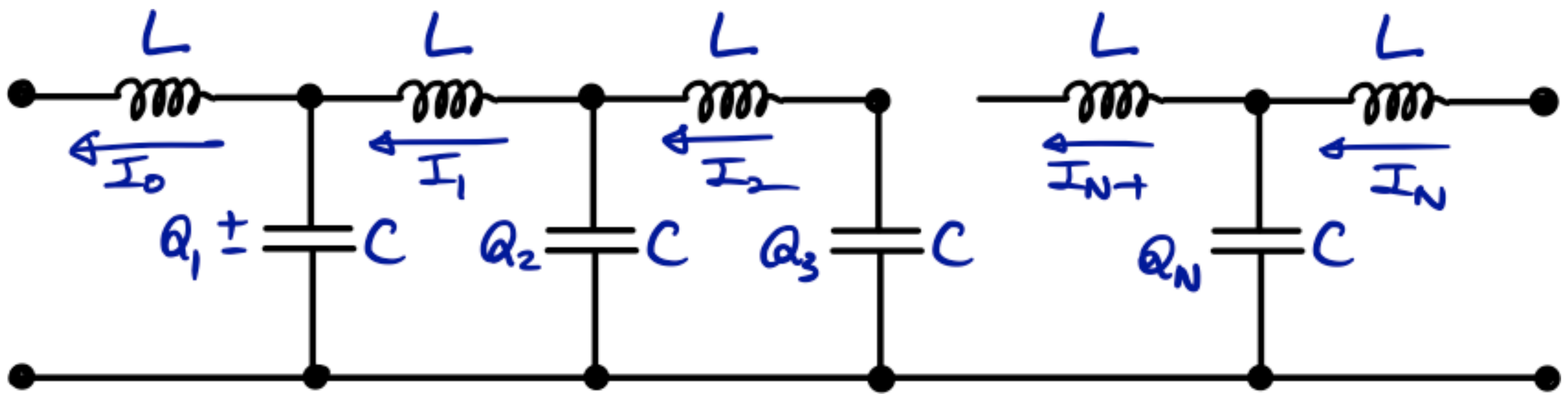
Blais *et al.* (PRA, 2004)
Wallraff *et al.* (Nature, 2004)

Superconducting Transmission Line



Blais *et al.* (PRA, 2004)
Wallraff *et al.* (Nature, 2004)

EQUIVALENT CIRCUIT



$$
\begin{aligned}
I_0 &= 0 \\
I_1 &= I_0 + \dot{Q}_1 \\
I_2 &= I_1 + \dot{Q}_1 \\
&\vdots \\
I_N &= I_{N-1} + \dot{Q}_N \\
I_N &= 0
\end{aligned}
$$

$$\Phi_0 := 0\,, \quad \Phi_n := \sum_{j=1}^{n} Q_j$$

$$Q_n = \Phi_n - \Phi_{n-1}\,, \quad I_n = \dot{\Phi}_n$$

$$\mathscr{L} = \sum_n \left[\frac{1}{2} L \dot{\Phi}_n^2 - \frac{1}{2} L \omega_{LC}^2 (\Phi_n - \Phi_{n-1})^2\right]$$

$$\mathscr{L} = \int dx\, \frac{1}{2}\left[(\partial_t \phi)^2 - v^2 (\partial_x \phi)^2\right]\,, \quad v := \frac{1}{\sqrt{\ell c}}$$

Quantization

$$\mathscr{L} = \int dx \, \frac{1}{2} \left[(\partial_t \phi)^2 - v^2 (\partial_x \phi)^2 \right] , \quad v := \frac{1}{\sqrt{\ell c}}$$

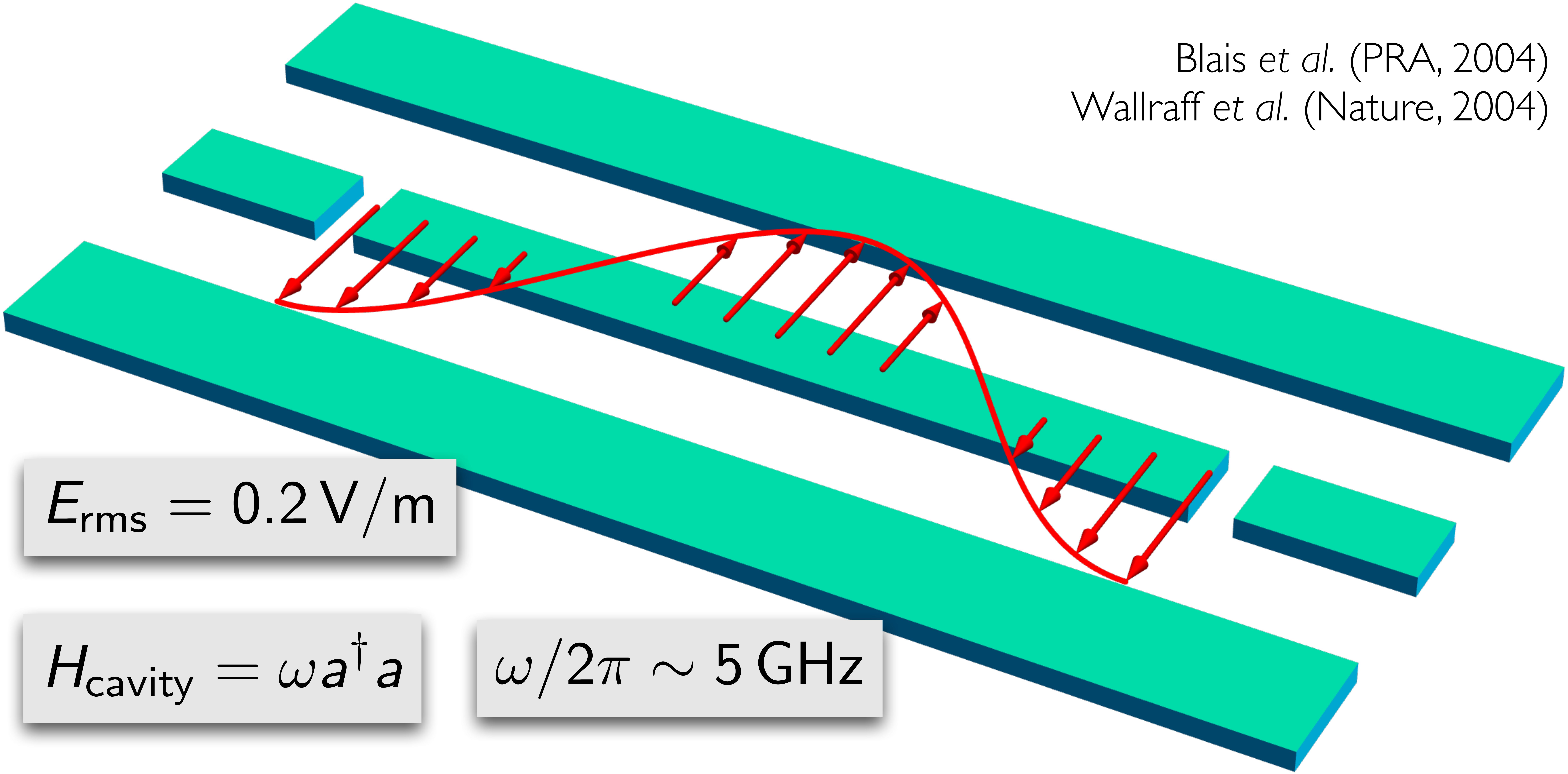
$$\hat{\phi}(x) = \sum_k e^{ikx - i\omega t} \hat{a}_k + e^{-ikx + i\omega t} \hat{a}_k^\dagger$$

$$i\hat{\pi}(x) = \sum_k e^{ikx - i\omega t} \hat{a}_k - e^{-ikx + i\omega t} \hat{a}_k^\dagger$$

$$\hat{H} = \sum_k \hbar \omega_k \hat{a}_k^\dagger \hat{a}_k , \quad \omega_k := vk$$

Resonator (Cavity)

Blais *et al.* (PRA, 2004)
Wallraff *et al.* (Nature, 2004)



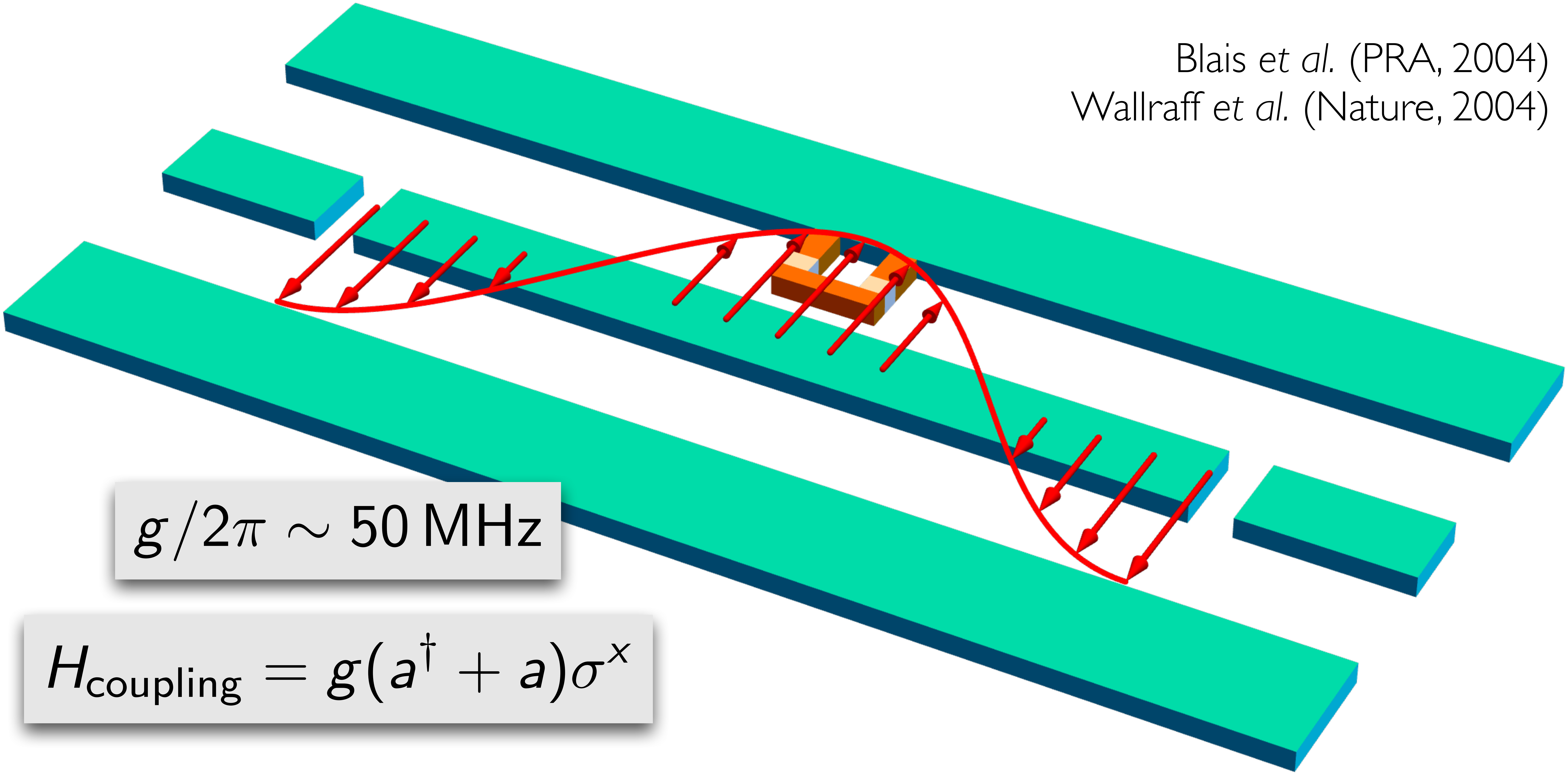
$E_{\mathrm{rms}} = 0.2\,\mathrm{V/m}$

$H_{\mathrm{cavity}} = \omega a^\dagger a$

$\omega/2\pi \sim 5\,\mathrm{GHz}$

Circuit QED System

Blais *et al.* (PRA, 2004)
Wallraff *et al.* (Nature, 2004)



$$g/2\pi \sim 50\,\text{MHz}$$

$$H_{\text{coupling}} = g(a^\dagger + a)\sigma^x$$

Rabi Hamiltonian

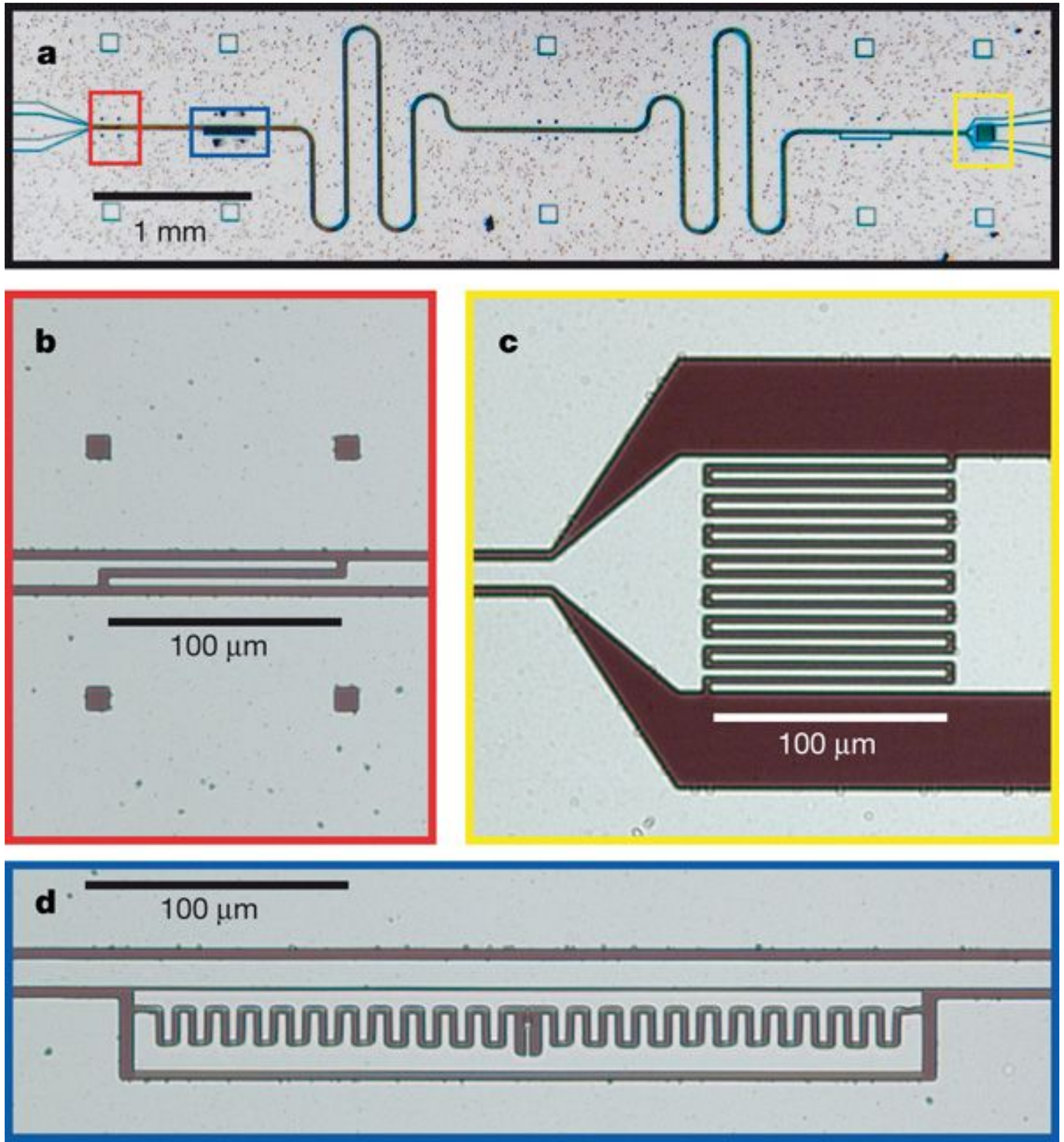
$\omega/2\pi \sim 5\,\mathrm{GHz}$

$g/2\pi \gtrsim 50\,\mathrm{MHz}$

$$H_{\mathrm{Rabi}} = \overbrace{\omega a^\dagger a}^{\text{resonator}} + \underbrace{\frac{1}{2}\Omega\sigma^z}_{\text{qubit}} + \overbrace{g(a + a^\dagger)\sigma^x}^{\text{coupling}}$$

$\Omega/2\pi \sim 5\,\mathrm{GHz}$

Circuit QED System



$$g/2\pi \approx 100\,\mathrm{MHz}$$

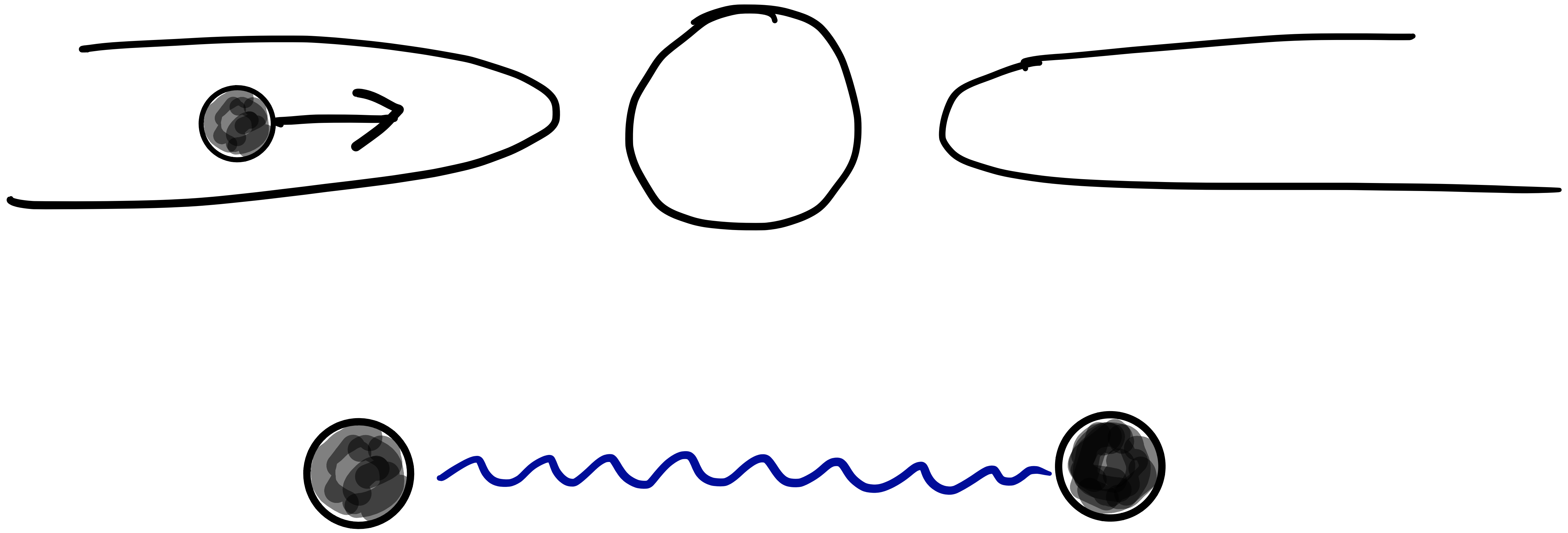
$$H_{\mathrm{coupling}} = g(a^\dagger + a)\sigma^x$$

Houck *et al.* (Nature, 2007)

Exotic Quantum States of Photons

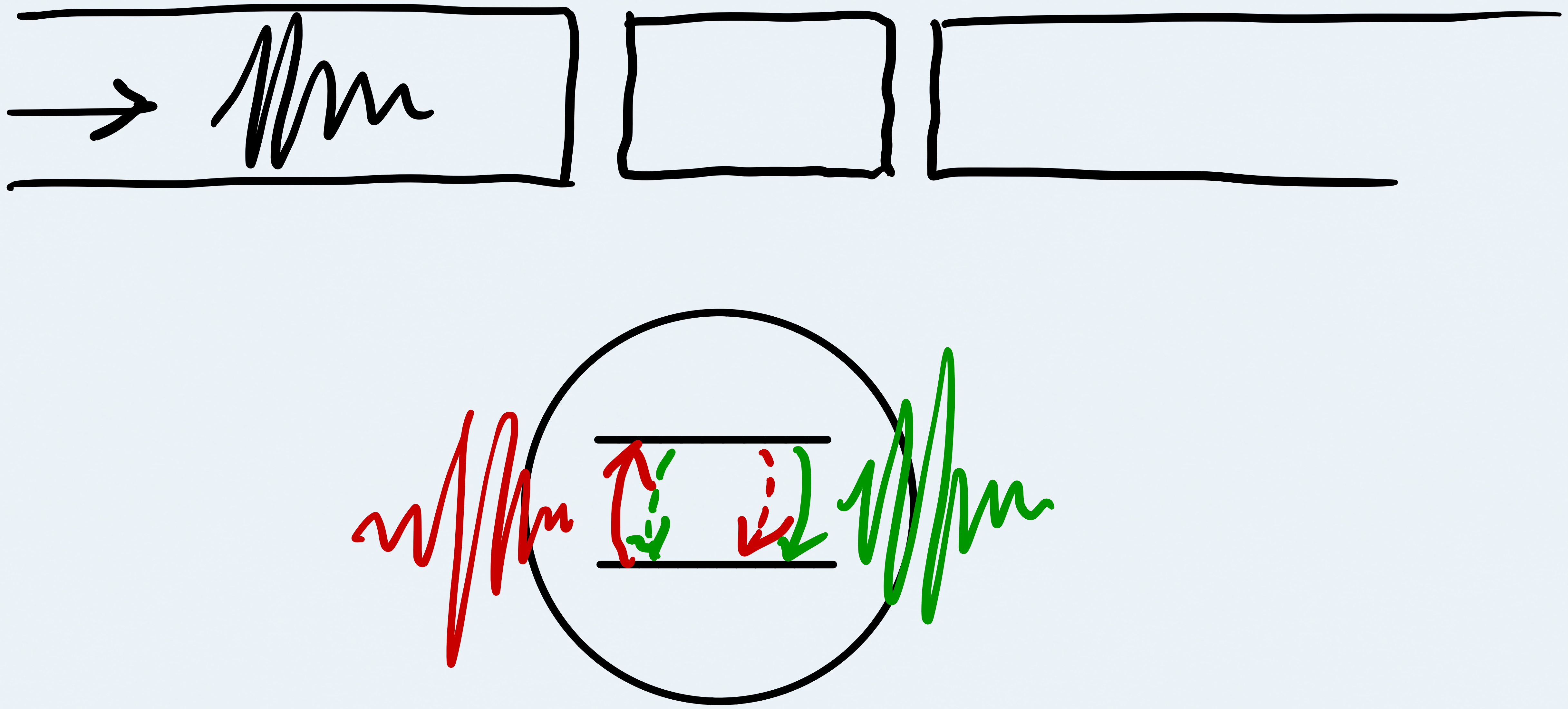
Photon Blockade and Delocalization

Phys. Rev. Lett. 116, 153601 (2016)



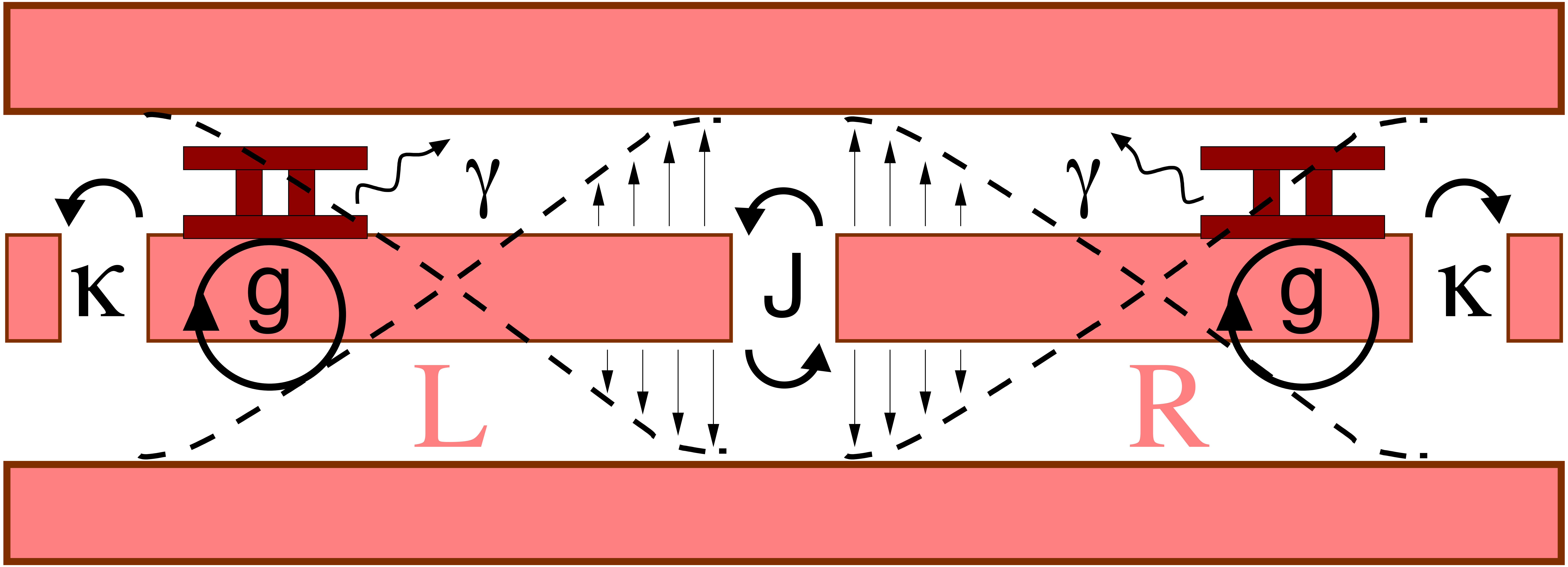
$$\sim \frac{e^2}{2C} \equiv E_c$$

COULOMB BLOCKADE



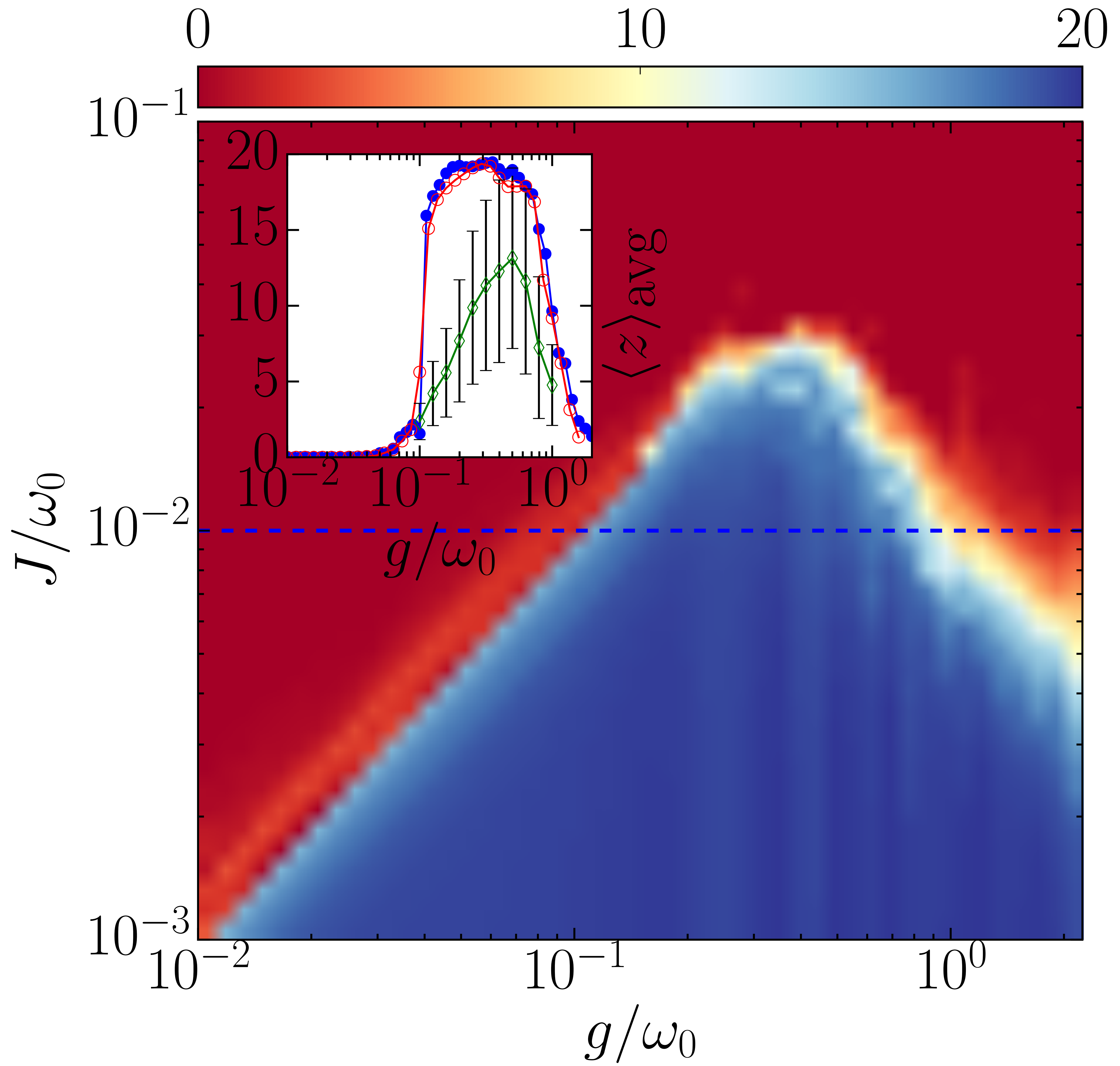
$$\sim g\,(a^{\dagger}+a)(\sigma^{+}+\sigma^{-})$$

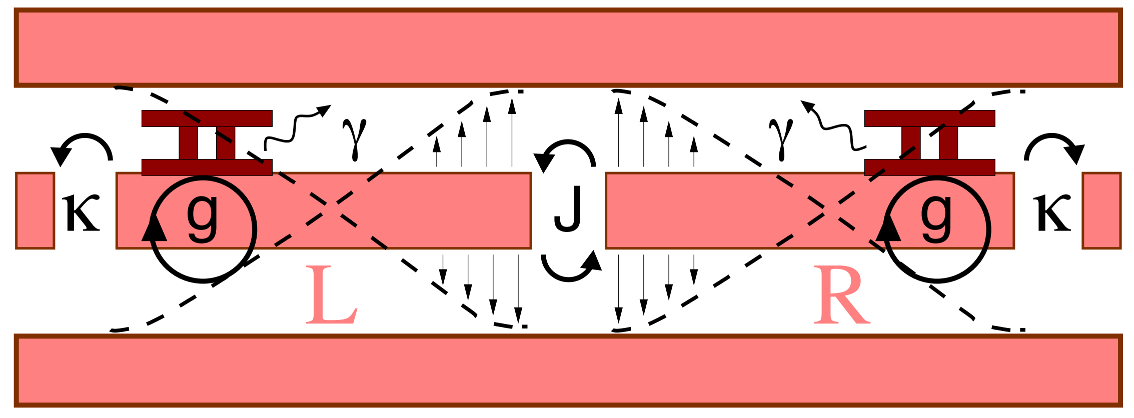
PHOTON BLOCKADE



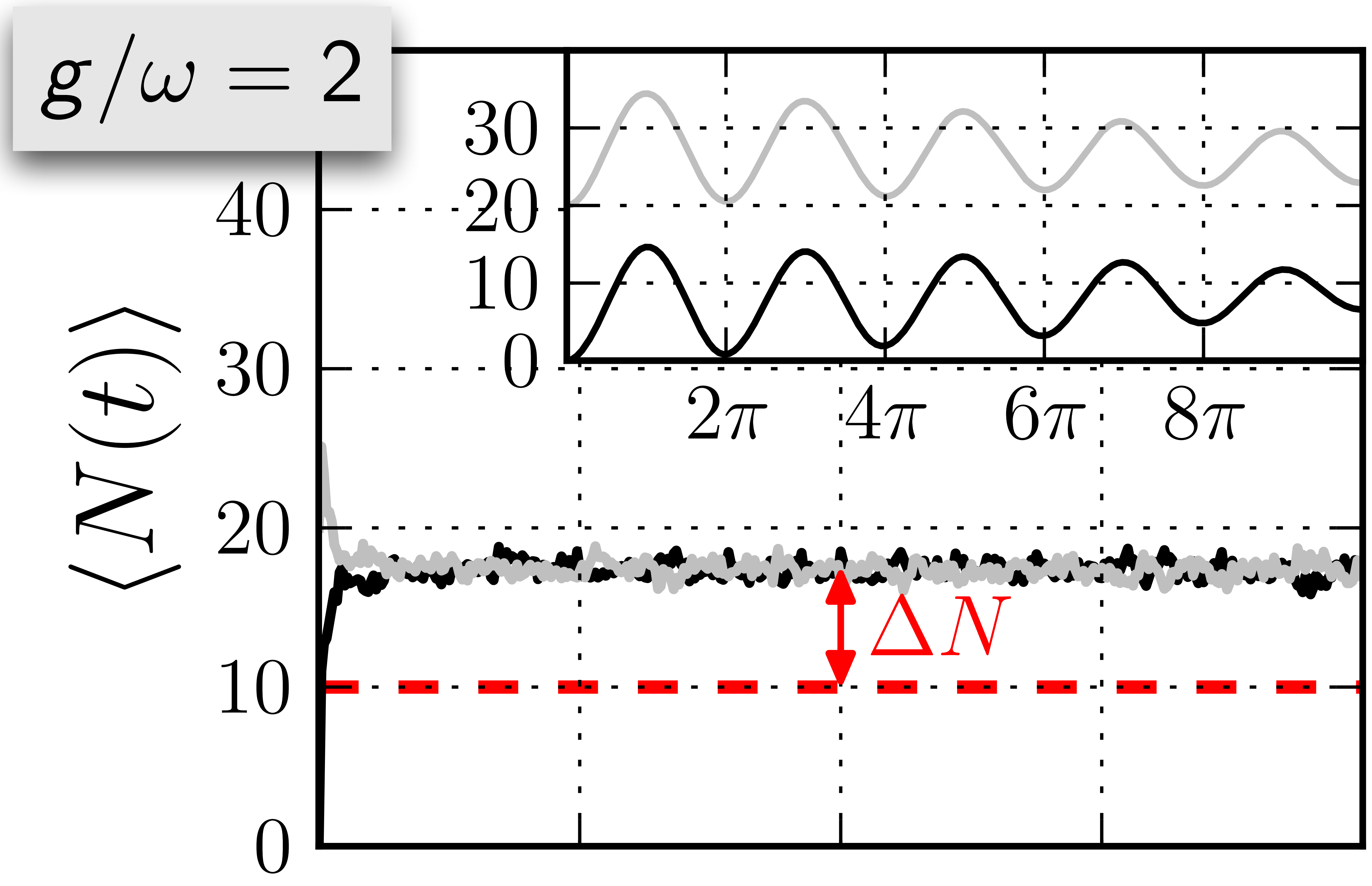
picture from Schmidt *et al.* (PRB, 2010)

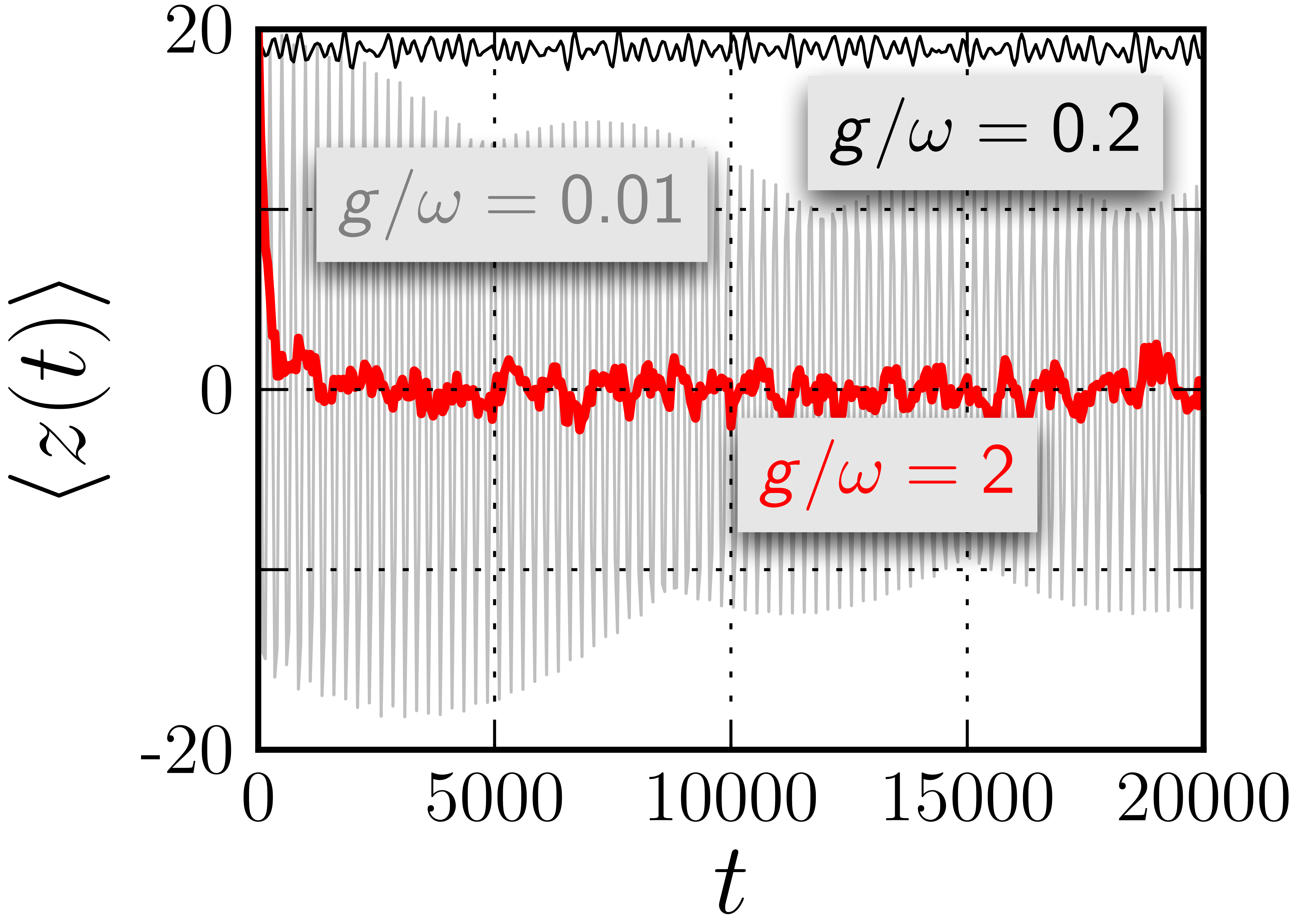
Recurrent Delocalization





Quasi-Equilibrium





Back to the Rabi Model I

from the weak-coupling limit

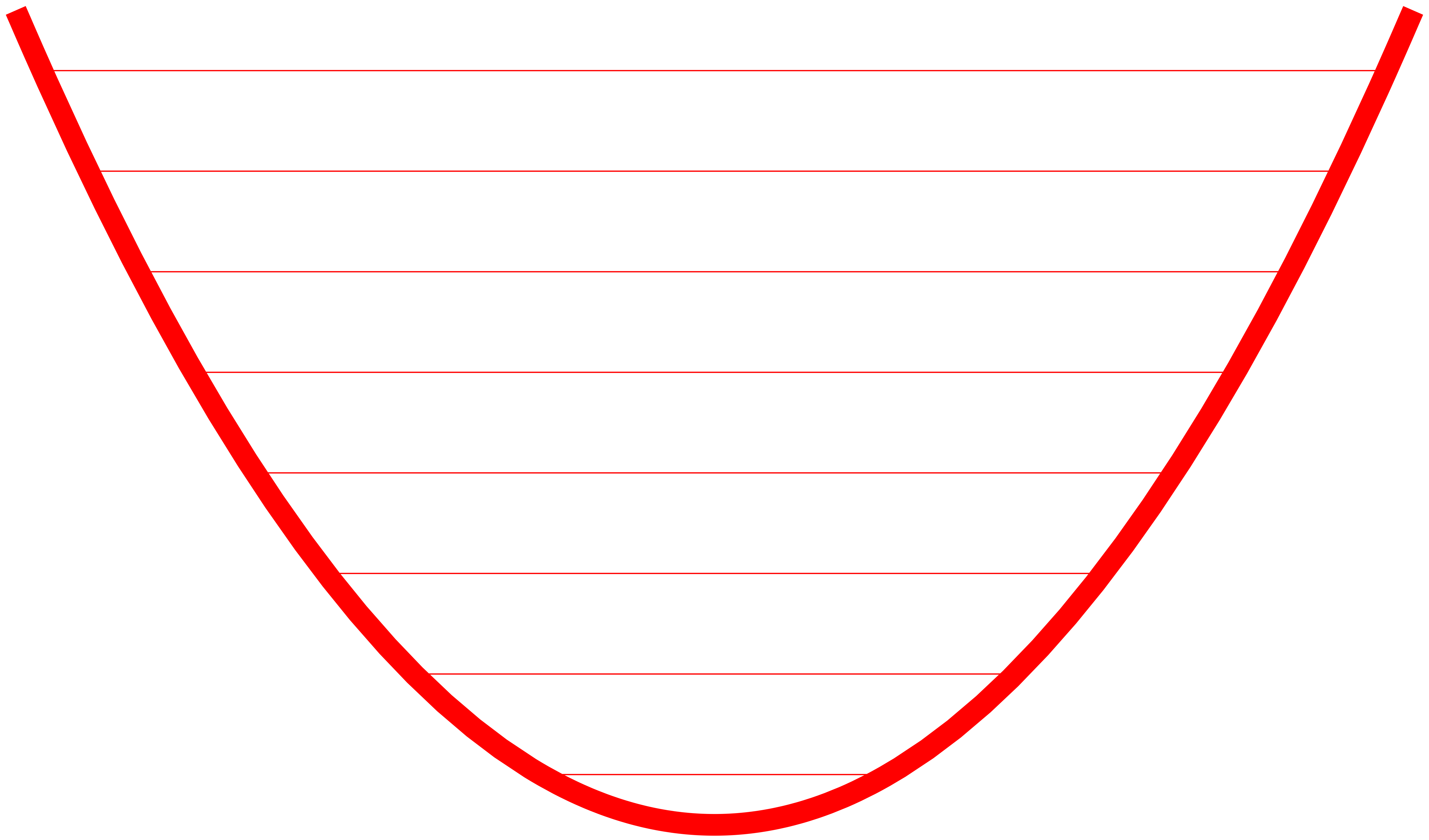
The Rabi Model

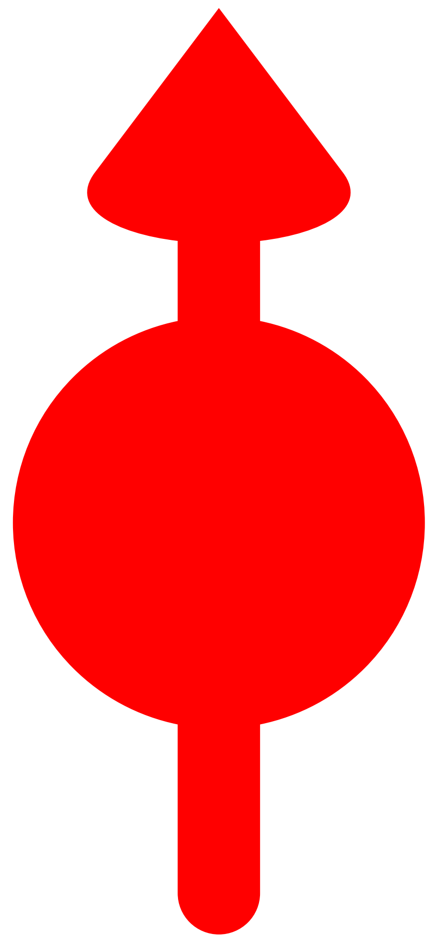
$$H_{\text{Rabi}} = \omega a^\dagger a + \frac{1}{2}\Omega\sigma^z + g(a^\dagger + a)\sigma^x$$

- κ : cavity photon loss rate
- γ : qubit decoherence rate

Weak-Coupling Limit

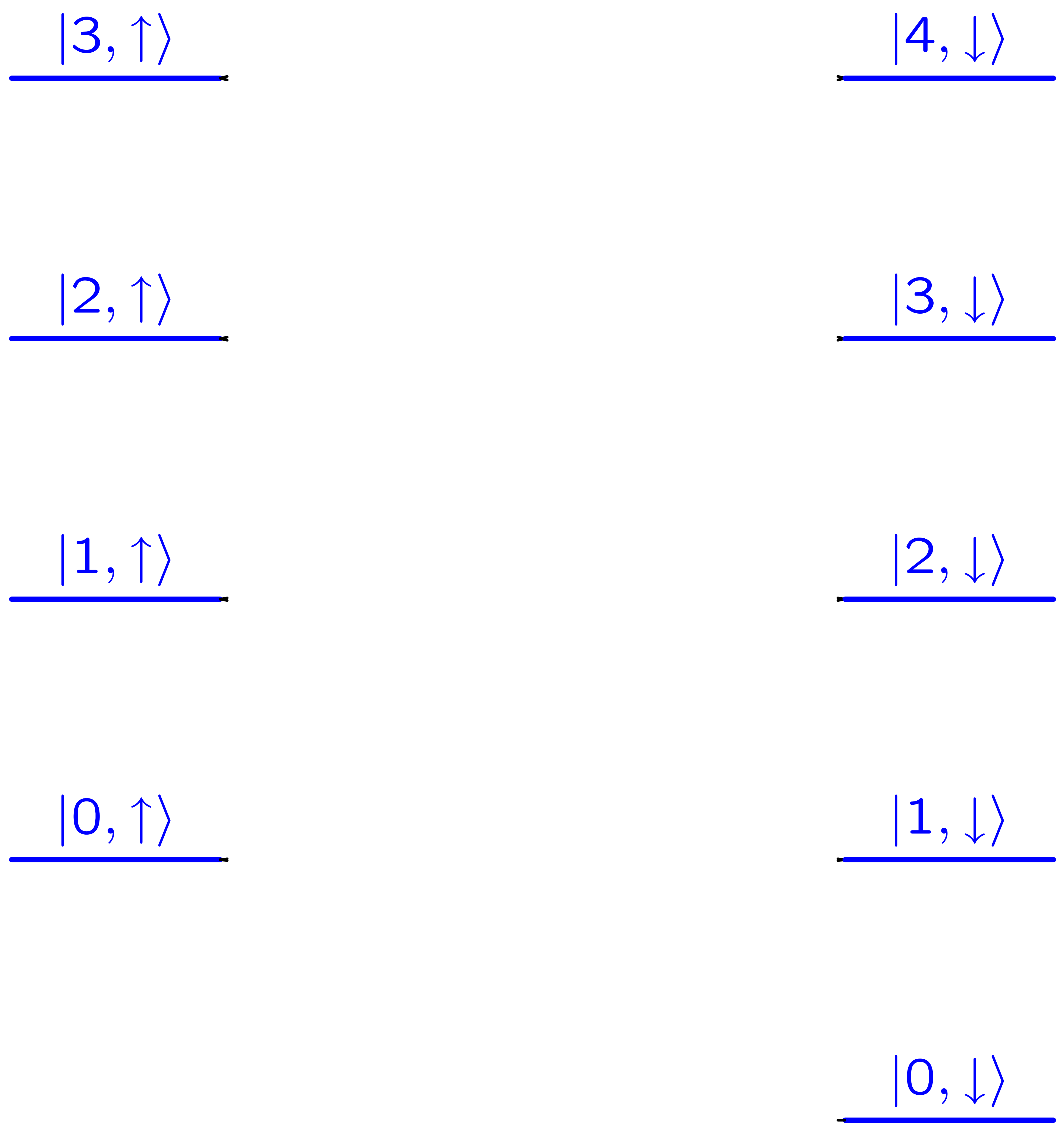
$$g = 0$$





Weak-Coupling Limit

$g = 0$



"Strong" Coupling

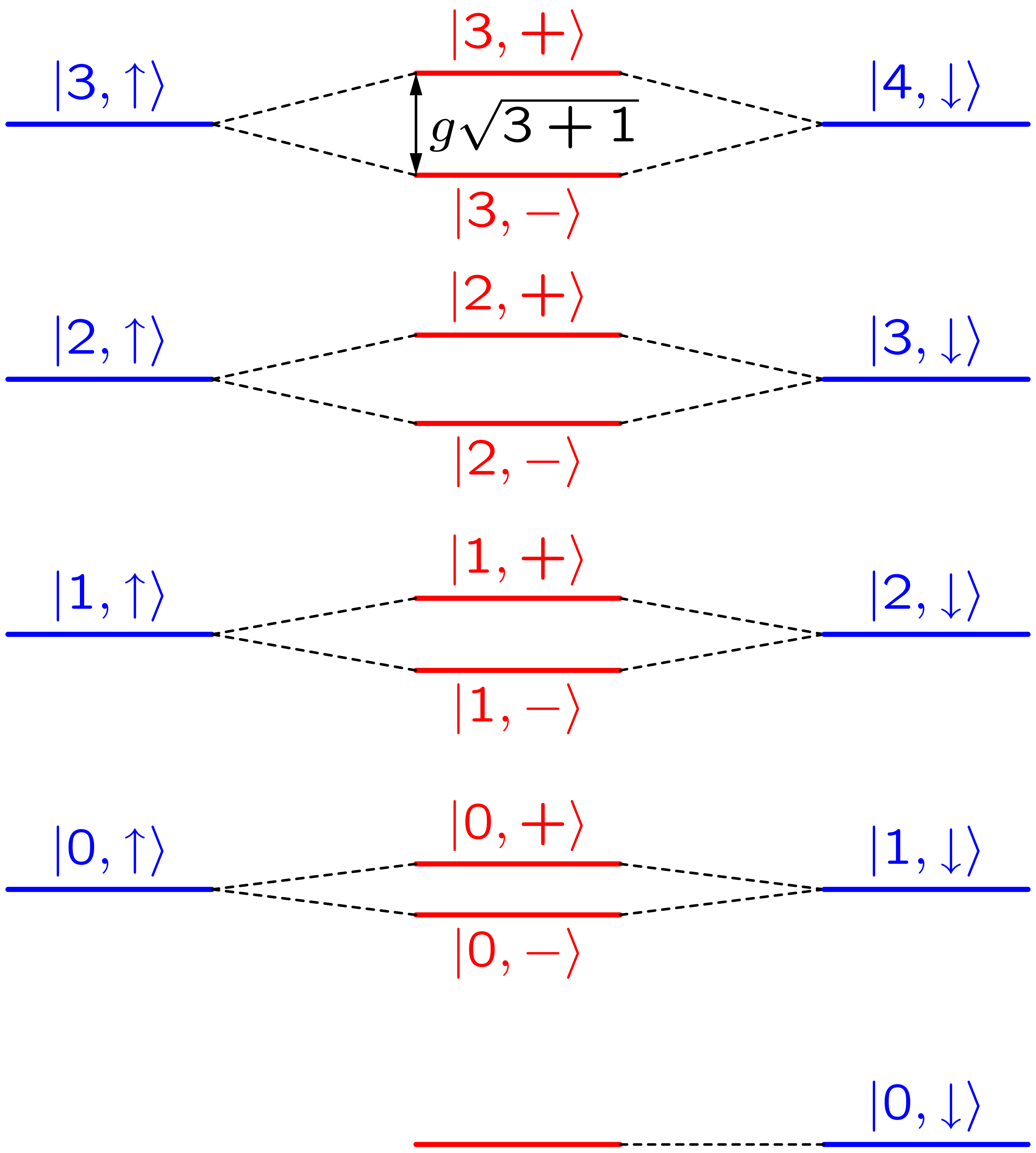
Jaynes-Cummings Model (Rotating Wave Approximation)

$$\kappa, \gamma \ll g \ll \omega$$

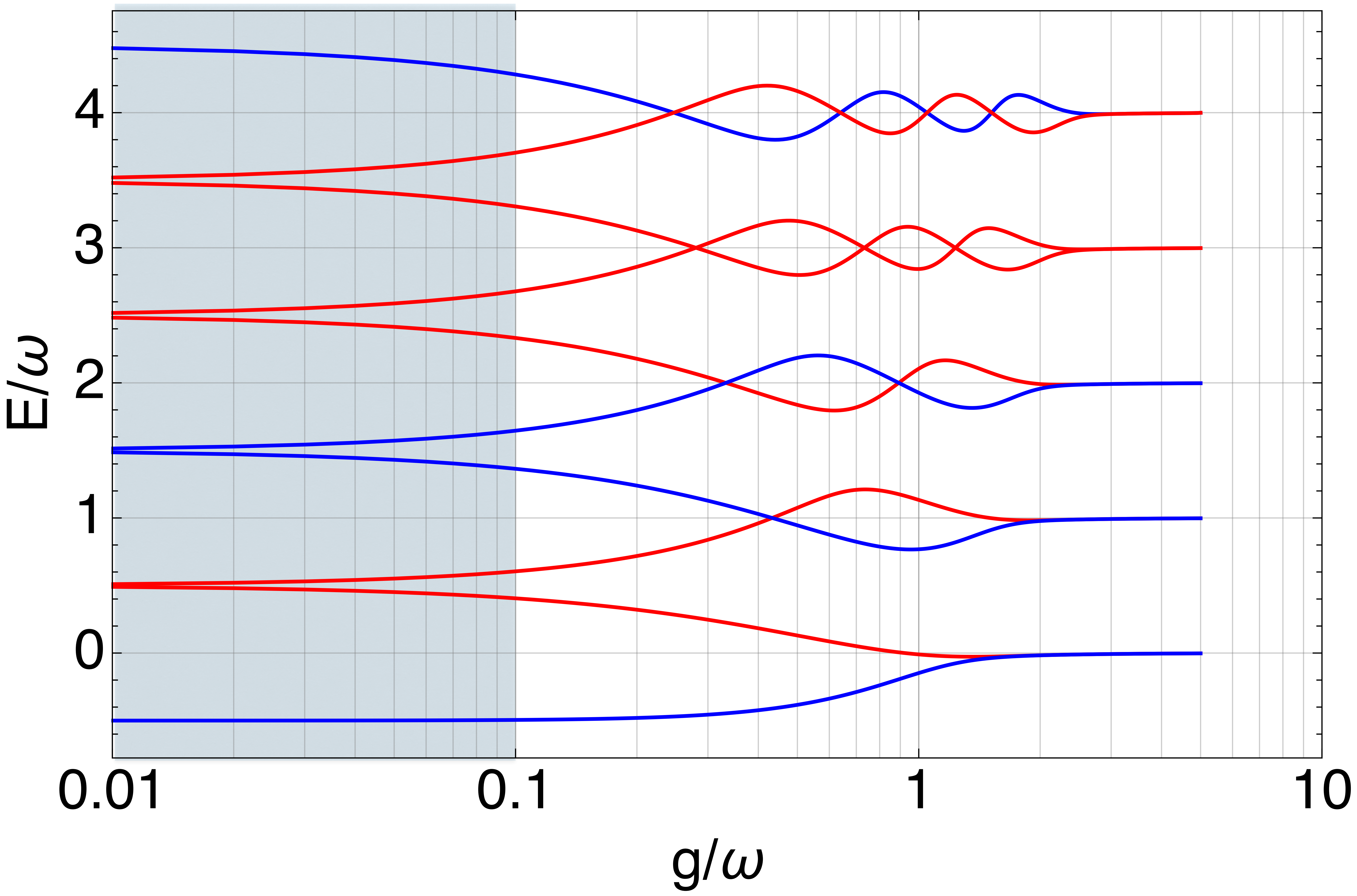
$$(a^\dagger + a)(\sigma^+ + \sigma^-) = \underbrace{(a^\dagger\sigma^- + a\sigma^+)}_{\text{rotating}} + \underbrace{(a^\dagger\sigma^+ + a\sigma^-)}_{\text{counter-rotating}}$$

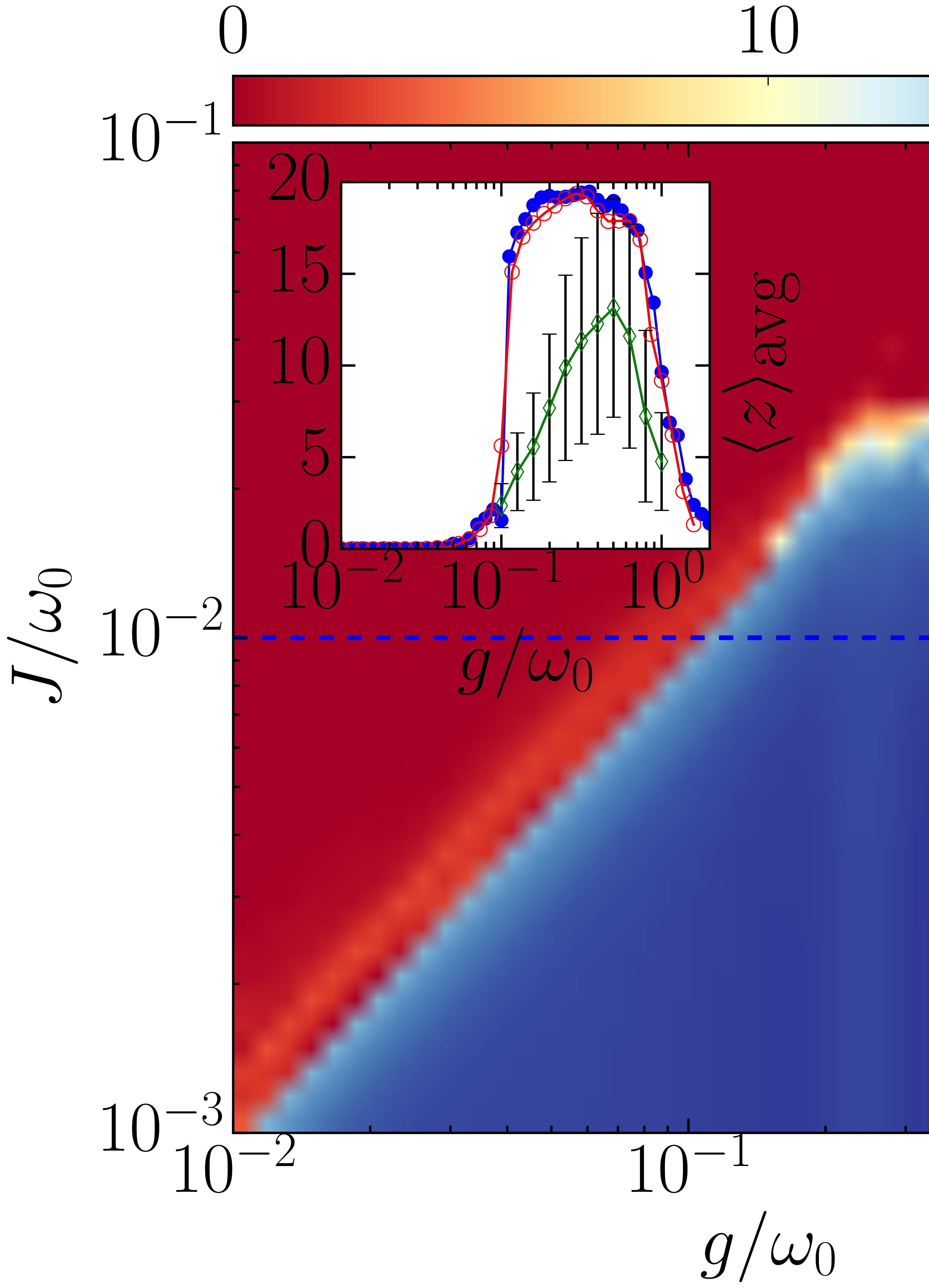
"STRONG" COUPLING

Jaynes-Cummings Model (Rotating Wave Approximation)



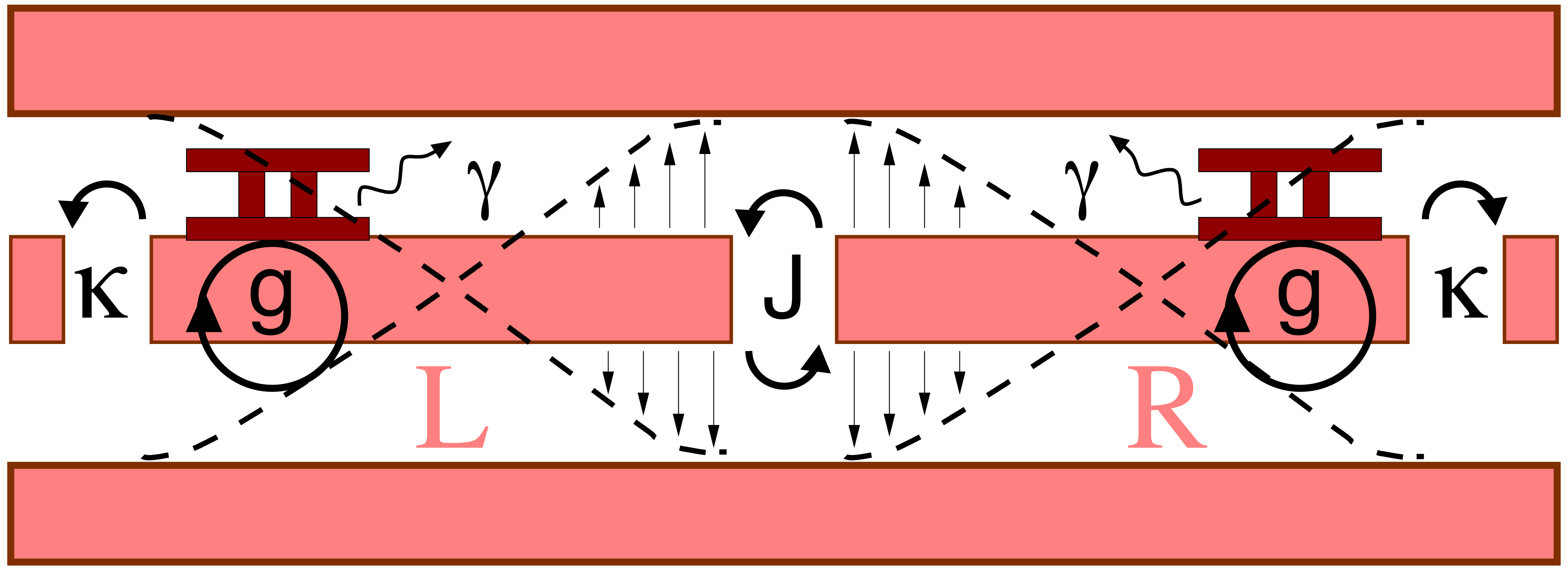
$$g \ll \omega$$

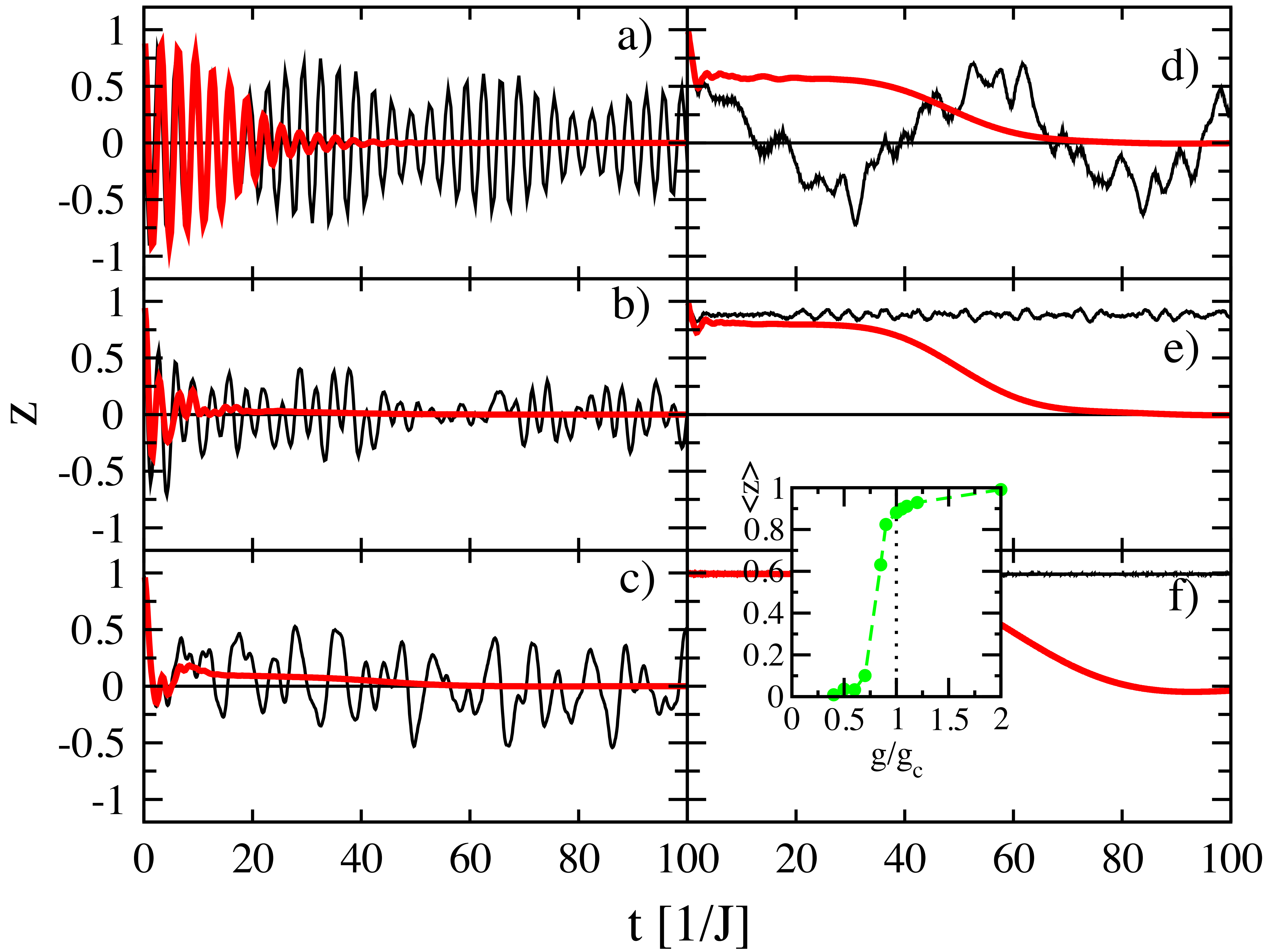

4
3
2
1
0
E/ω
0.01
0.1
1
10
g/ω


0
10
10^{-1}
10^{-2}
10^{-3}
J/ω_0
10^{-2}
10^{-1}
g/ω_0
20
15
10
5
0
$\langle z \rangle$avg
10^{-2}
10^{-1}
10^{0}
g/ω_0

Photon Blockade

Schmidt *et al.*, (PRB, 2010); Raftery *et al.*, (PRX, 2014)





red curve: no decoherence, no photon loss
black curve: small decoherence+photon loss

Back to the Rabi Model II

from the (infinitely) strong-coupling limit

INFINITELY STRONG-COUPLING LIMIT

$$g \to \infty$$

$$H_{\text{Rabi}} = \omega a^\dagger a + \frac{1}{2}\Omega\sigma^z + g(a^\dagger + a)\sigma^x$$

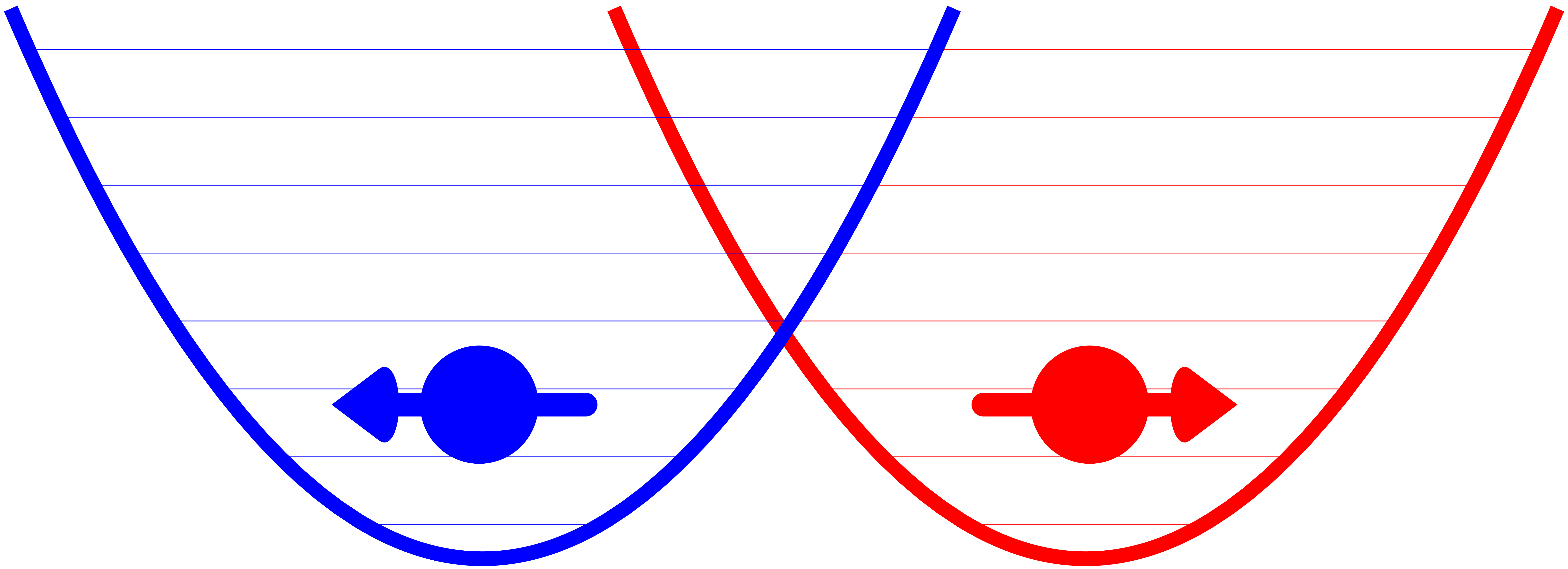
$$|n, \pm\rangle = D(\pm g/\omega)\,|n\rangle \times |\pm\rangle$$

$$D(z) \equiv \exp(z a^\dagger - z^* a)$$

$$E_{n,\pm} \approx \omega n + \mathcal{O}(e^{-g^2/2\omega^2})$$

Infinitely Strong-Coupling Limit

$g \to \infty$



(Franck-Condon effect in polaron physics)

Parity Conserving Representation

Hwang & Choi (PRA, 2010; PRB, 2013)

$$H_{\text{Rabi}} = \omega a^\dagger a + \frac{1}{2}\Omega\sigma^z + g(a^\dagger + a)\sigma^x$$

$$\tau^z \equiv \cos(\pi a^\dagger a)\sigma^z$$

$$b \equiv a\sigma^x$$

$$H_{\text{Rabi}} = \omega b^\dagger b - g(b^\dagger + b) + \frac{1}{2}\Omega\cos(\pi b^\dagger b)\tau^z$$

Dressed States

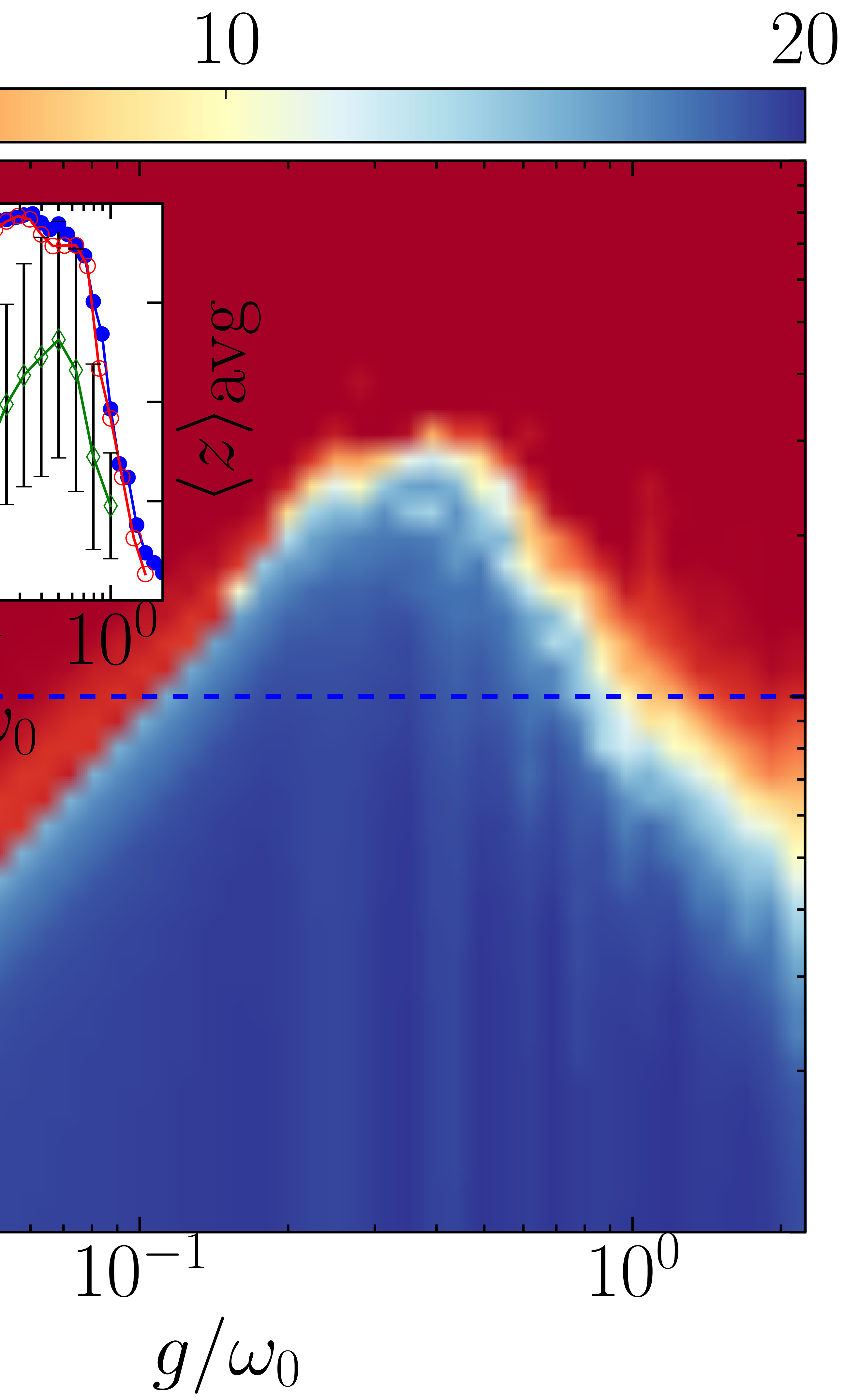
(Generalized Rotating Wave Approximation)

Feranchuk et al. (JPA, 1996); Irish (PRL, 2007)
Hwang & Choi (PRA, 2010; PRB, 2013)

$$|n, e/o\rangle = D_b(g/\omega)\,|n\rangle_b \times |e/o\rangle$$

$$E_{n,e/o} \approx \omega n + \mathcal{O}(e^{-g^2/2\omega^2})$$

$$g \gg \omega$$


10
20
$\langle z \rangle$avg
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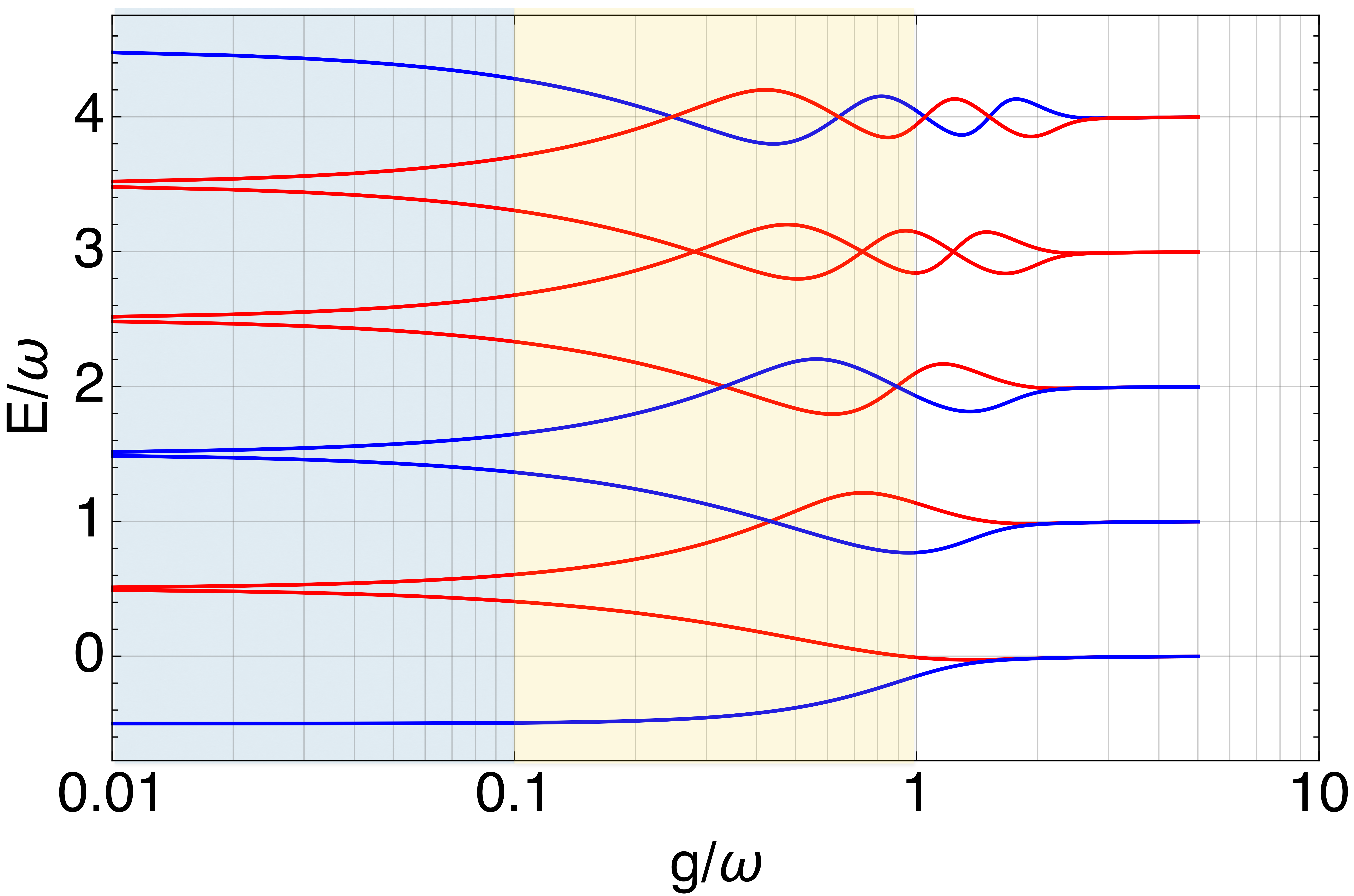
10^{-1}
10^0
g/ω_0

E/ω
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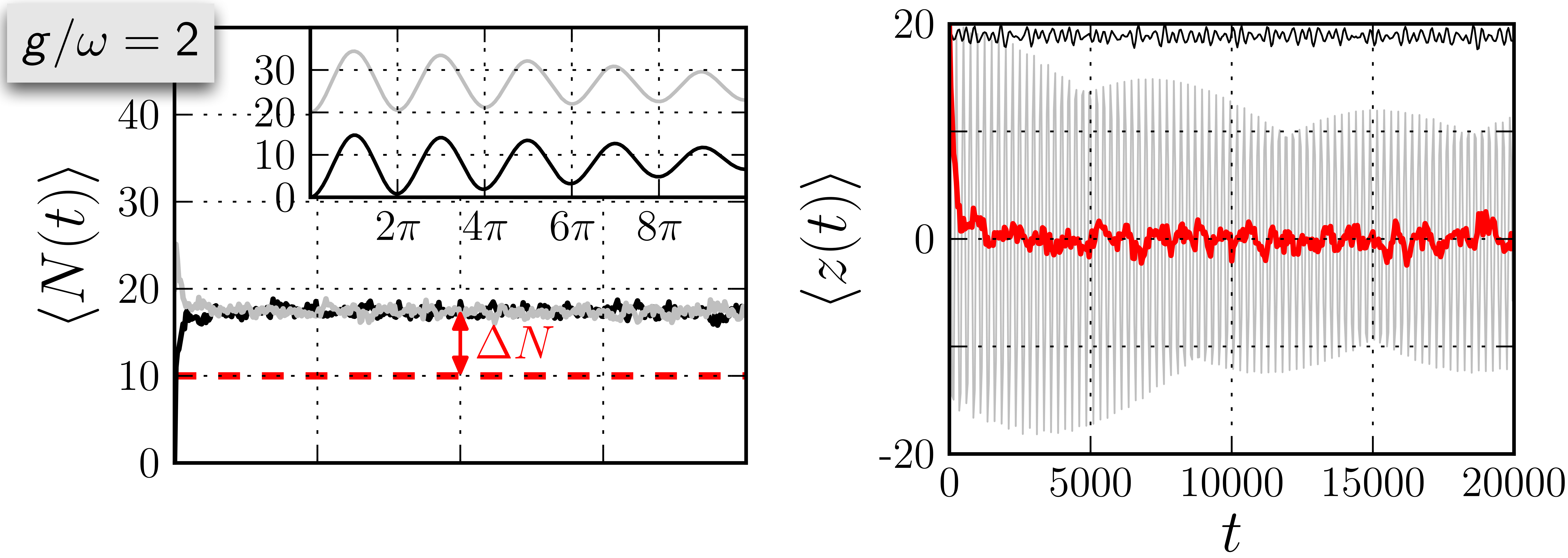
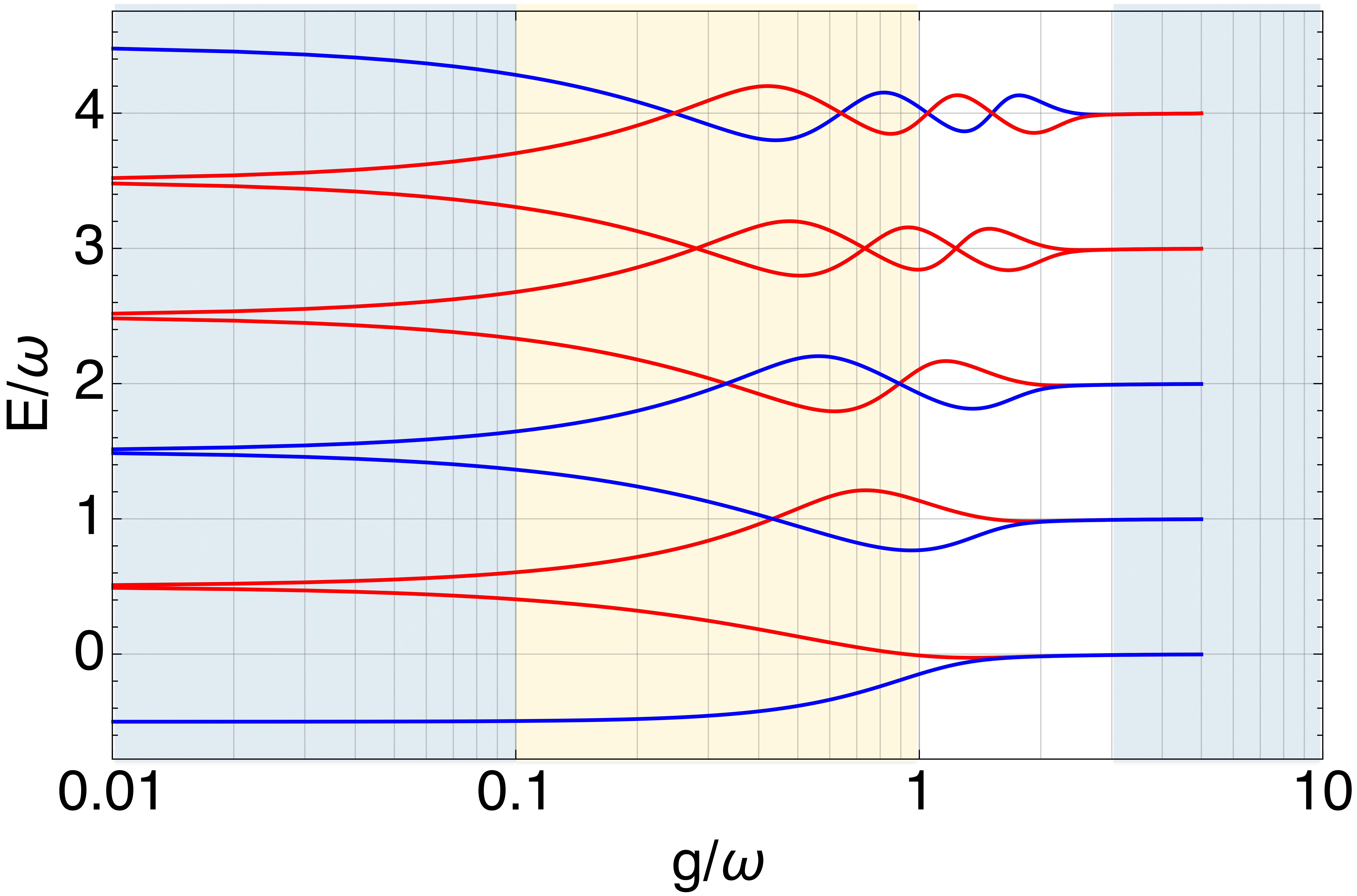
g/ω

Quasi-Equilibrium



One more transition!



E/ω
4
3
2
1
0
0.01
0.1
1
10
g/ω

DUALITY

DUALITY

Hwang & Choi (PRA, 2010)
Hwang, Puebla, & Plenio (PRL, 2015)

$$H \to \widetilde{H} = D^\dagger(\alpha) H D(\alpha)\,, \quad \alpha \equiv \frac{1}{2}\sqrt{\frac{\Omega}{\omega}\left(\frac{4g^2}{\omega\Omega} - \frac{\omega\Omega}{4g^2}\right)}$$

$$\widetilde{H} \approx \omega a^\dagger a + \frac{1}{2}\tilde{\Omega}\tilde{\sigma}^z - \tilde{g}(a^\dagger + a)\tilde{\sigma}^x$$

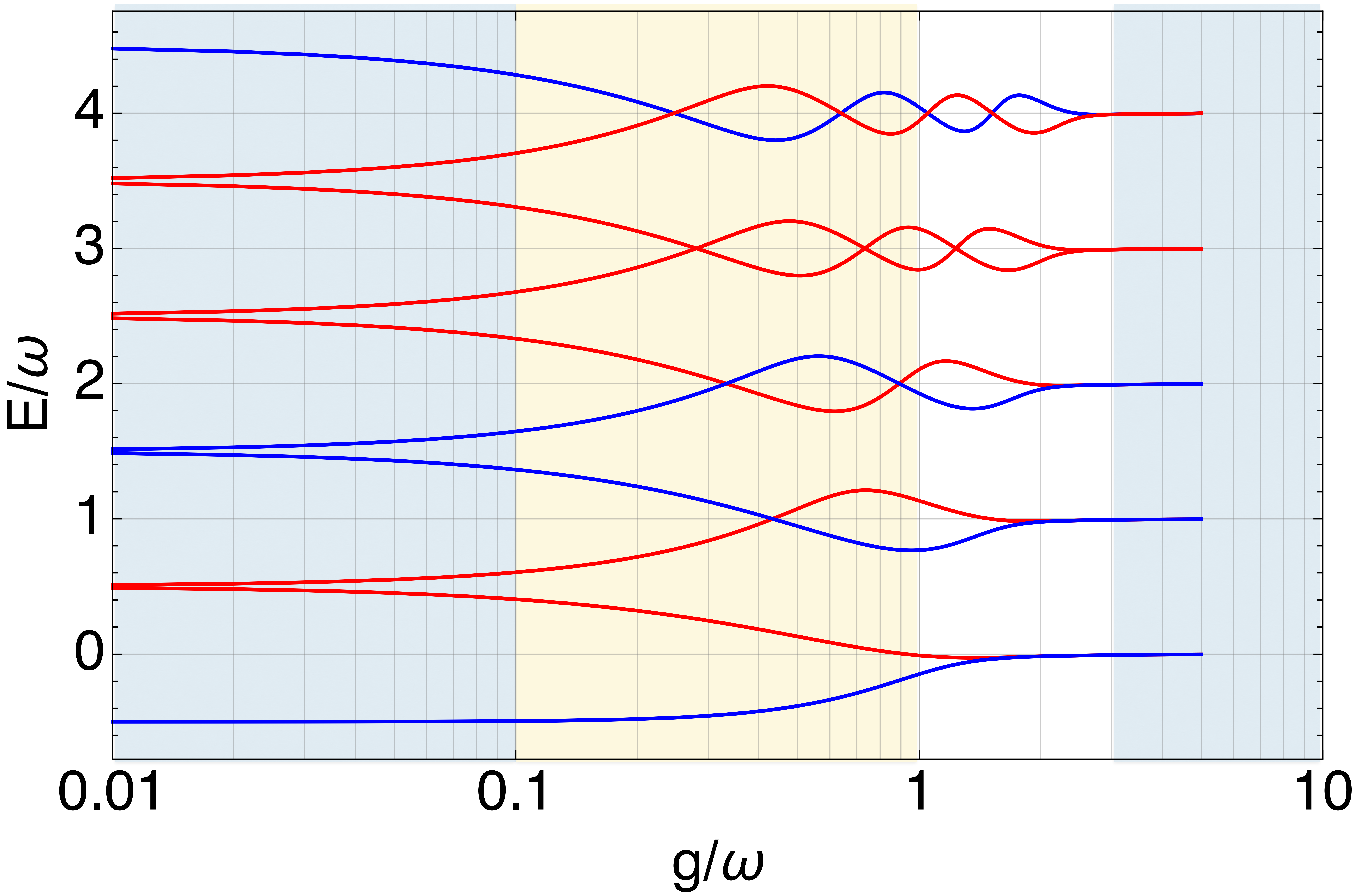
$$\frac{\tilde{\Omega}}{\Omega} \equiv \frac{4g^2}{\omega\Omega} \qquad \frac{\tilde{g}}{g} \equiv \frac{\omega\Omega}{4g^2}$$

Duality!

Hwang & Choi (PRA, 2010)
Hwang, Puebla, & Plenio (PRL, 2015)

$$\widetilde{H} = \omega a^\dagger a + \frac{1}{2}\tilde{\Omega}\tilde{\sigma}^z - \tilde{g}(a^\dagger + a)\tilde{\sigma}^z$$

$$\frac{2\tilde{g}^2}{\omega\tilde{\Omega}} = \frac{\omega\Omega}{2g^2}$$


4
3
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E/ω
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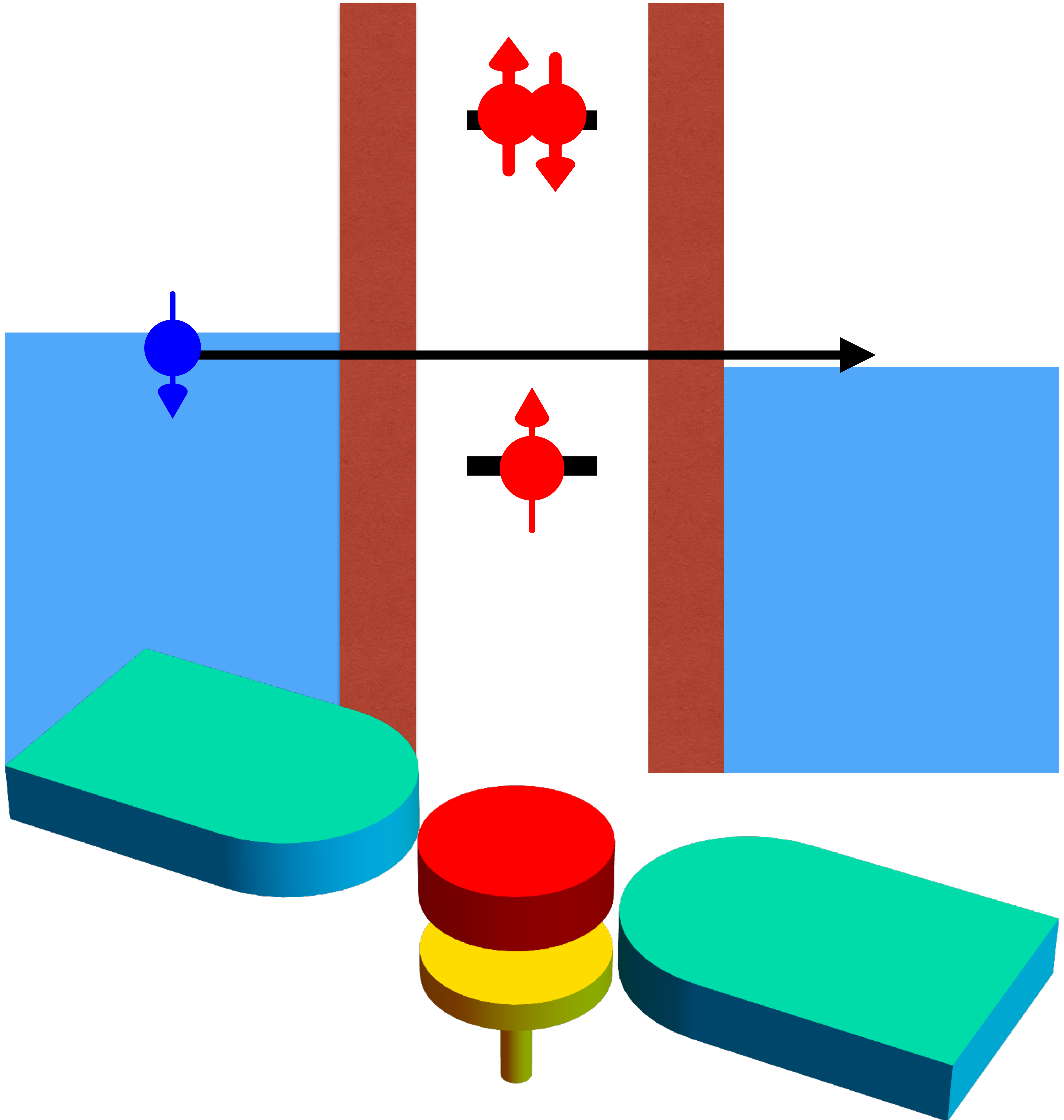
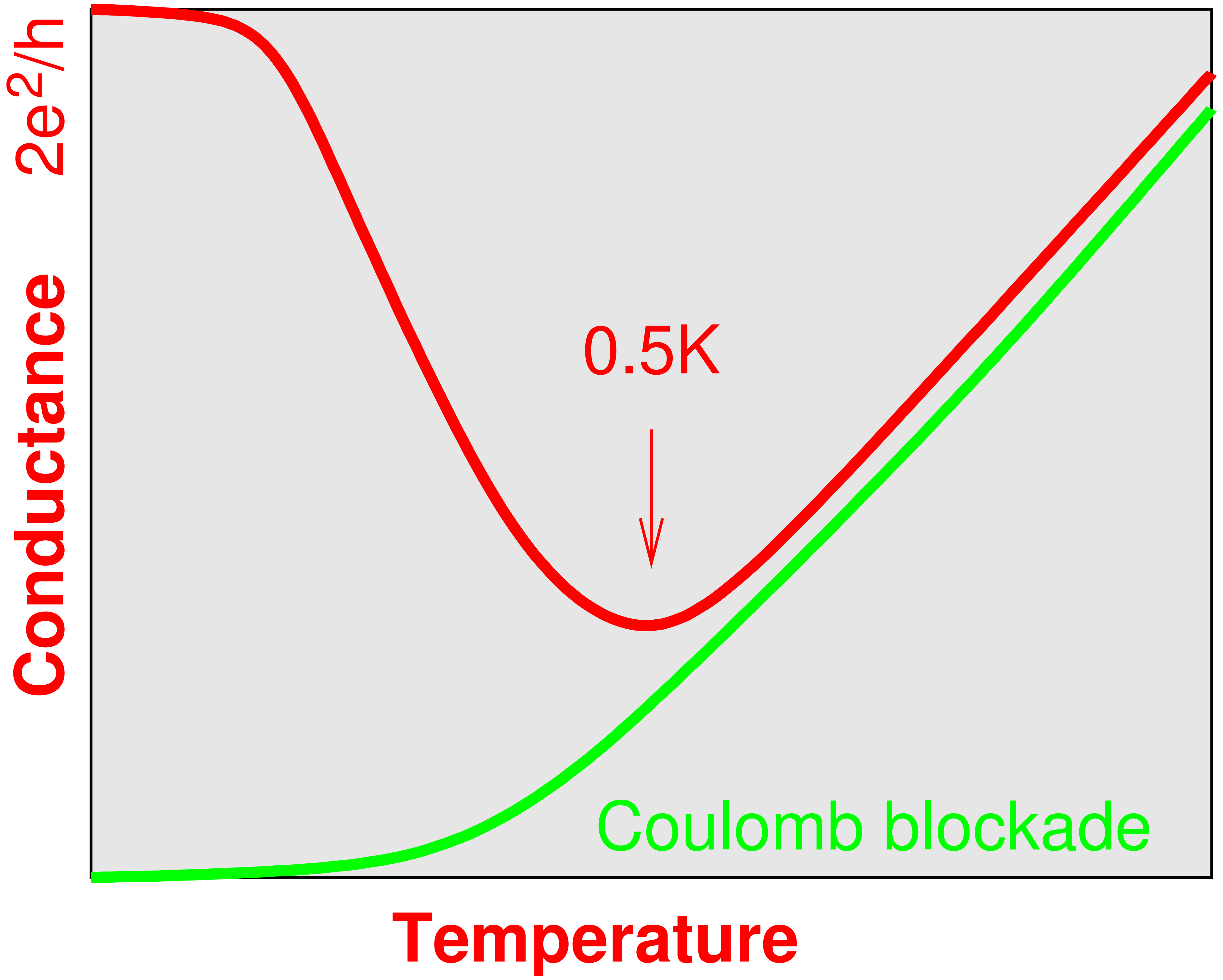
10
g/ω

Quantum Sensing

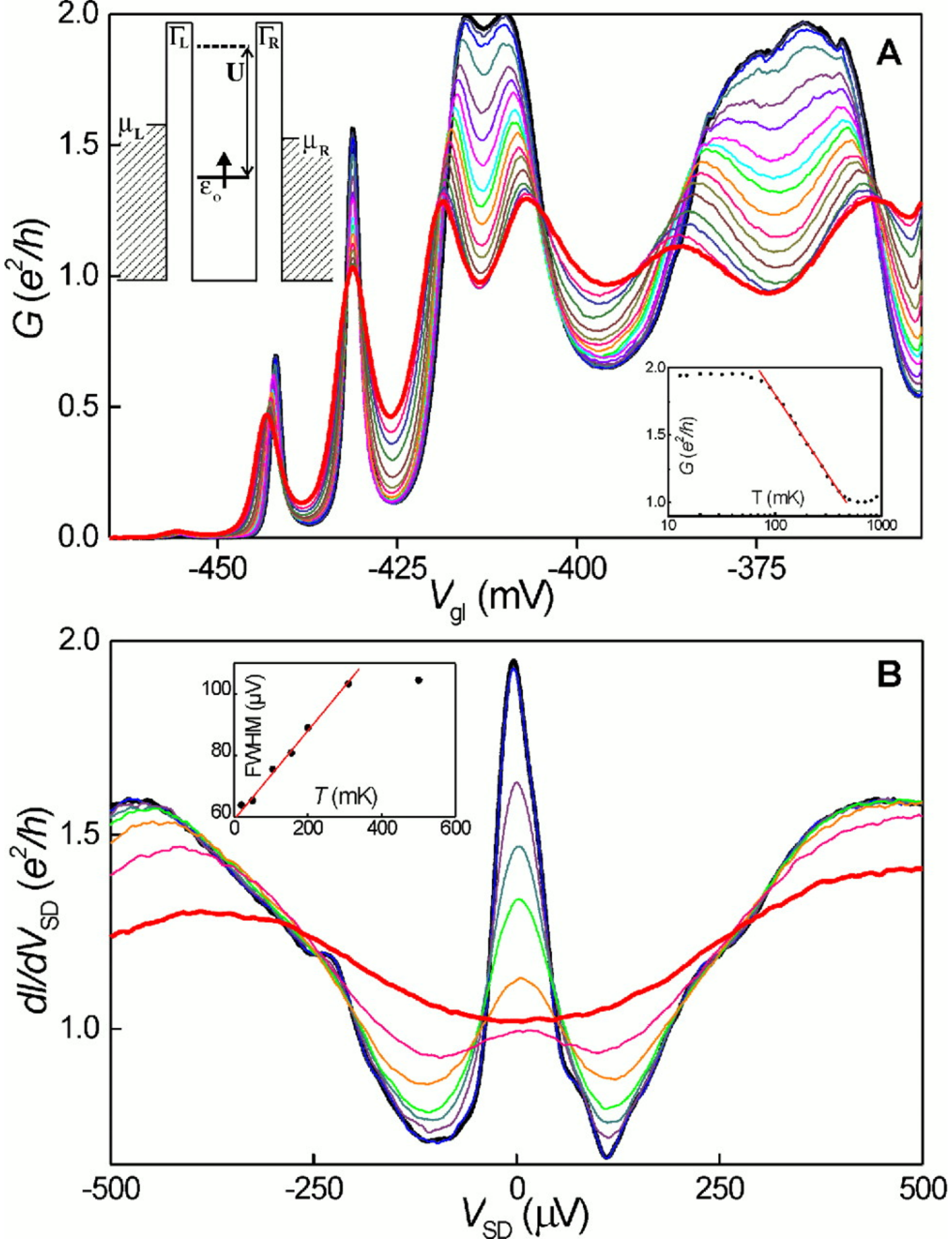
Kondo Physics

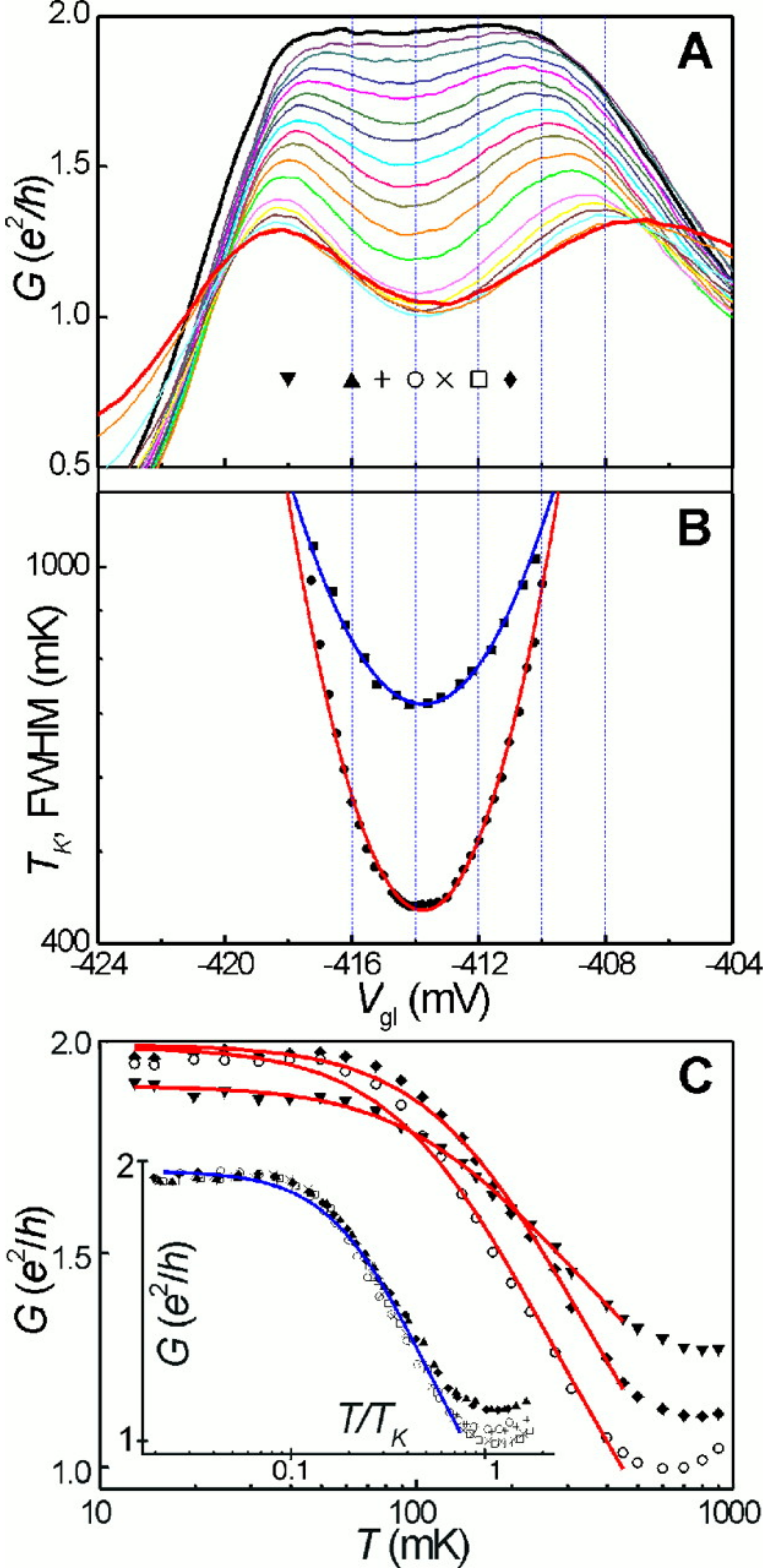
Kondo Effect in Nano-Devices





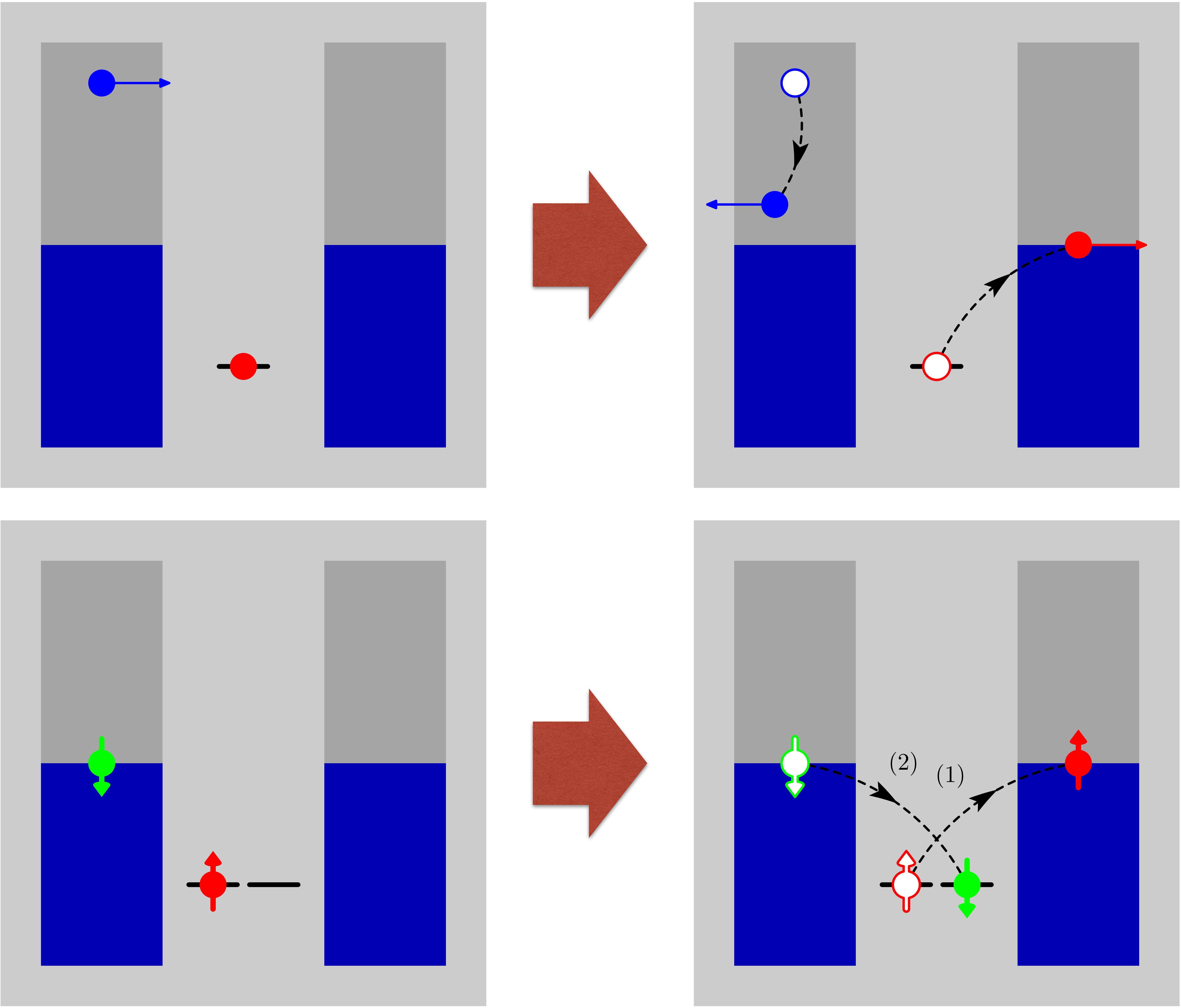
- Glazman & Raikh, Pis'ma Zh. Eksp. Teor. Fiz. (1988),
- Ng & Lee, Phys. Rev. Lett. (1988), ...
- Goldhaber-Gordon et al., Nature (1998),
- Cronenwett et al., Science (1998),
- Wiel et al., Science (2000), ...





van der Wiel et al. (Science, 2000)

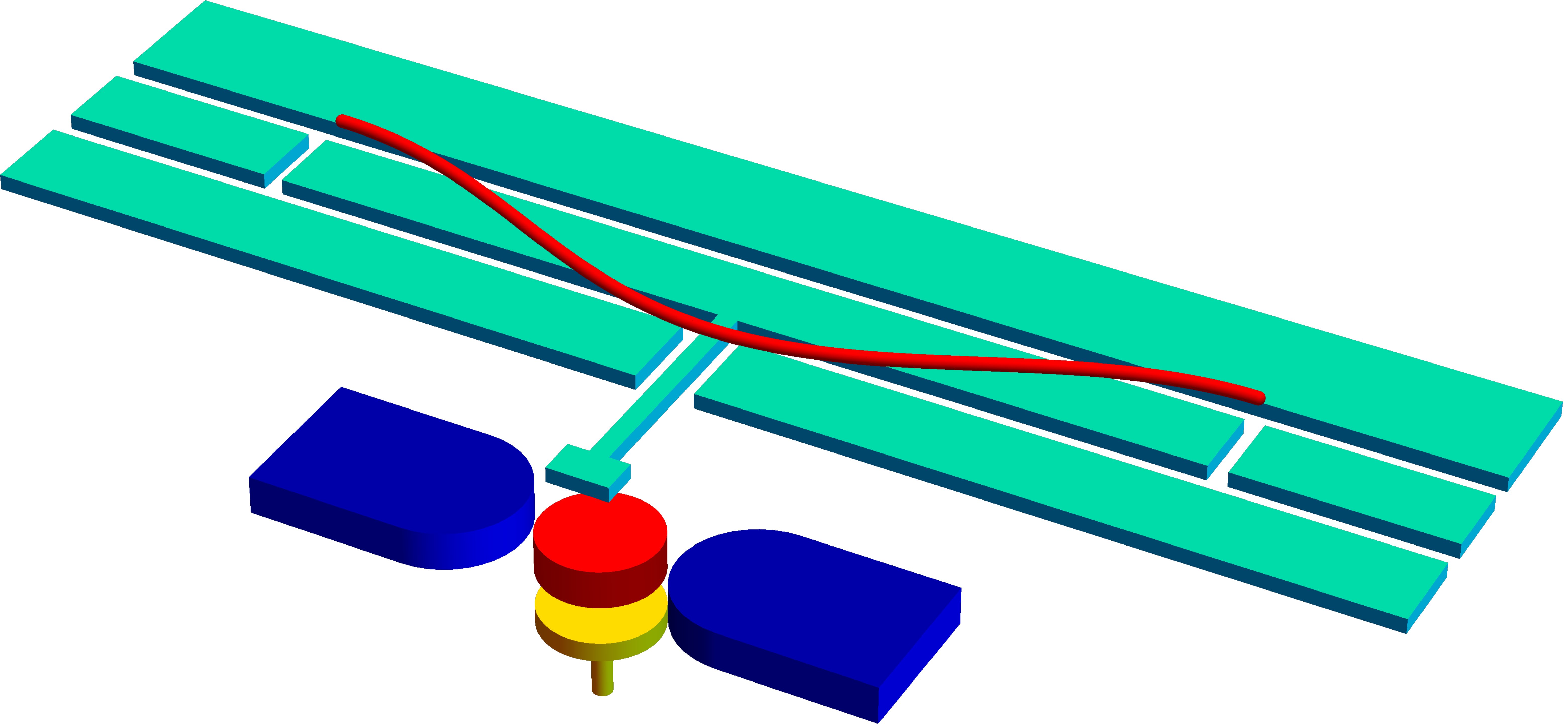
Charge vs Spin Fluctuations



New Experiment on Kondo

Nature 545, 71 (2017)

M. Desjardins, J. Viennot, M. Dartiailh, L. Bruhat, M. Delbecq,
M. Lee, M.-S. Choi, A. Cottet, T. Kontos



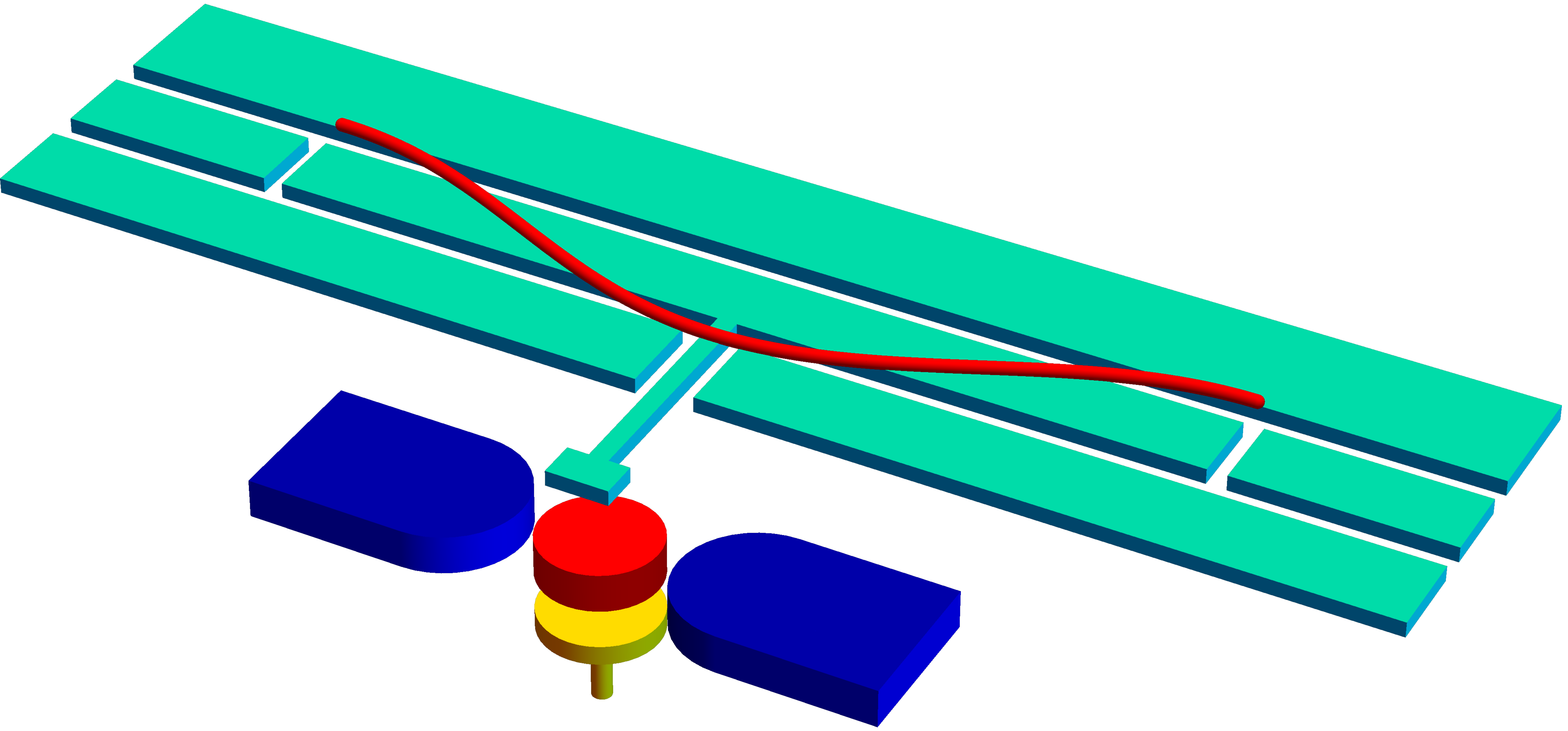
$$\chi := \frac{\partial N}{\partial \mu} = \langle\langle n_d ; n_d \rangle\rangle_\omega$$

$$H = H_{QD} + \omega_0 a^\dagger a + g n_d (a + a^\dagger) + H_{\text{photon loss}}$$

$$D^R(\omega) \approx D_0^R(\omega) + g^2 D_0^R(\omega) \chi(\omega) D^R(\omega)$$

$$D^R(\omega) \approx \frac{1}{\omega - \omega_0 - g^2 \chi(\omega) + i\kappa}$$

Bruhat *et al.* (PRX, 2016)

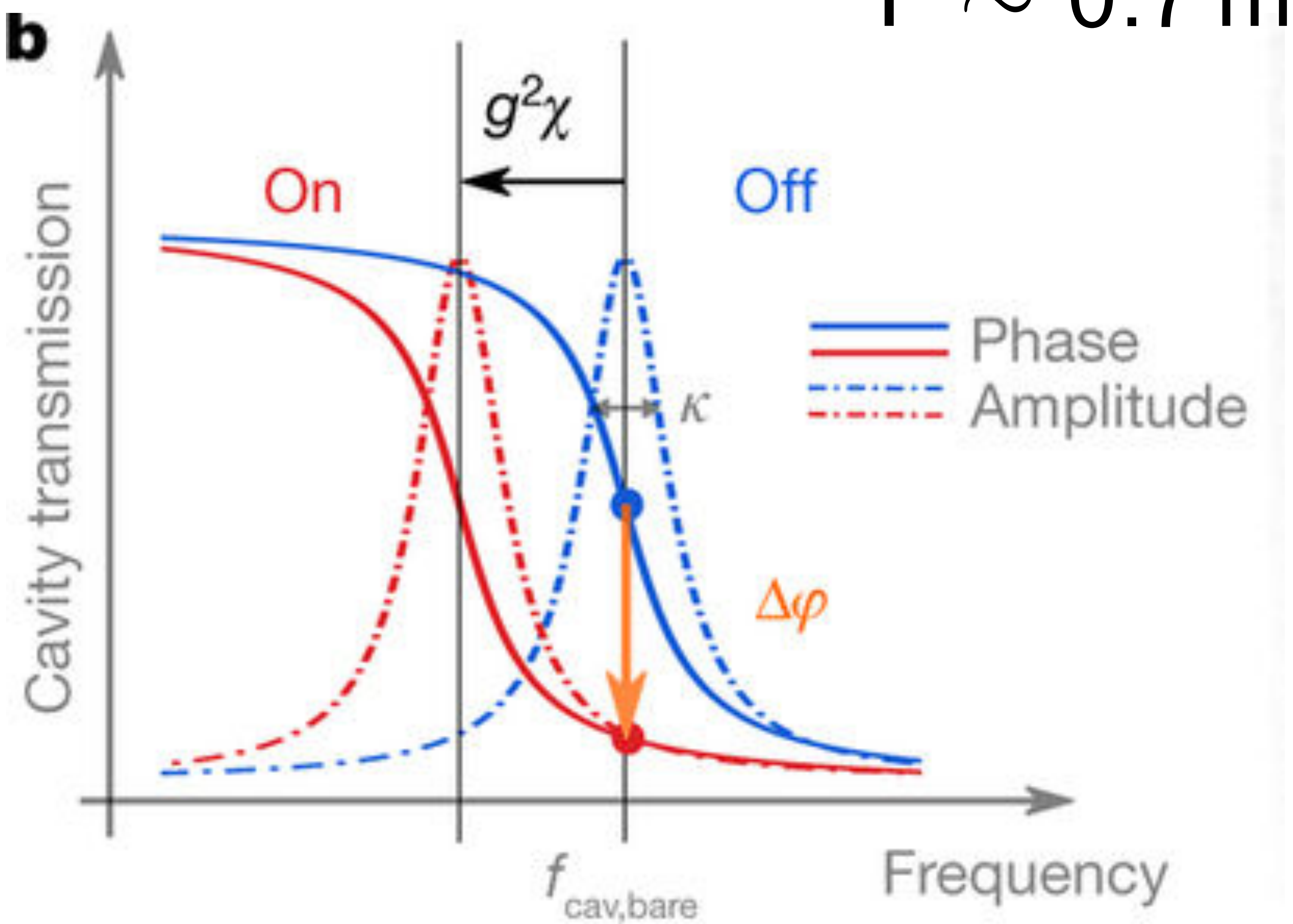


$$\omega_{mw}/2\pi \sim 6.7\,\text{GHz}$$

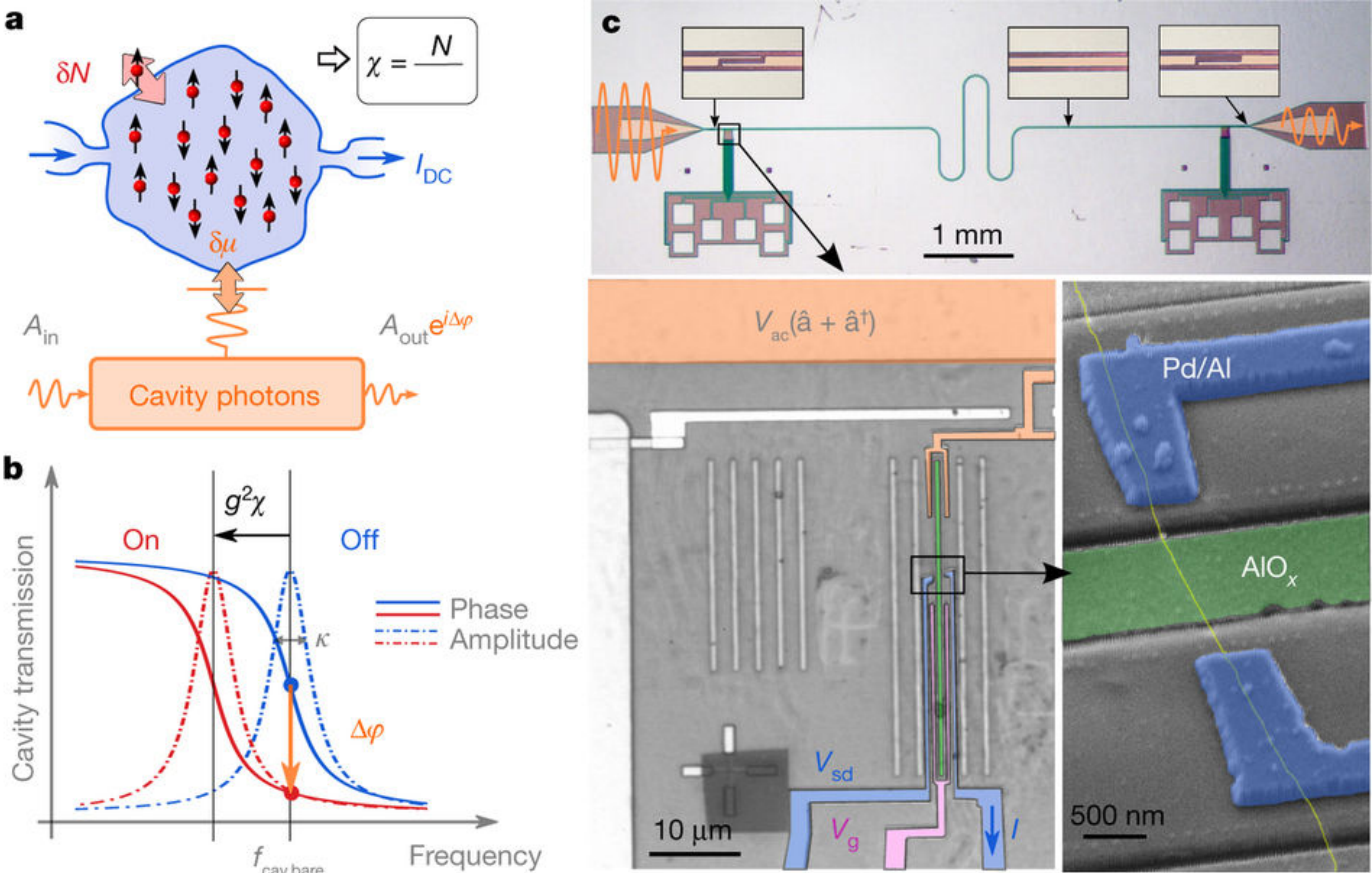
$$g/2\pi \sim 65\,\text{MHz}$$

$$T \sim 250\,\text{mK}$$

$$\Gamma \sim 0.7\,\text{meV}$$

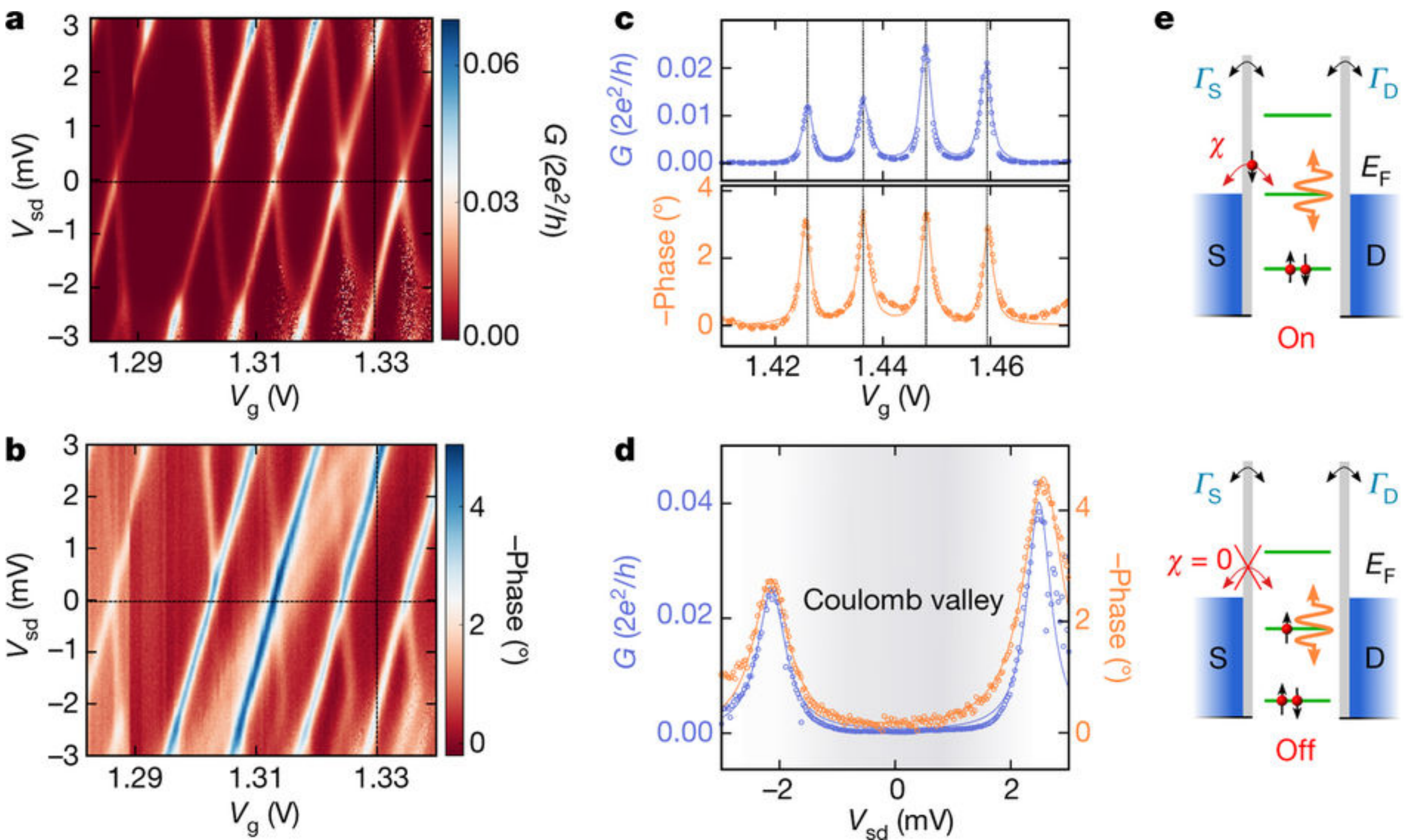


$$\chi := \frac{\partial N}{\partial \mu} = \langle\langle n_d; n_d \rangle\rangle_\omega$$

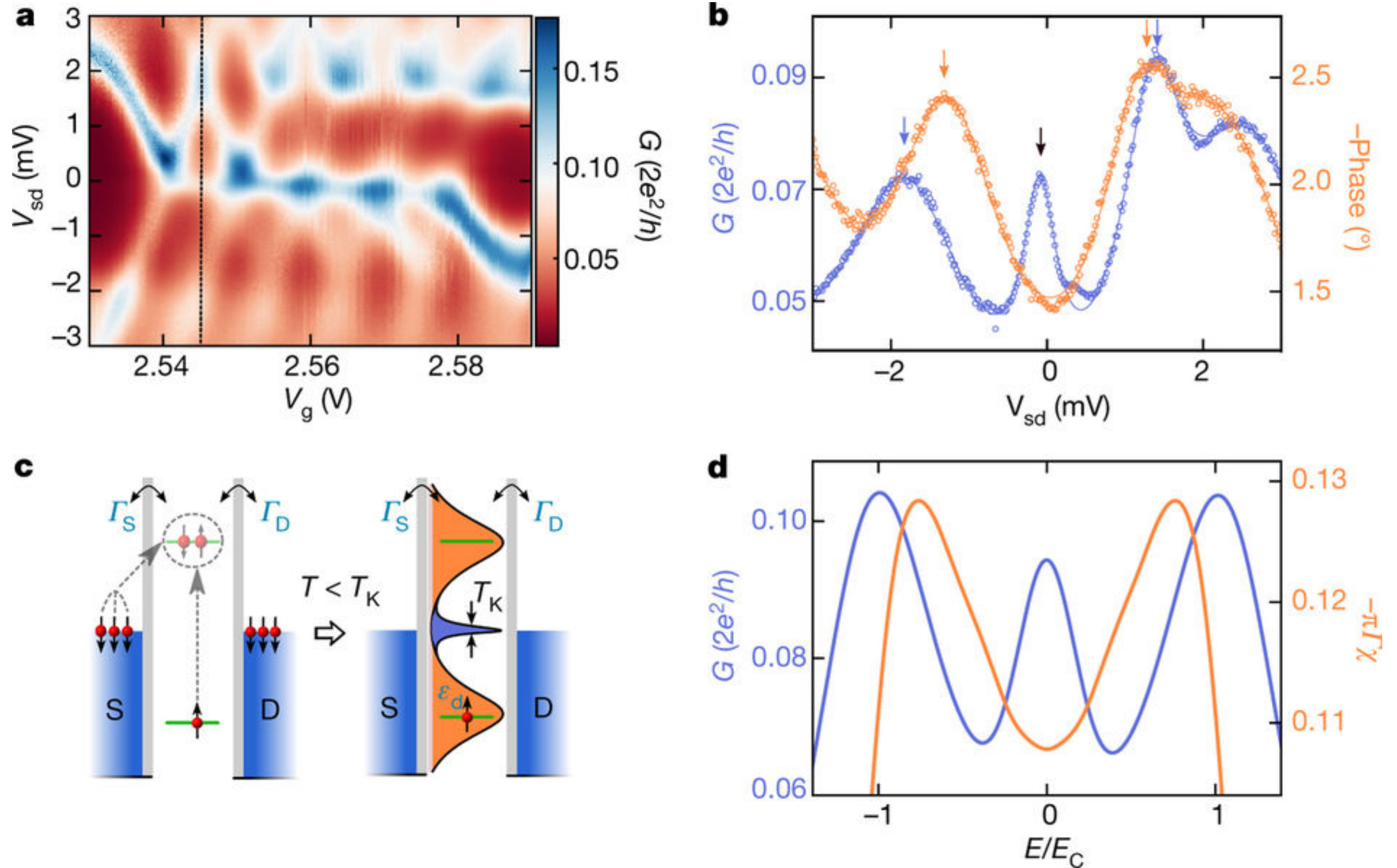


a
δN
χ = N
I_DC
δμ
A_in
A_out e^{iΔφ}
Cavity photons
b
g²χ
On
Off
κ
Δφ
Phase
Amplitude
Cavity transmission
Frequency
f_cav,bare
c
1 mm
V_ac(â + â†)
V_sd
V_g
I
10 μm
Pd/Al
AlO_x
500 nm

Coulomb blockade Regime

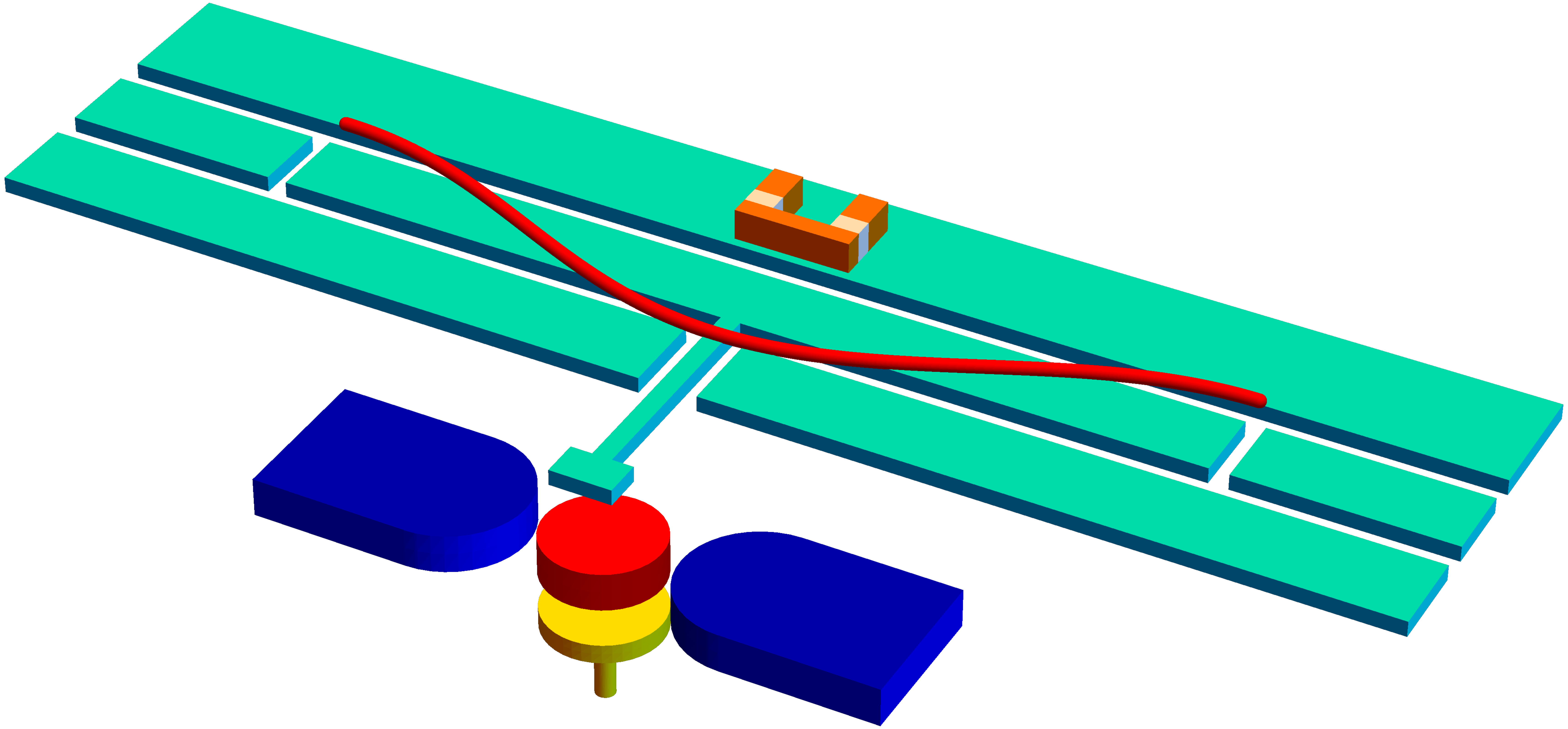


Kondo Ridge



a
b
c
d
V_{sd} (mV)
V_g (V)
G (2e²/h)
-Phase (°)
Γ_S
Γ_D
S
D
$T < T_K$
T_K
ε_d
E/E$_C$
$-\pi\Gamma\chi$

Manipulation of Decoherence

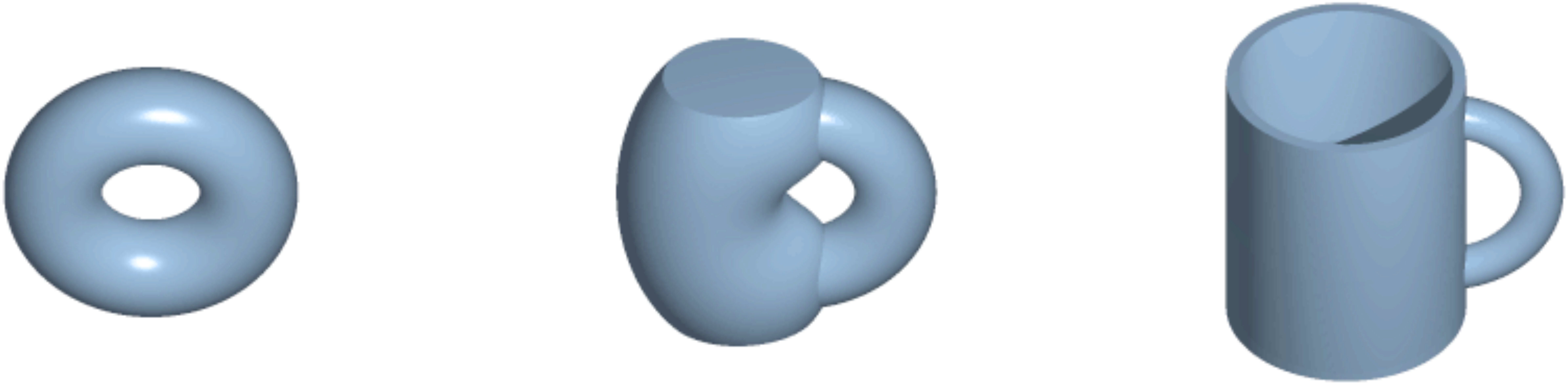


Fundamental Interaction?

Minimal Model for Light-Topological Matter Interaction?

What is the Wikipedia animation about?

Homeomorphism



Topological states cannot be studied by homeomorphism!

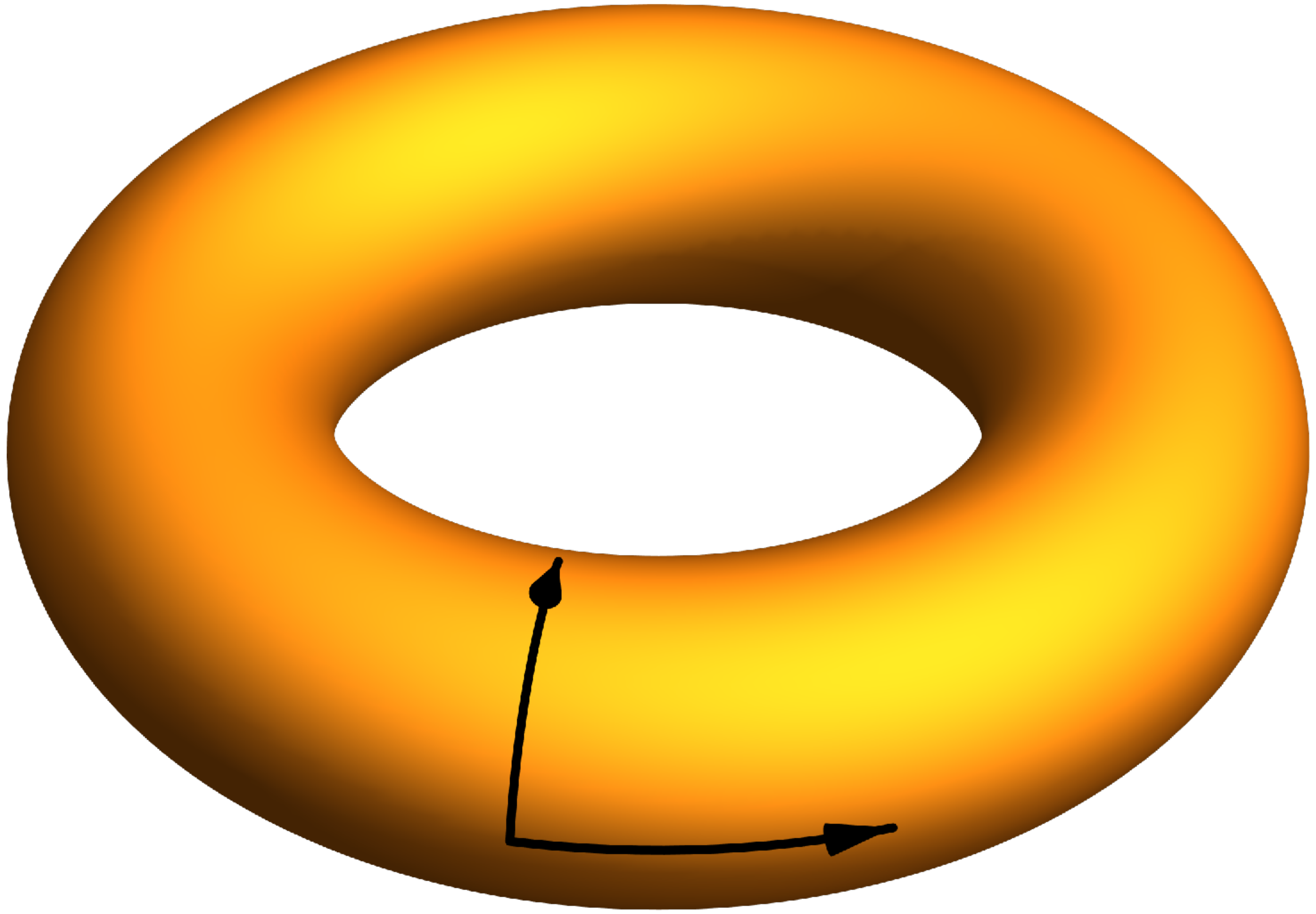
Homeomorphism vs Homotopy

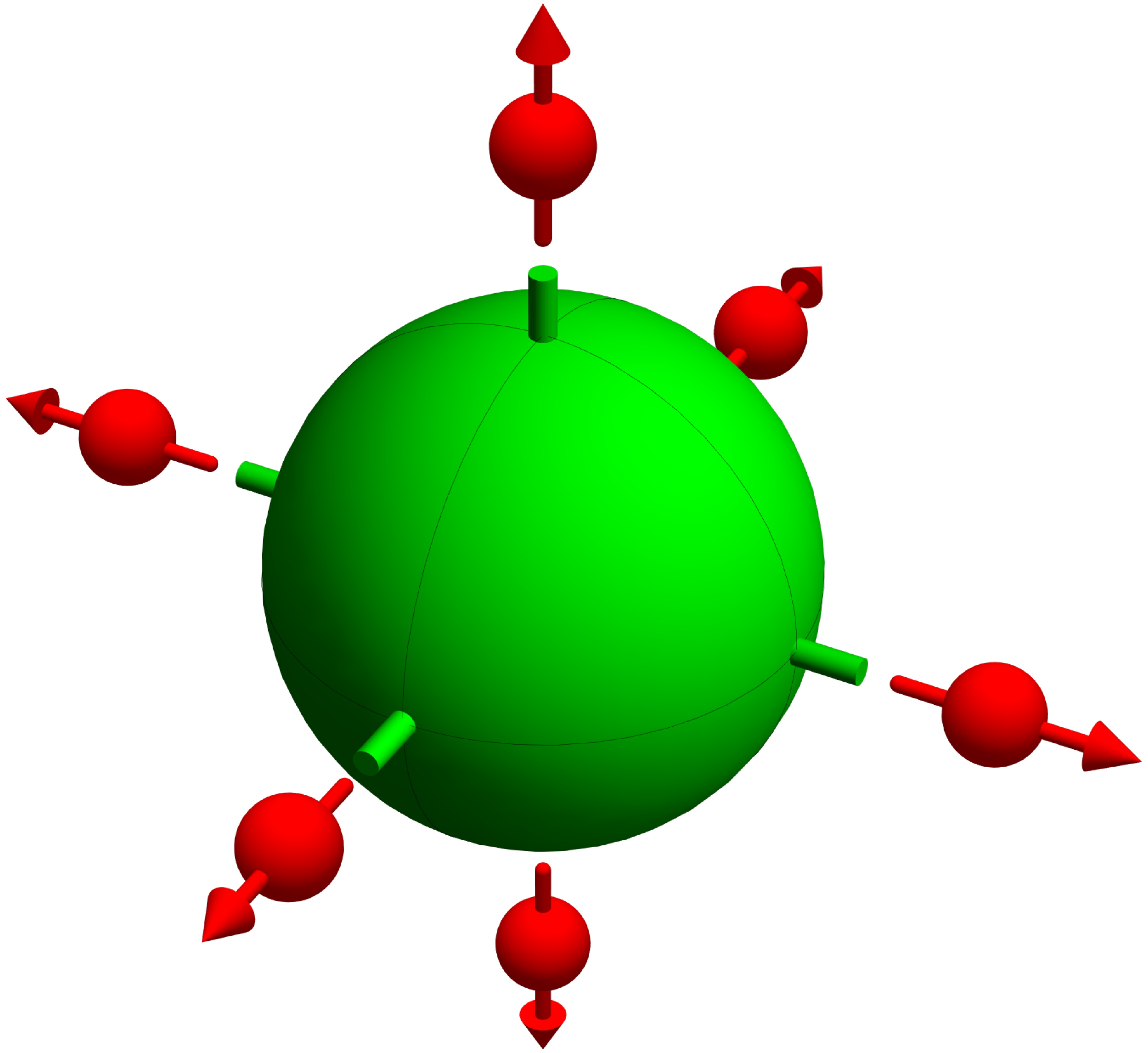
- “Homeomorphism” concerns about the continuous deformation of the topological space.

- “Homotopy” distinguishes continuous variations of continuous maps from one to another topological space.

From Momentum Space to Hilbert Space

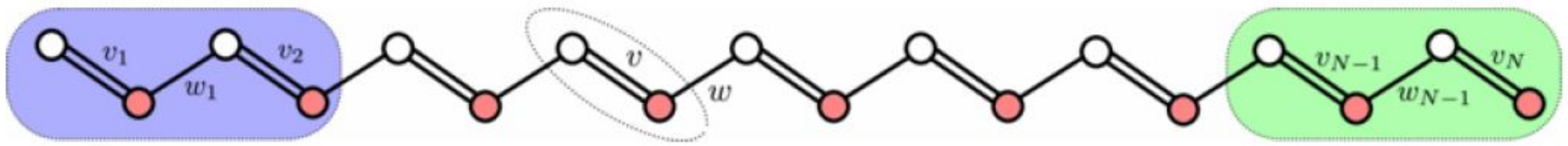
(From Lattice to Wave Functions)





Su-Schrieffer-Heeger Model

(1D Spinless Fermion on a Bipartite Lattice)

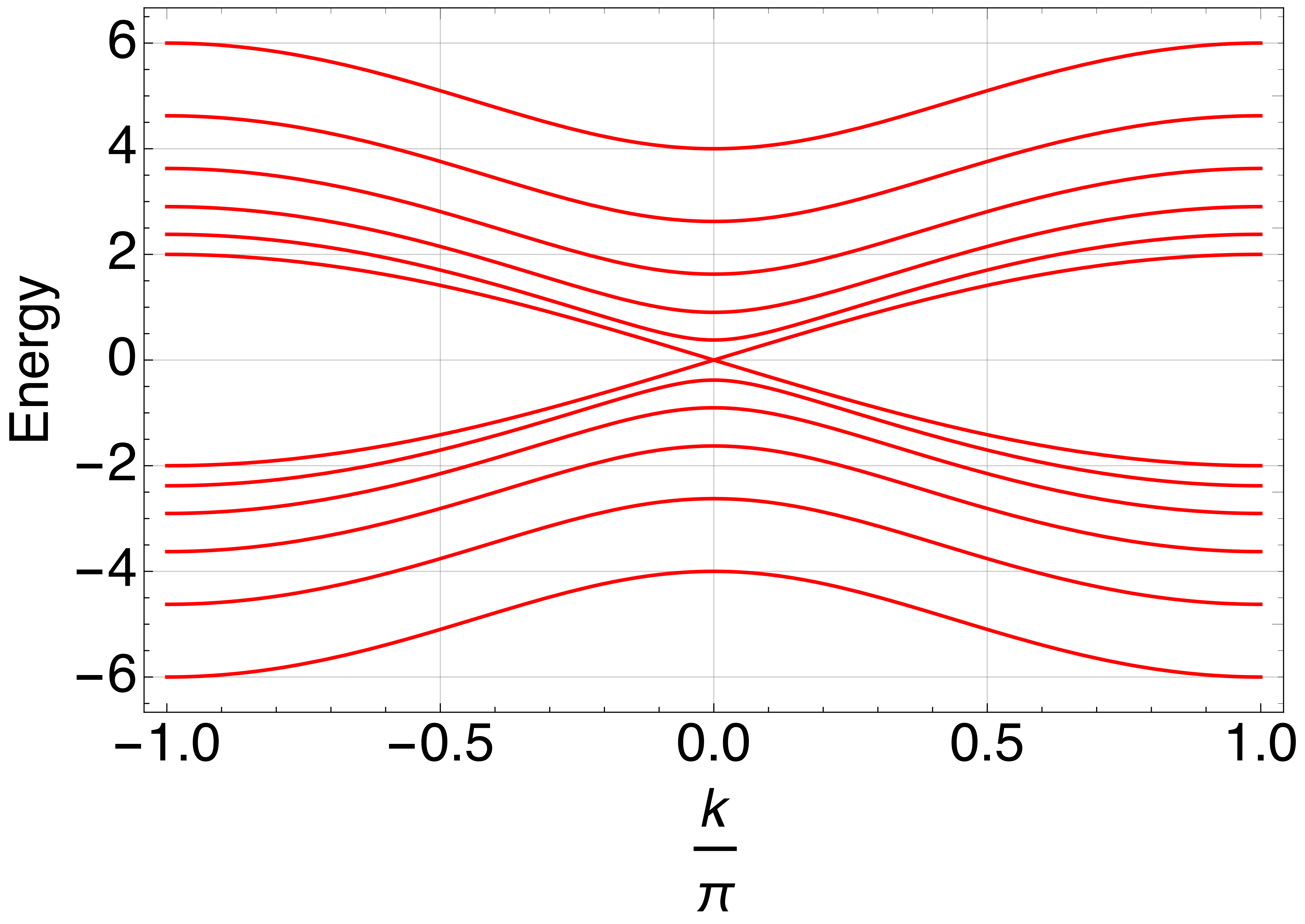


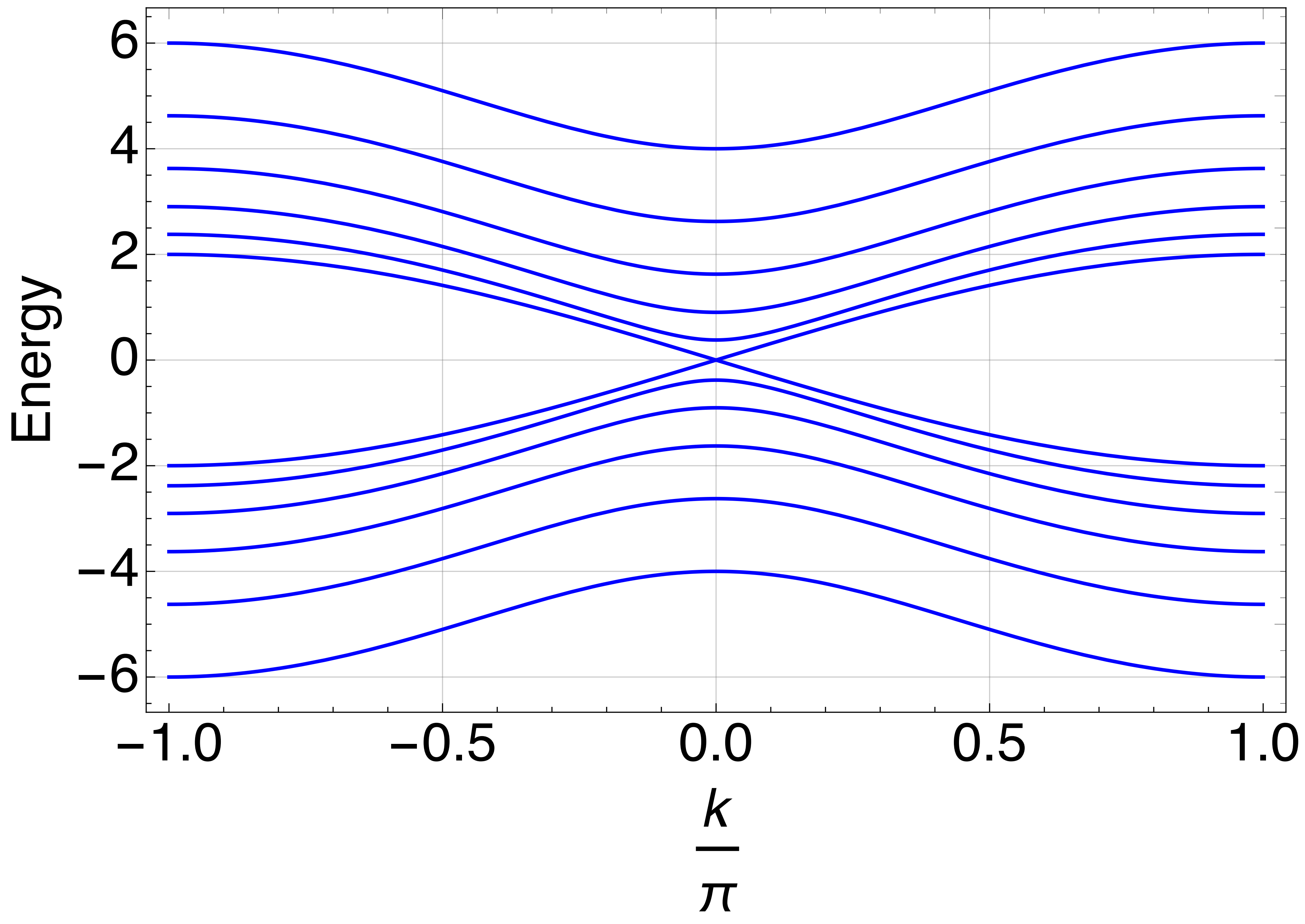
$$\hat{H} = \sum_{k \in \mathrm{BZ}} \hat{\psi}^\dagger(k) H(k) \hat{\psi}(k) \,, \quad H(k) := \begin{bmatrix} 0 & T(k) \\ T^*(k) & 0 \end{bmatrix} , \quad \hat{\psi}(k) := \begin{bmatrix} \psi_A(k) \\ \psi_B(k) \end{bmatrix}$$

$$H(k) = \boldsymbol{T} \cdot \boldsymbol{\sigma} \,, \quad \boldsymbol{T}(k) := \begin{bmatrix} v - w\cos k \\ -w \sin k \\ 0 \end{bmatrix}$$

Su-Schrieffer-Heeger Model

(1D Spinless Fermion on a Bipartite Lattice)





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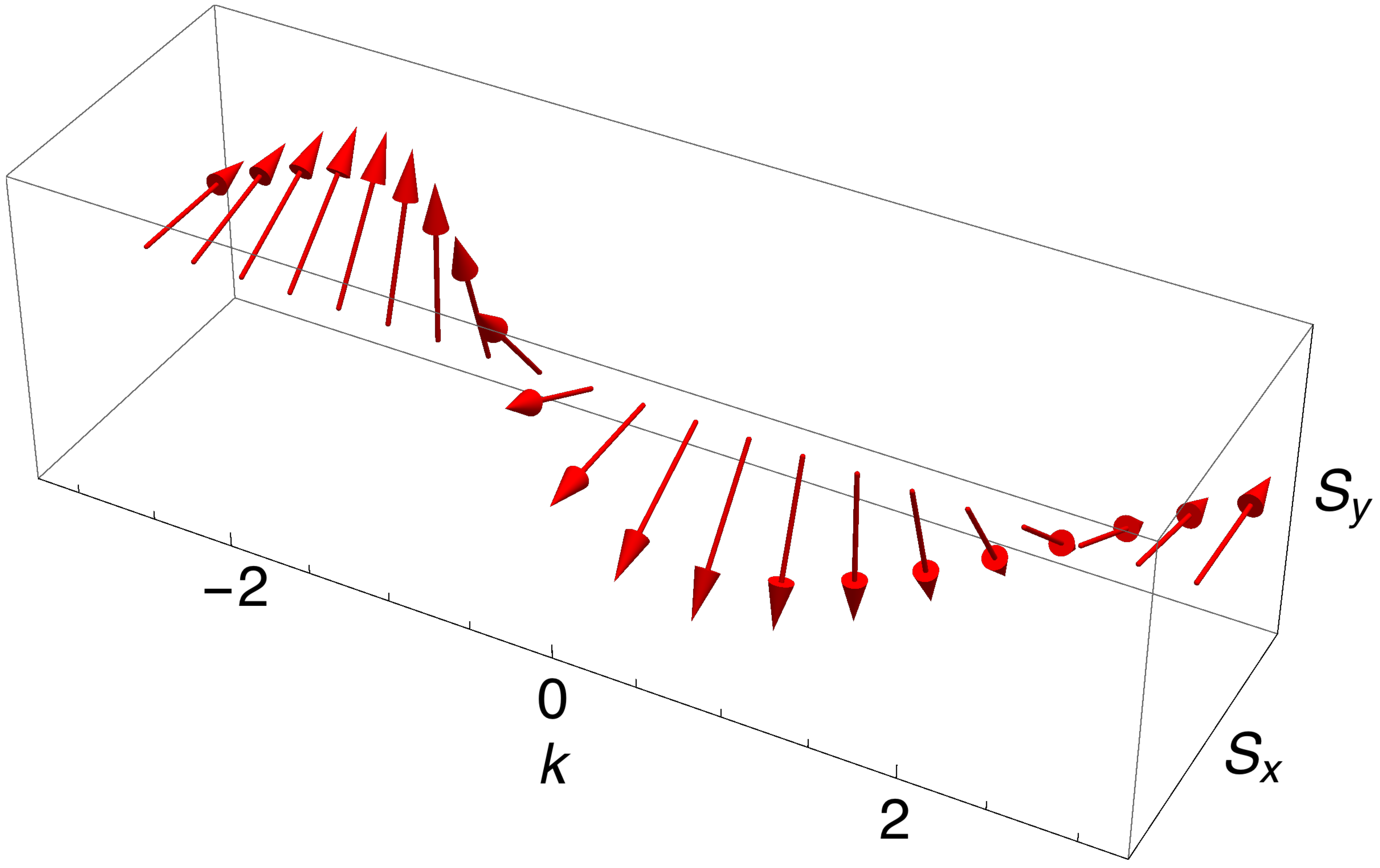

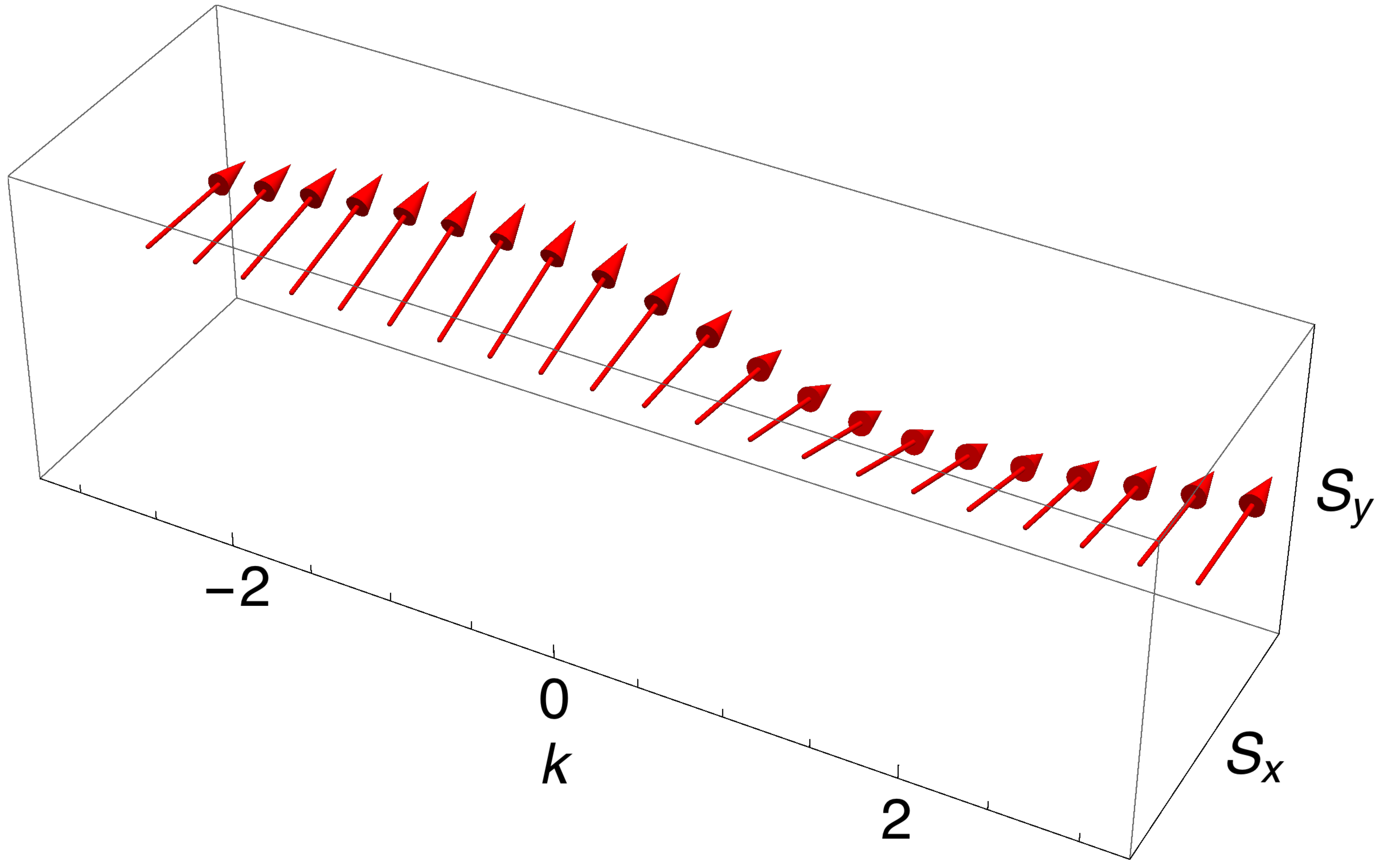

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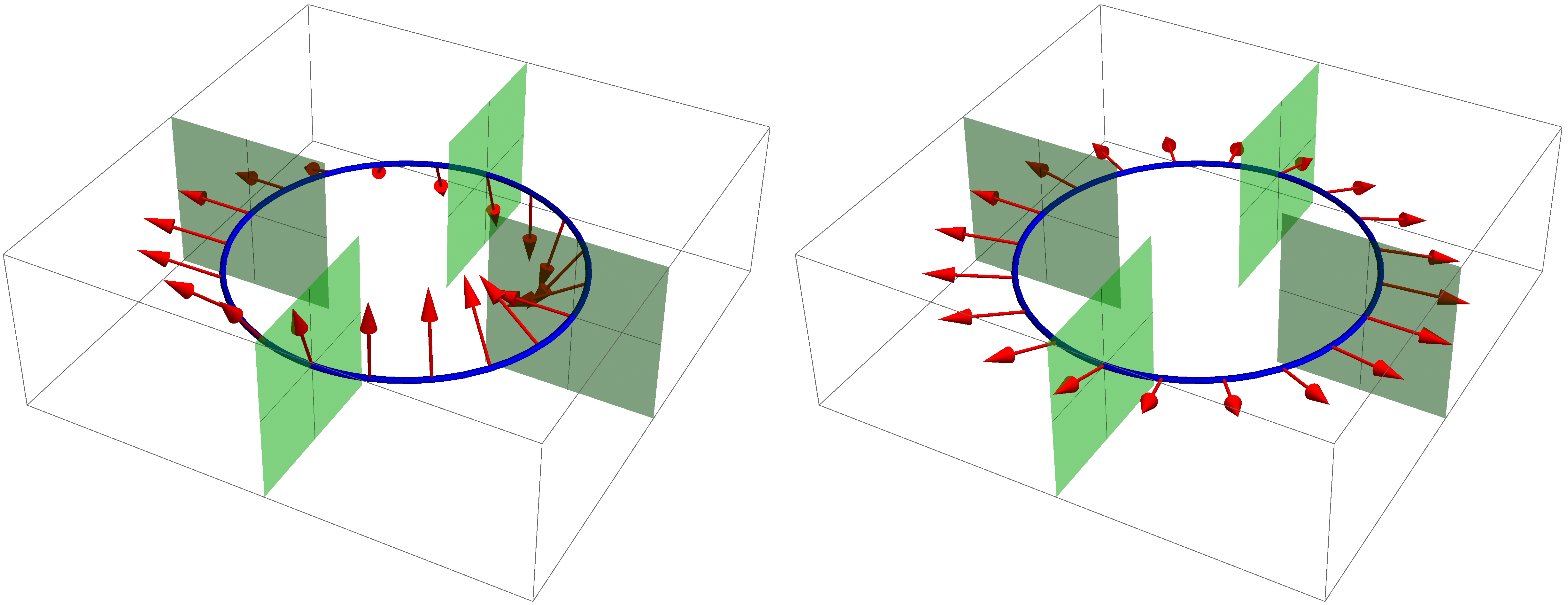
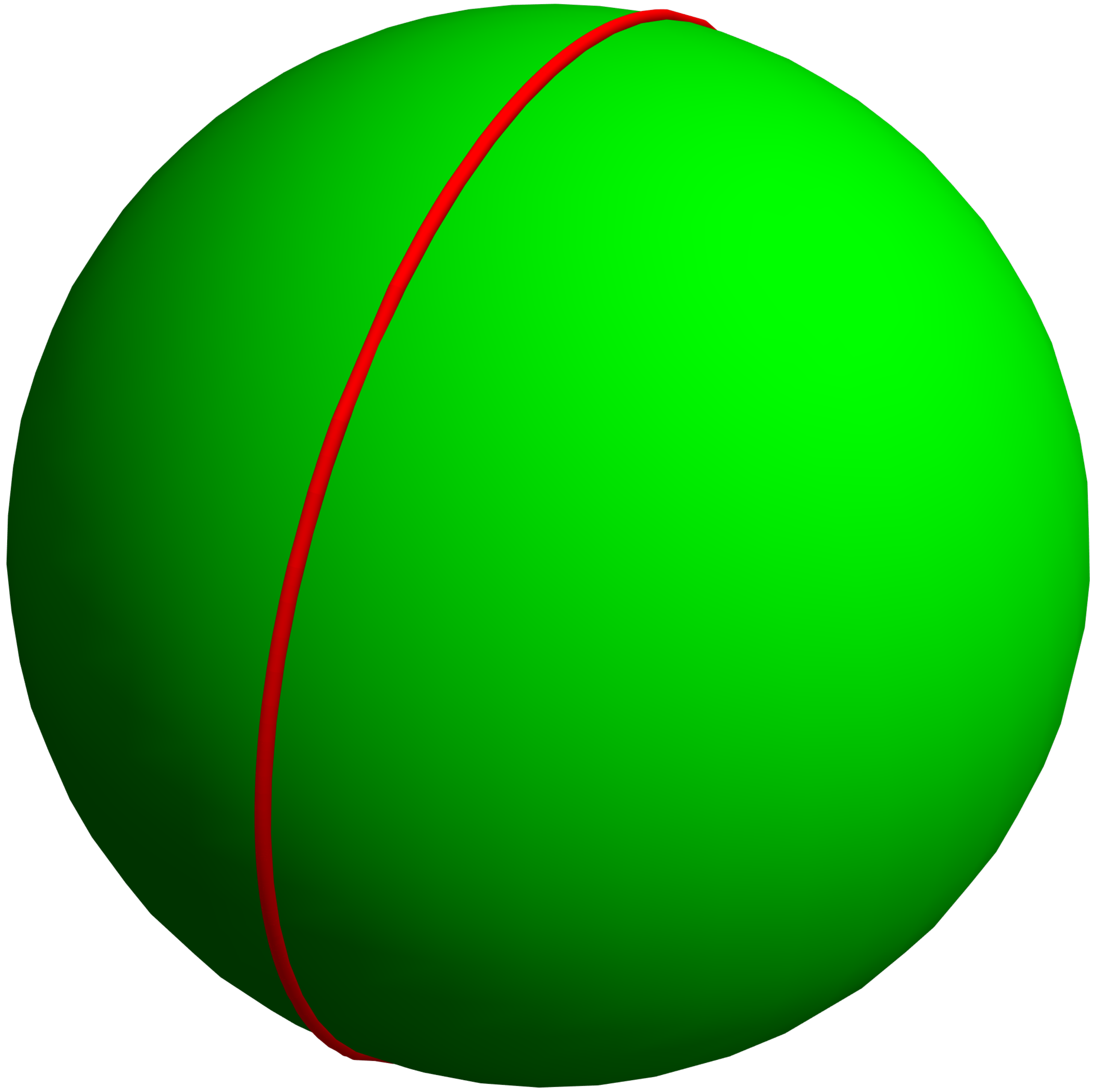
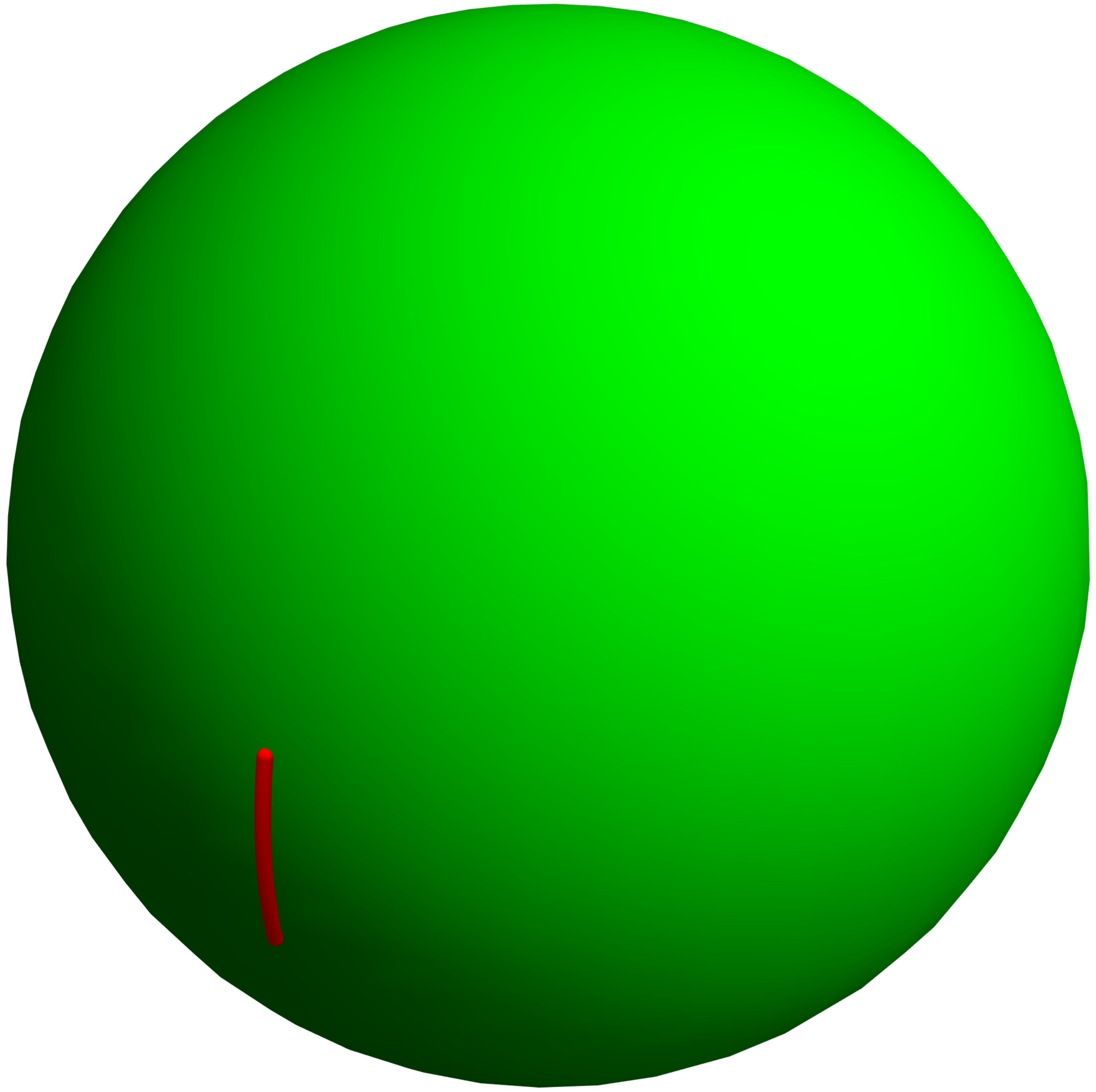


Su-Schrieffer-Heeger Model

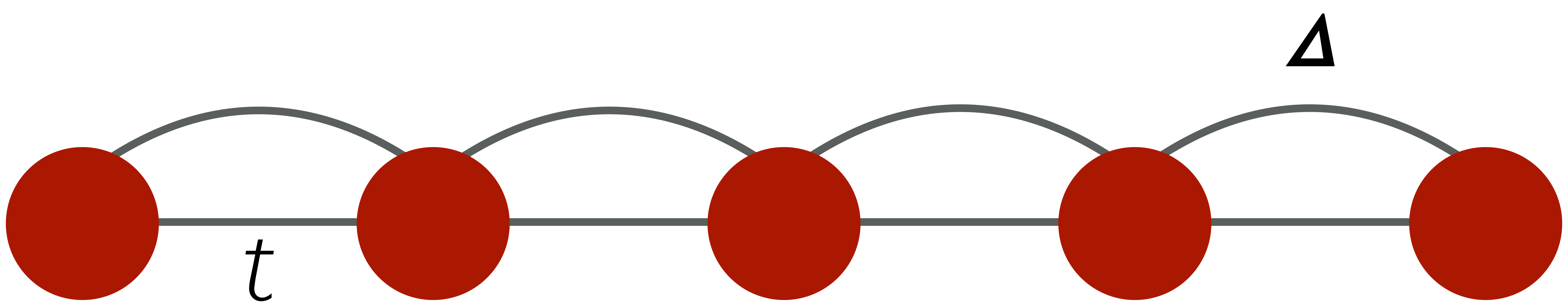
(1D Spinless Fermion on a Bipartite Lattice)



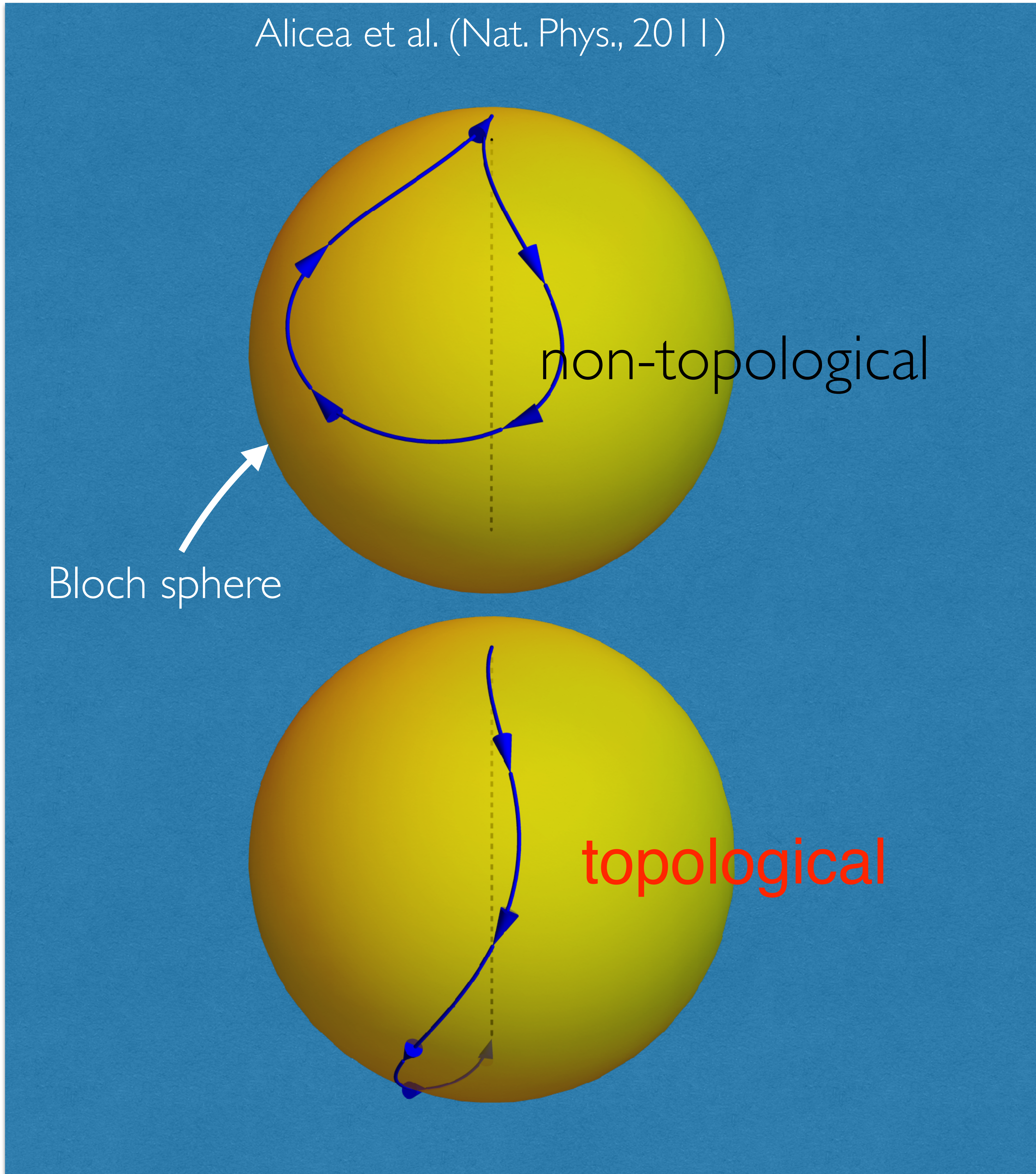


TOPOLOGICAL SUPERCONDUCTOR

Kitaev (cond-mat/0010440)

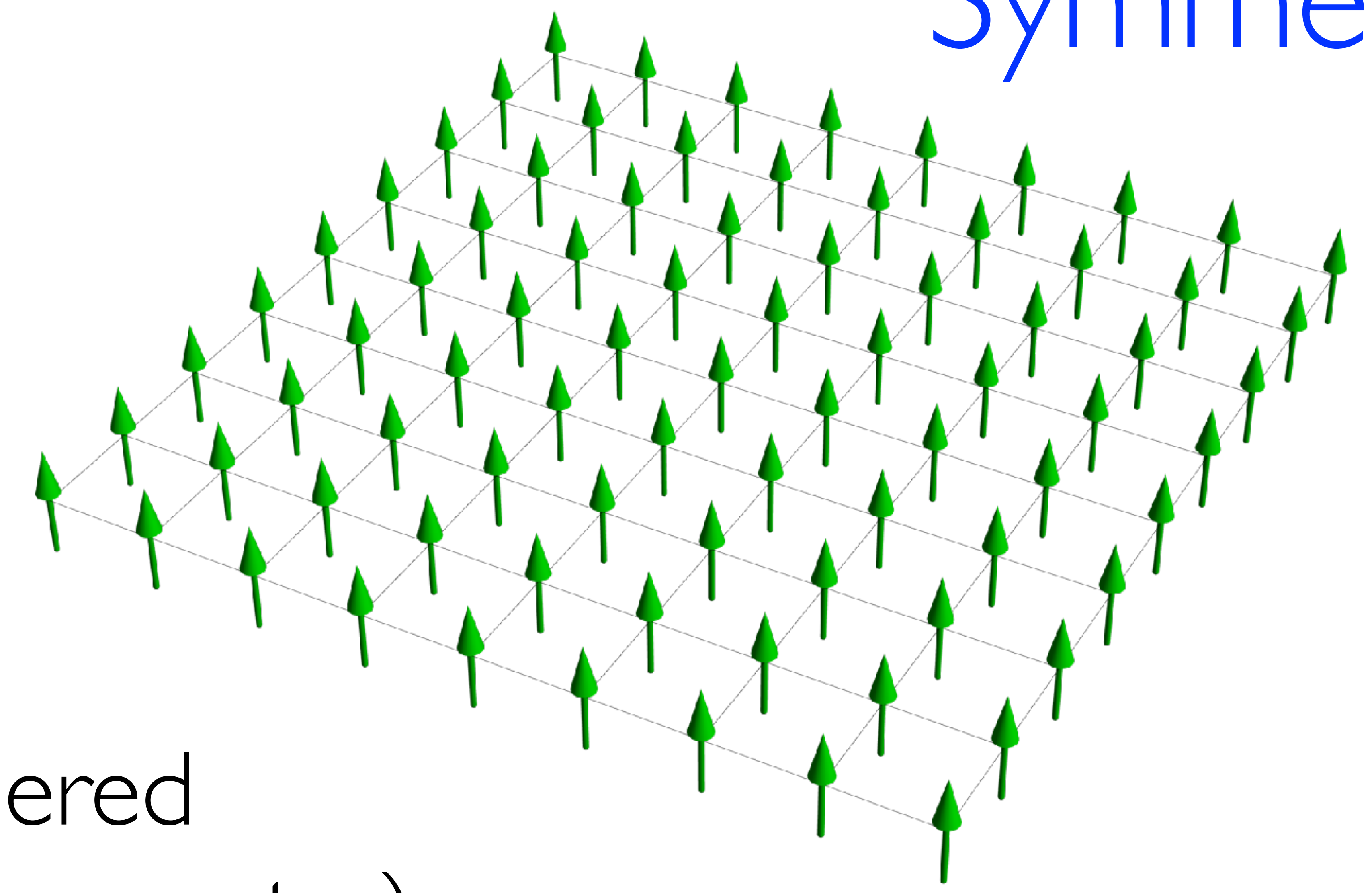


$$H = \sum_{\ell} \left[\left(t\, c_{\ell}^{\dagger} c_{\ell+1} + \Delta c_{\ell}^{\dagger} c_{\ell+1}^{\dagger} \right) + h.c. \right]$$

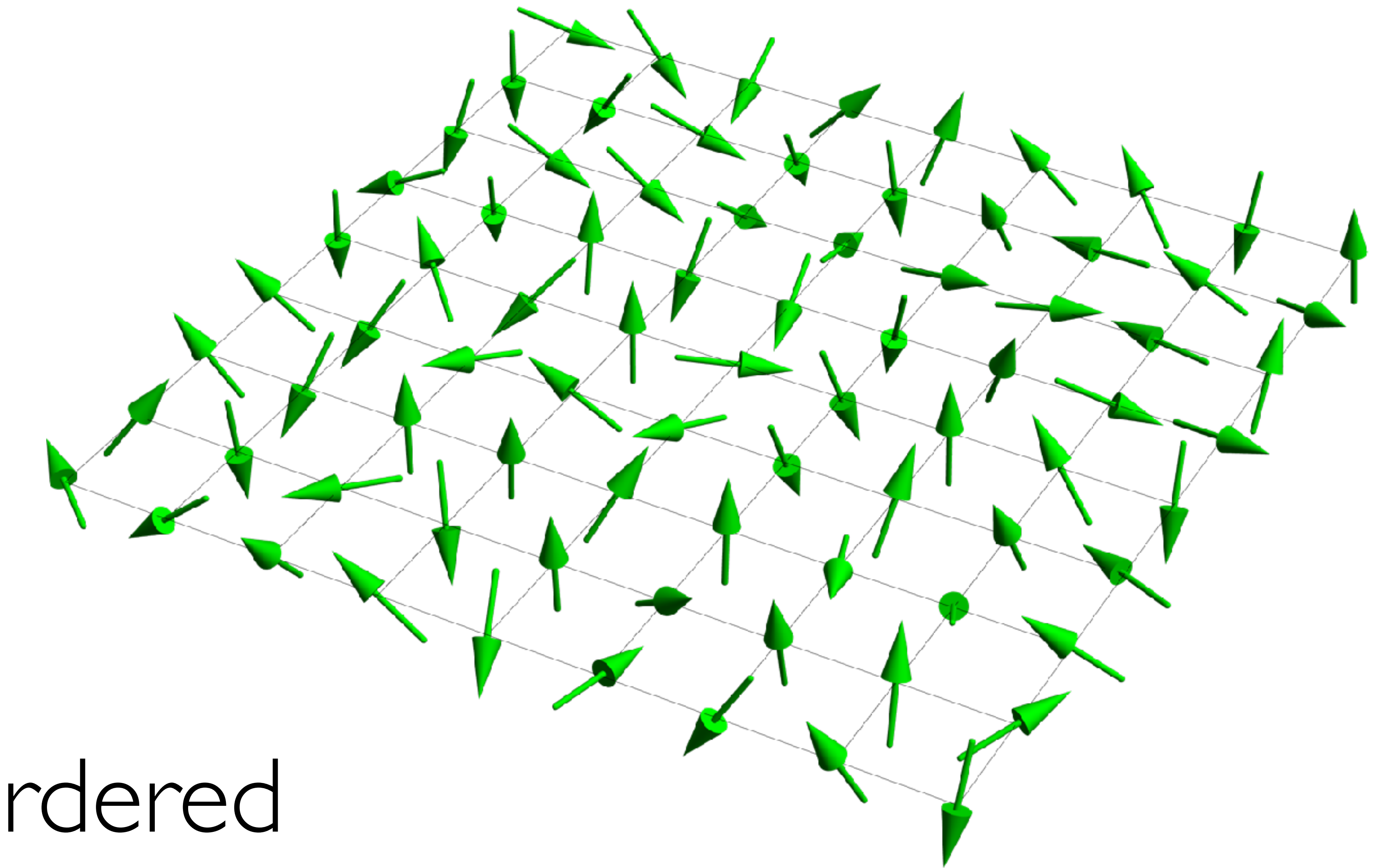


States of Matter

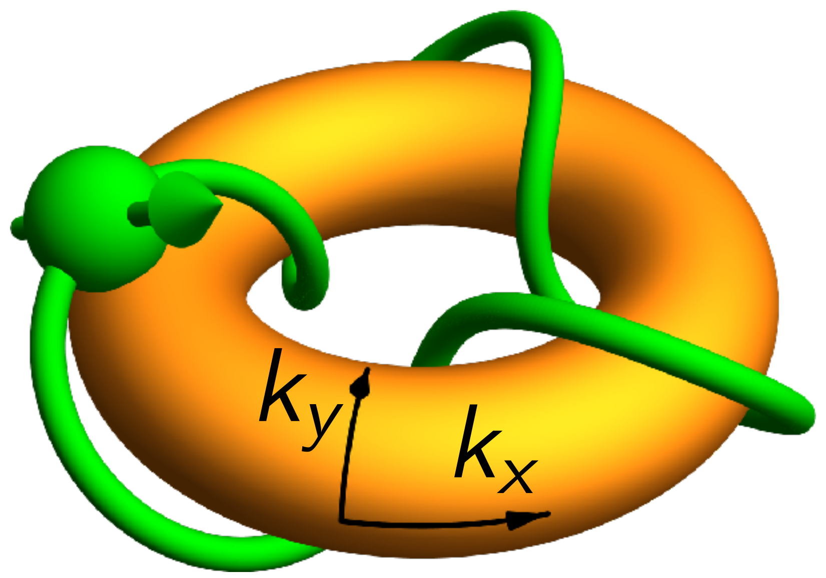
Symmetry vs. Topology



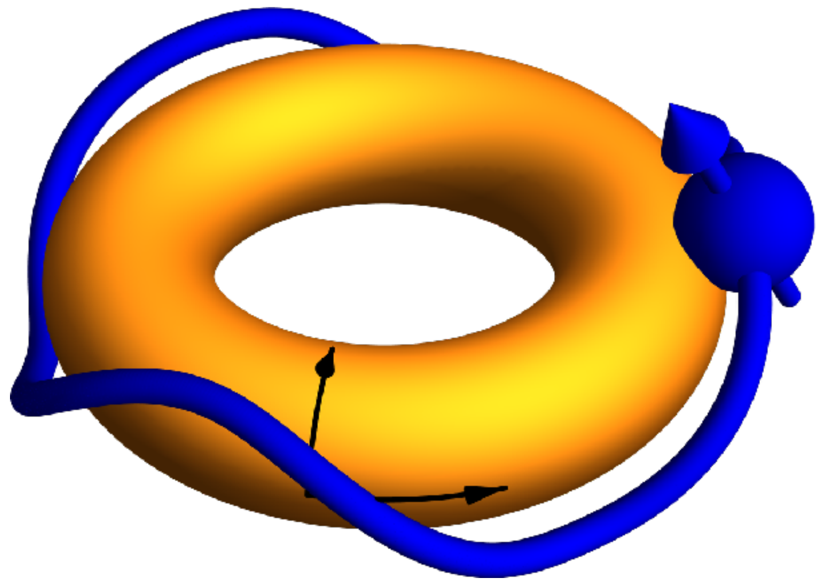
Ordered
(broken symmetry)



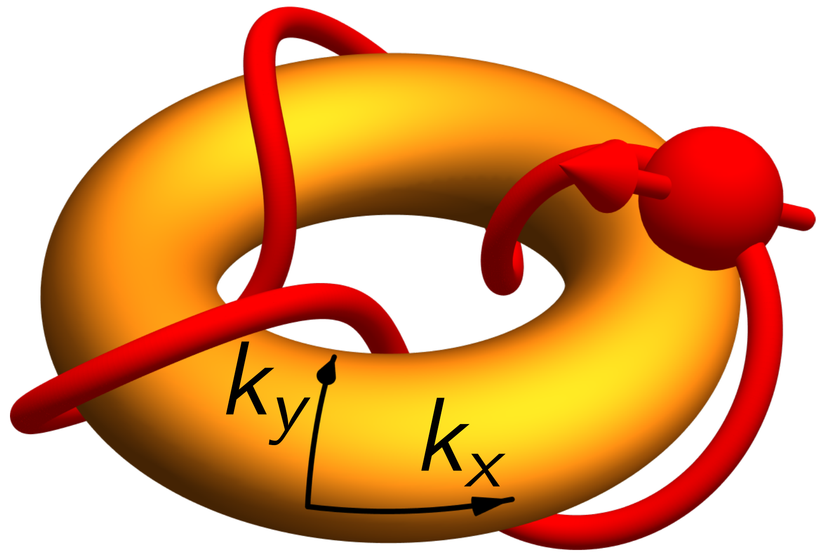
Disordered



Topological number = +1



Topological number = 0



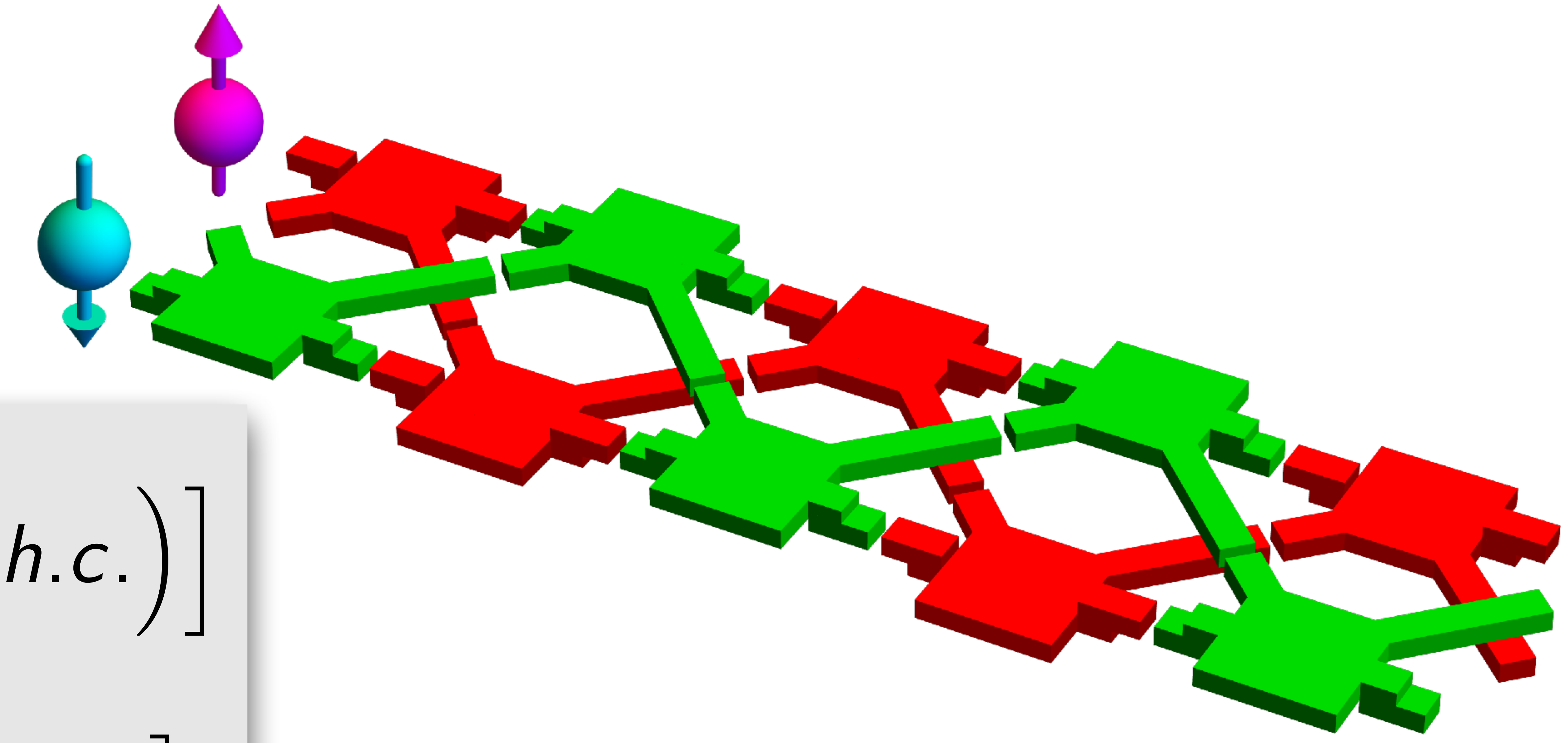
Topological number = -1

"Synthetic" Kitaev Quantum Wire

(Synthetic Spin-Orbit Coupling)

Synthetic Kitaev Quantum Wire

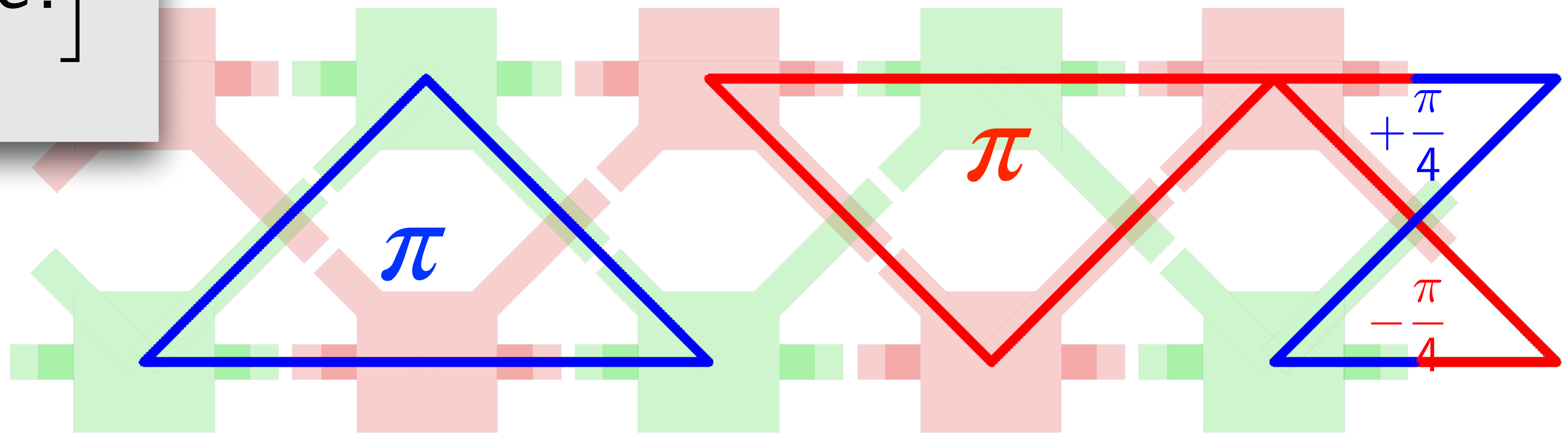
(Synthetic Spin-Orbit Coupling)



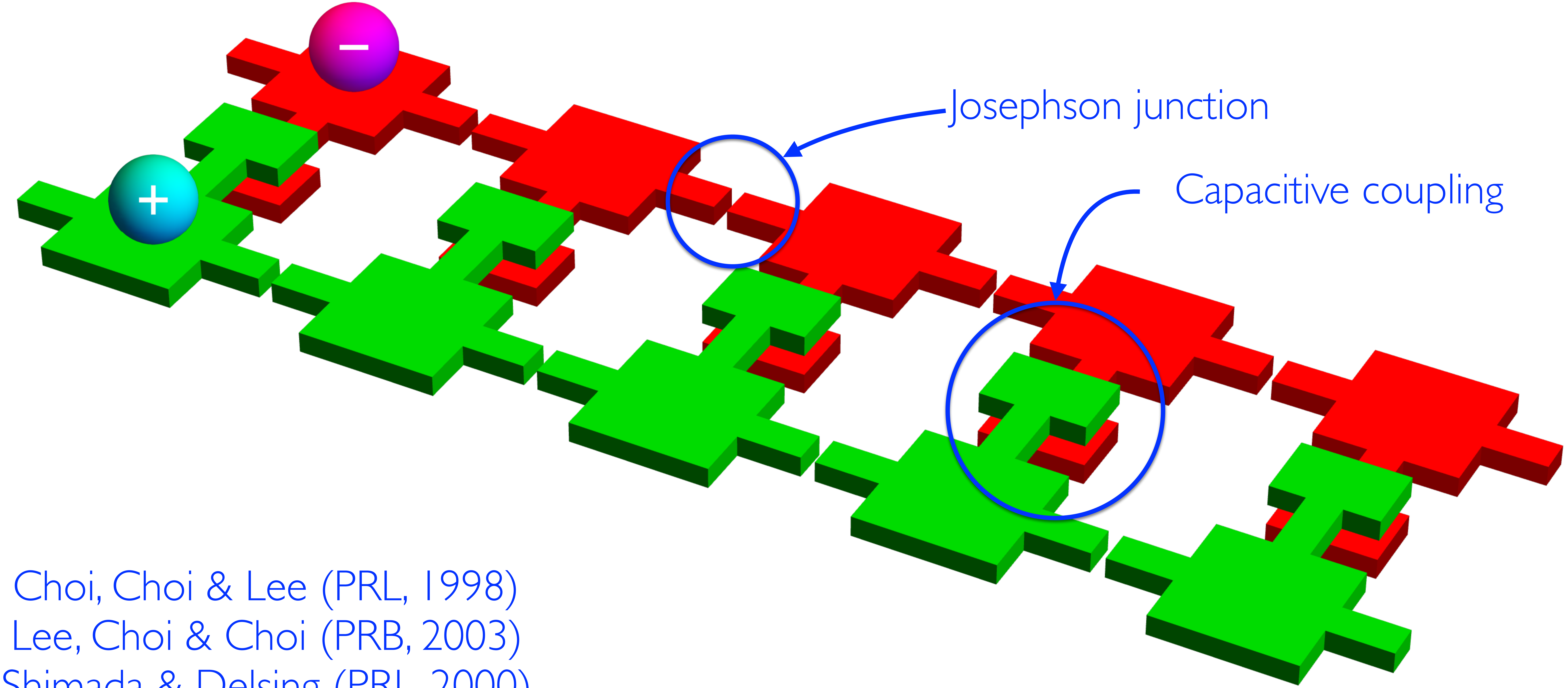
$$H = \sum_{\ell=0}^{L-1} \sum_{\sigma=\uparrow,\downarrow} \left[\epsilon_\ell b^\dagger_{\ell\sigma} b_{\ell\sigma} - t \left(b^\dagger_{\ell,\sigma} b_{\ell+1,\sigma} + h.c. \right) \right]$$

$$- \alpha t \sum_\ell \left[\left(b^\dagger_{\ell,\uparrow} b_{\ell+1,\downarrow} - b^\dagger_{\ell,\downarrow} b_{\ell,\uparrow} \right) + h.c. \right]$$

- Conceptually straightforward.
- The chain index plays the role of isospin.
- Dealing with Bosons (instead of Fermions).



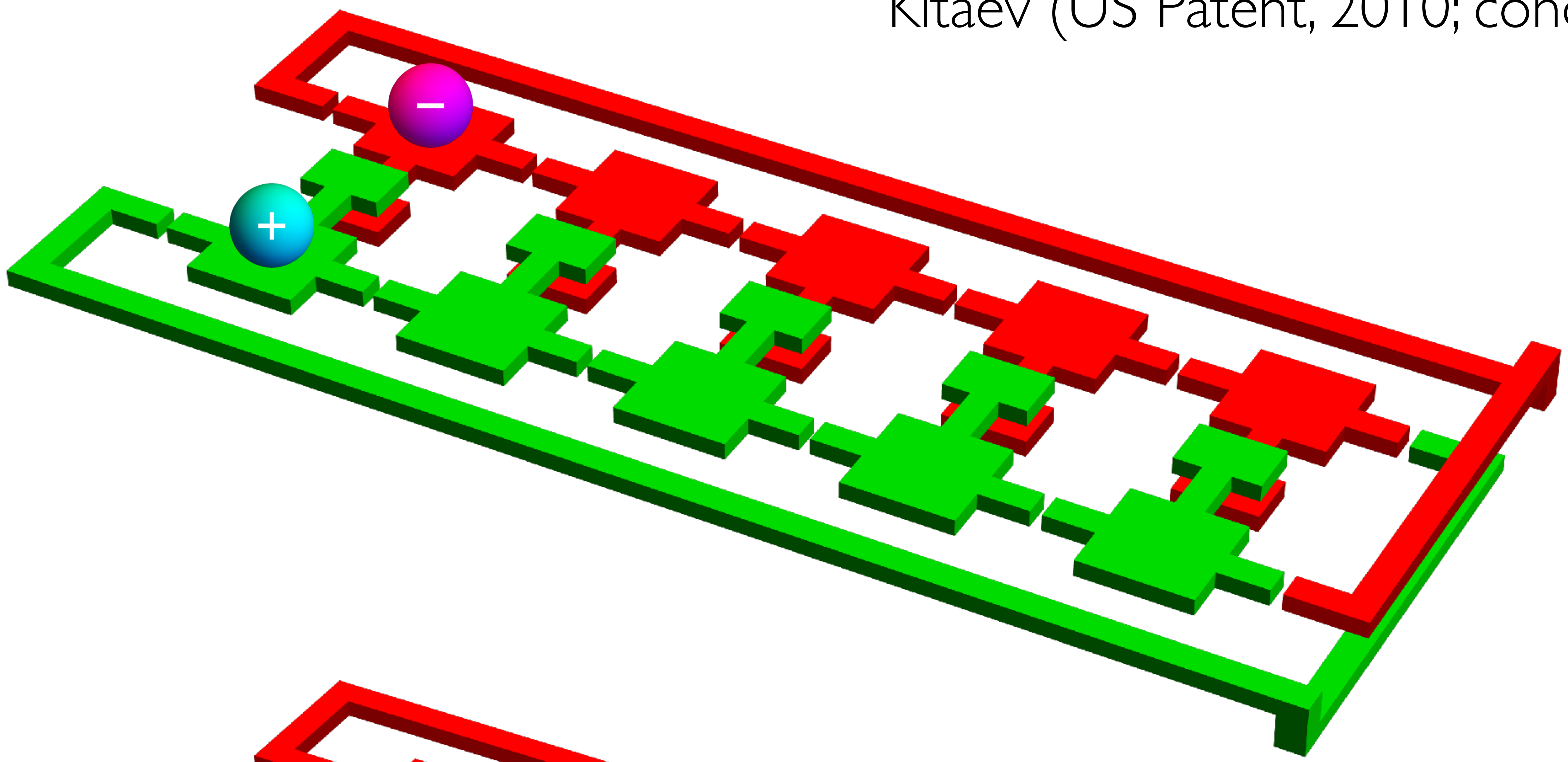
Capacitively Coupled Josephson Junction Ladder

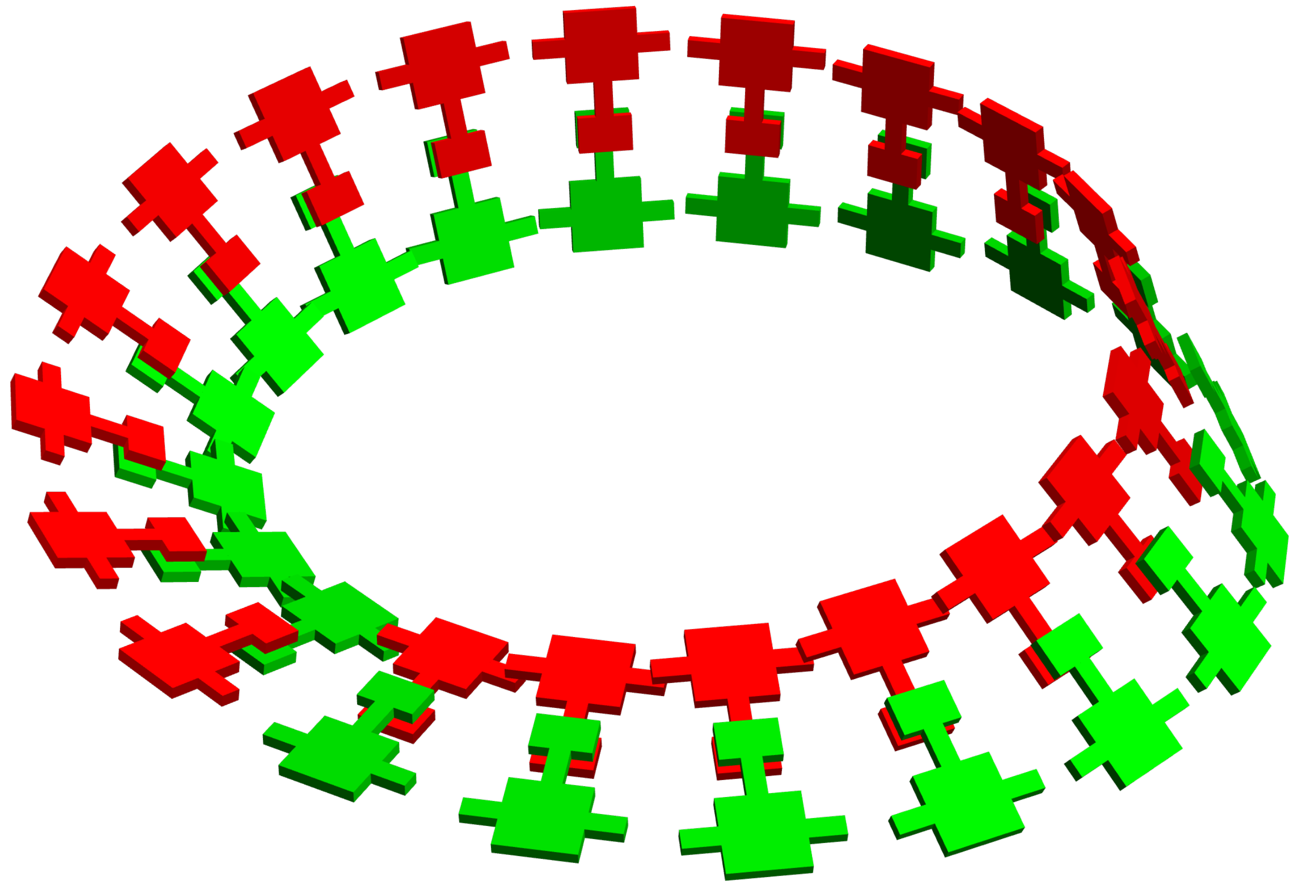


Choi, Choi & Lee (PRL, 1998)
Lee, Choi & Choi (PRB, 2003)
Shimada & Delsing (PRL, 2000)

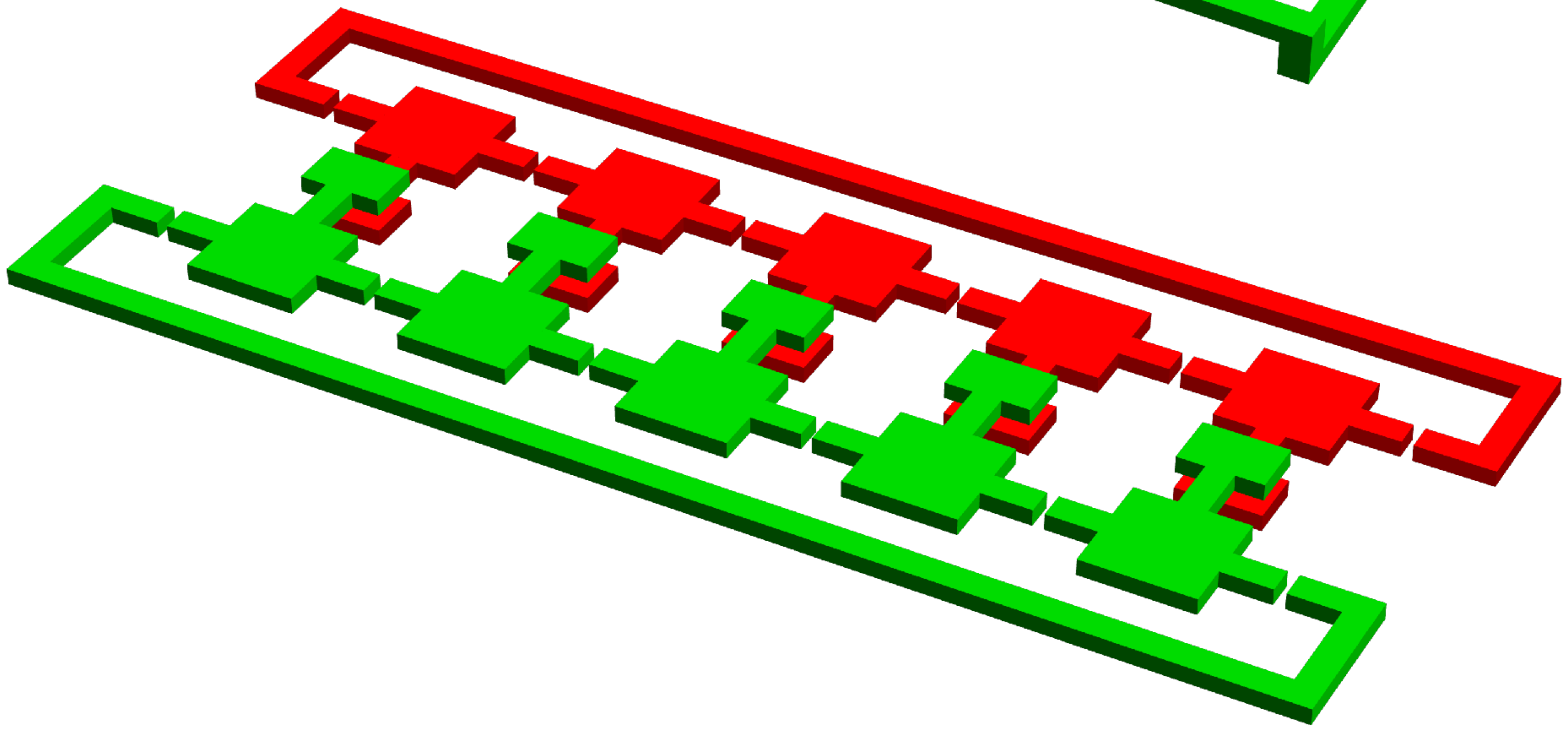
Moebius Josephson Ladder (MJL)

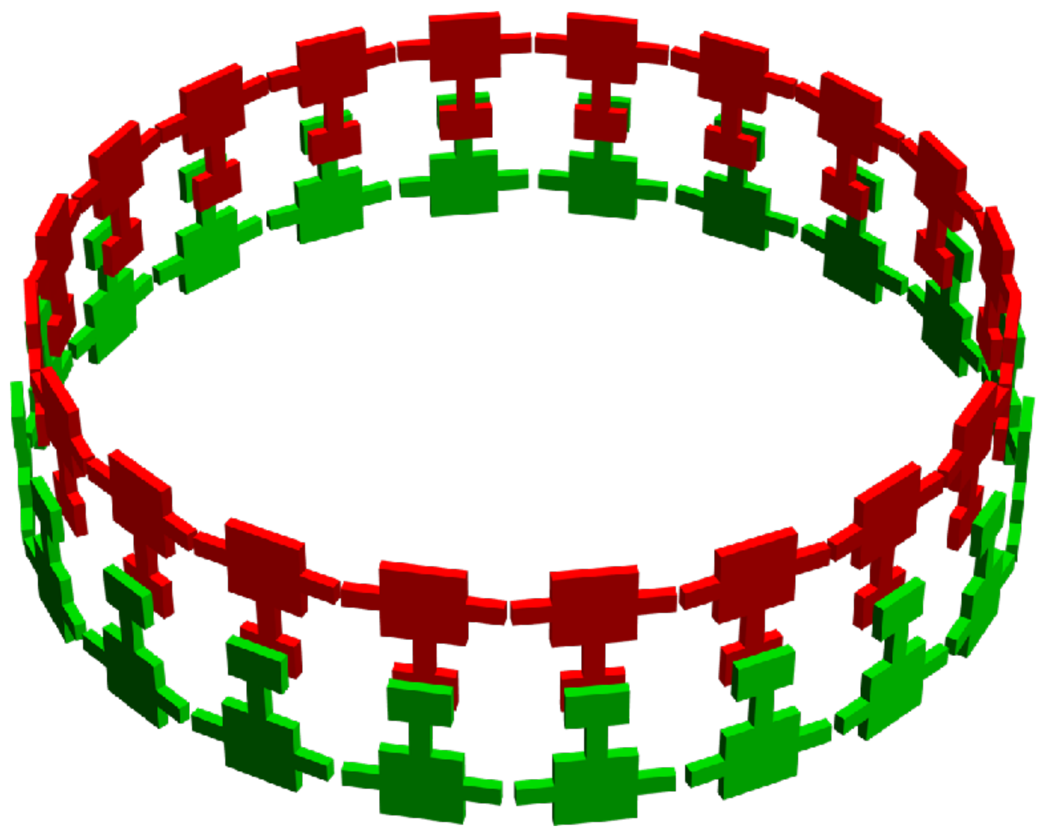
Kitaev (US Patent, 2010; cond-mat/0609441)





twisted
boundary
condition

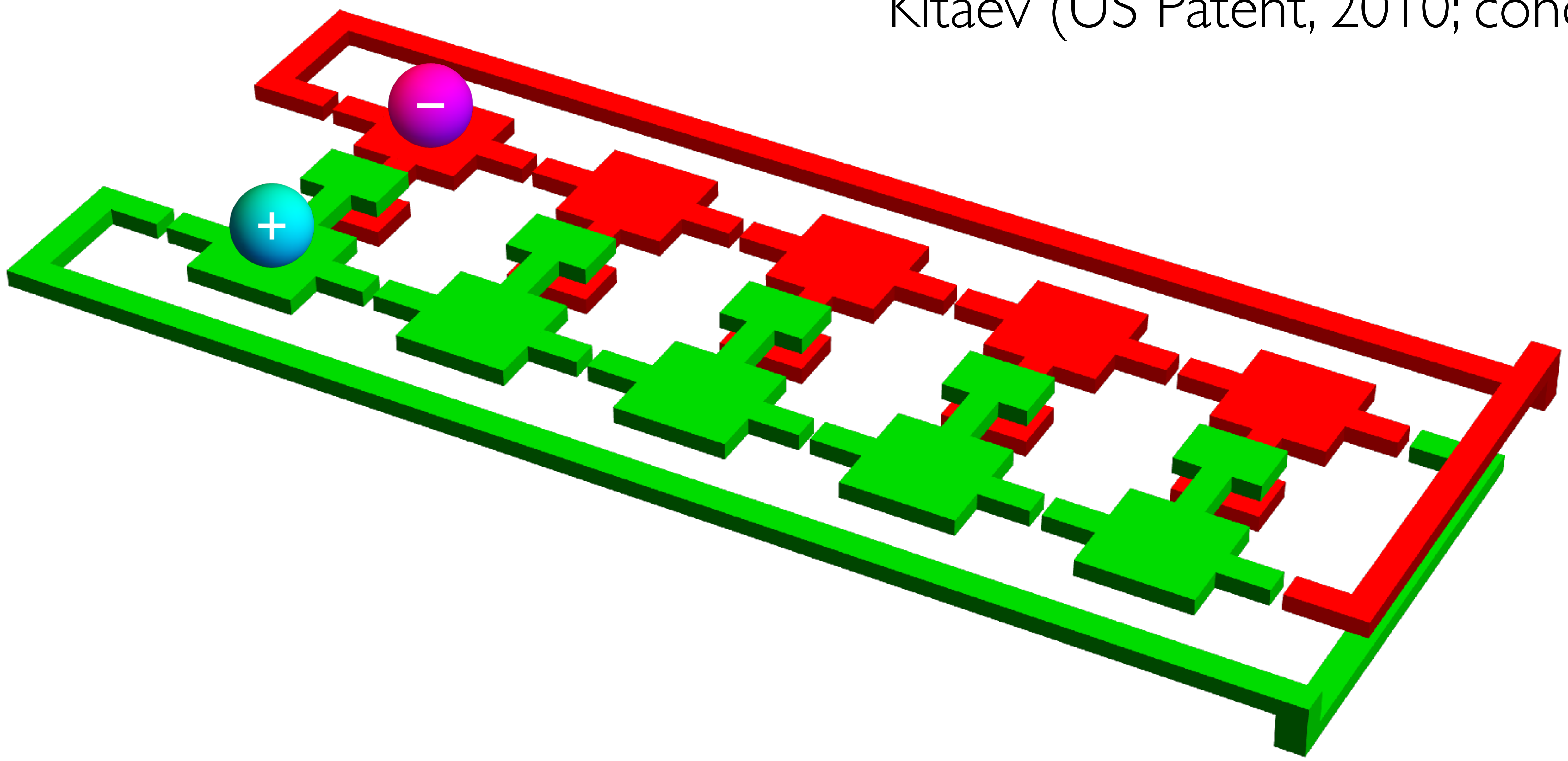


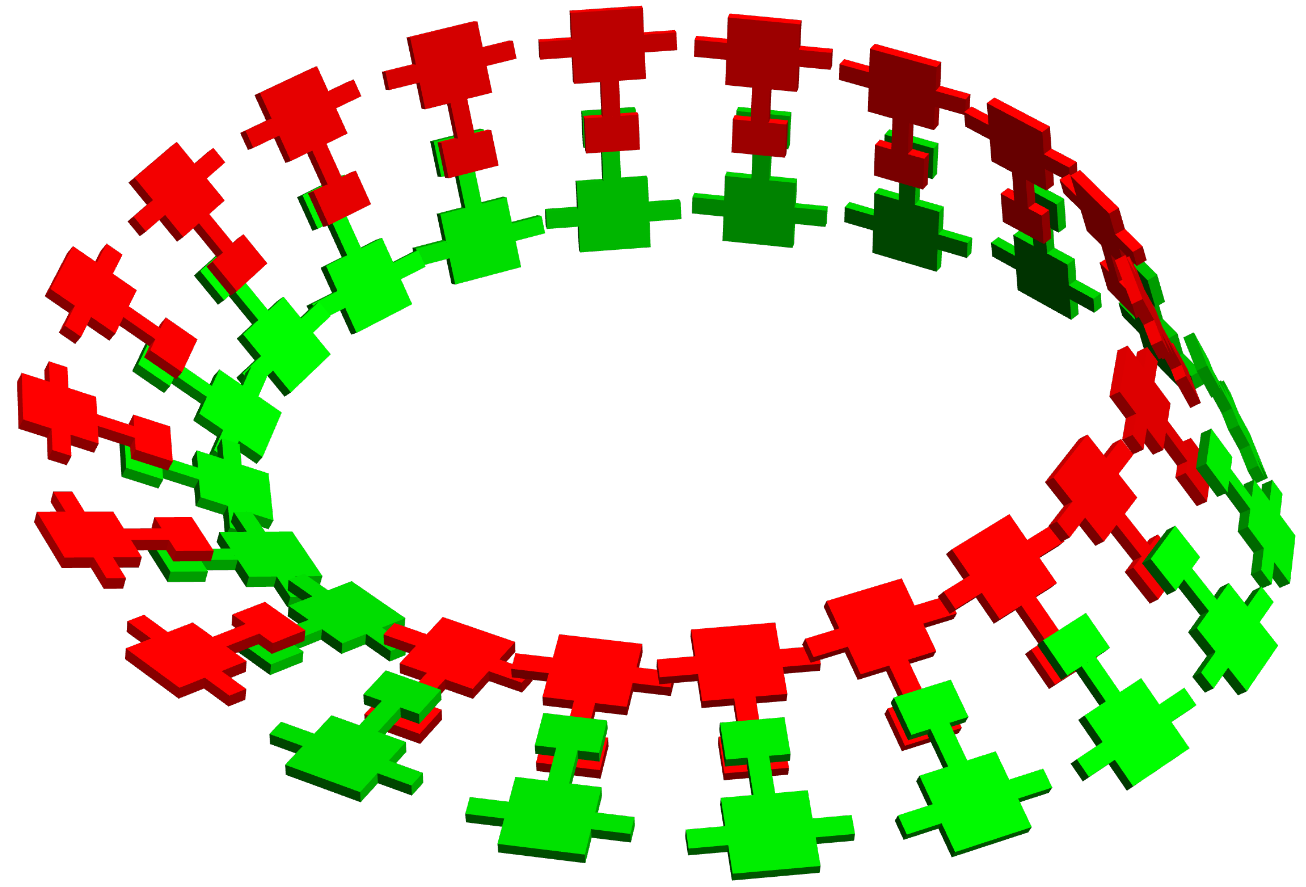


periodic
boundary
condition

MOEBIUS JOSEPHSON LADDER (MJL)

Kitaev (US Patent, 2010; cond-mat/0609441)





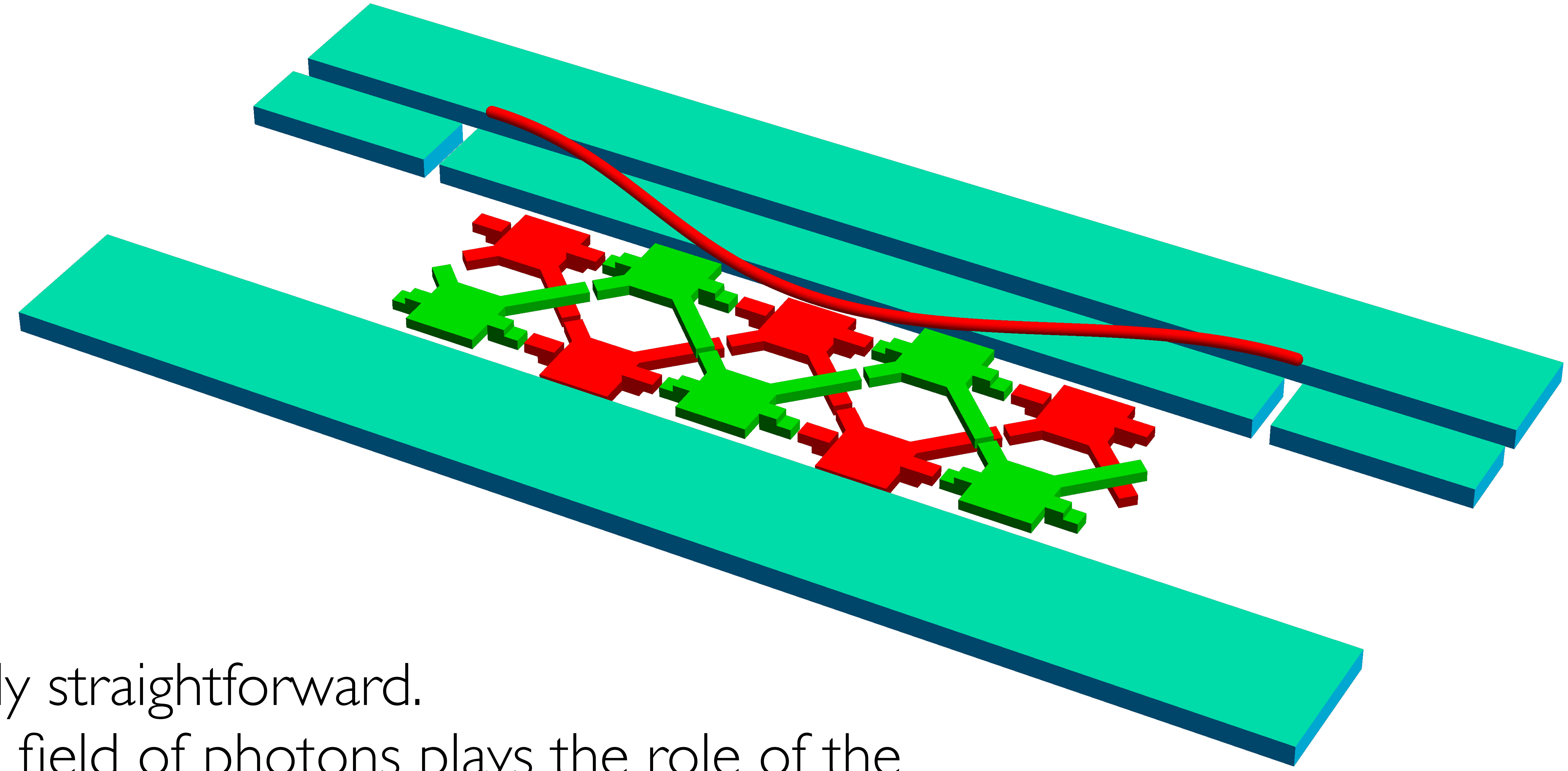
twisted boundary condition

- Topology in the real space (unlike most other topological materials)
- The energy scales as $\sim 1/L$.

Light-Topological Qubit Coupling Schemes

Prototypes Based on Josephson Junction Arrays

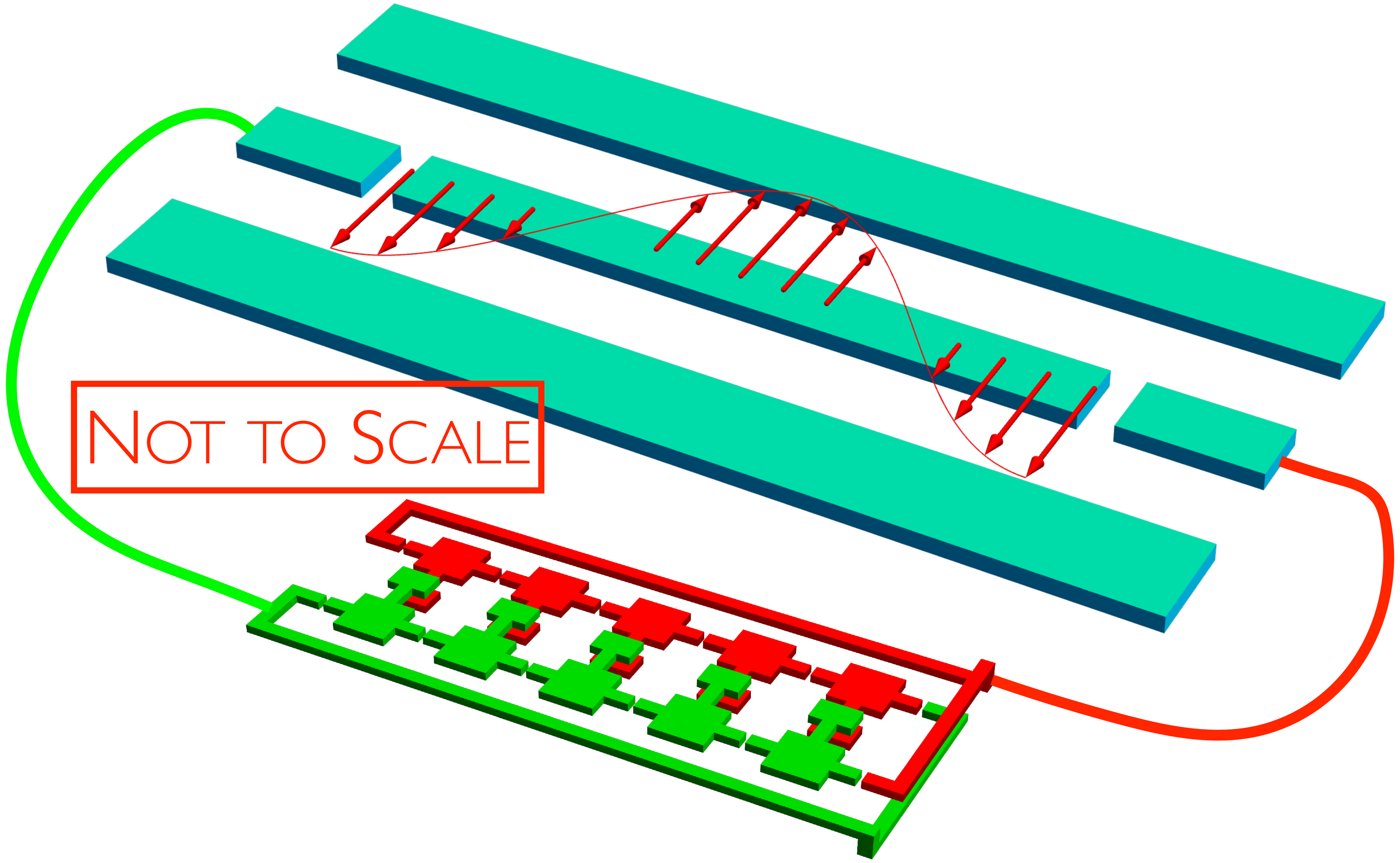
Synthetic Kitaev Quantum Wire

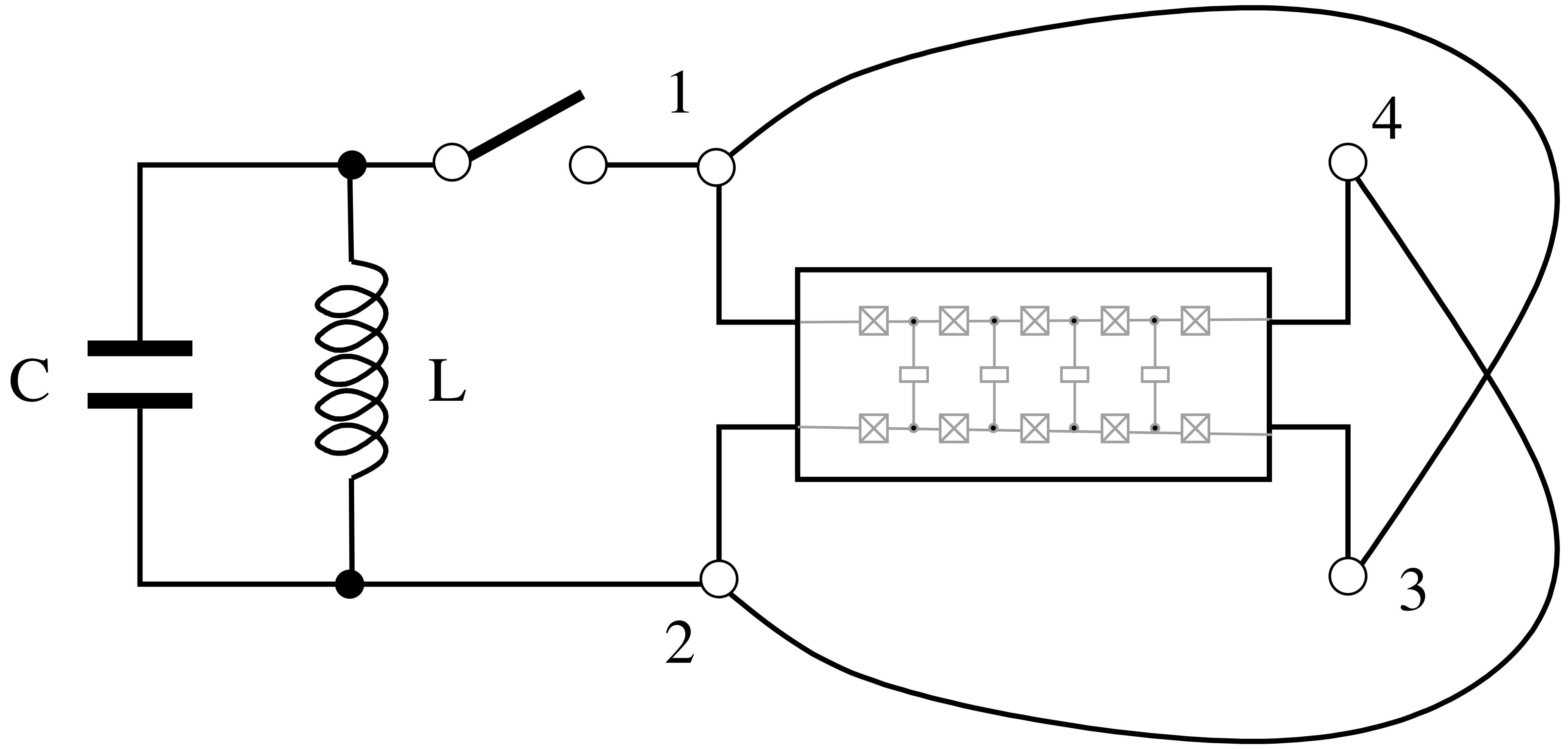


- Conceptually straightforward.
- The electric field of photons plays the role of the magnetic field for the synthetic Kitaev quantum wire.
- The coupling is thus very strong as the islands are *microscopically large* (still *macroscopically small*).

Moebius Josephson Ladder: I

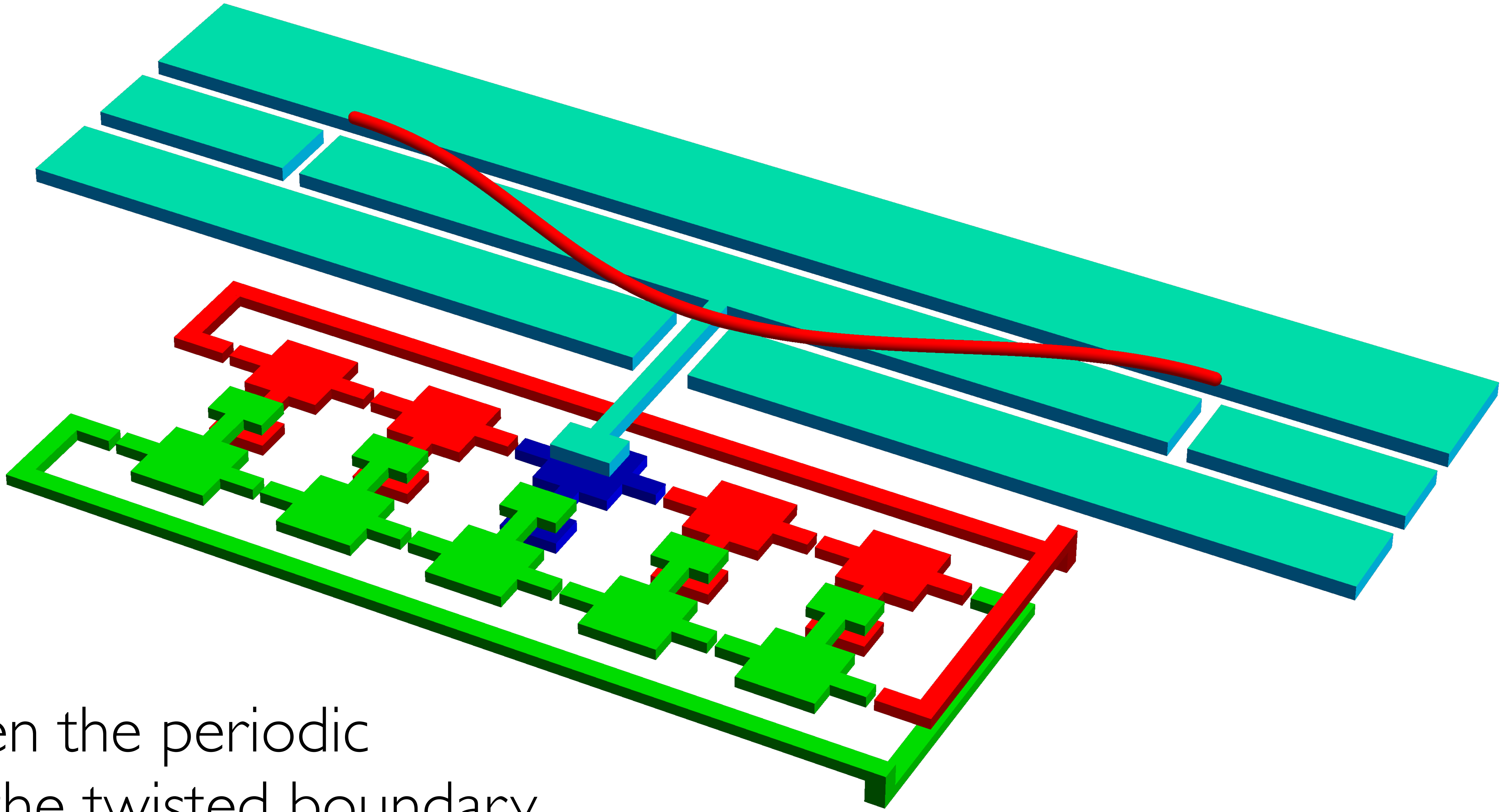
(Kitaev's Example, cond-mat/0609441)





- Modulations of quantum phases only.
- Big difference in scales (centimeters vs sub-millimeters)
- Spatiotemporal effects of the circuit resonator?

Moebius Josephson Ladder: II



- Directly switching between the periodic boundary condition and the twisted boundary condition.
- Effectively switching the topological states of the Moebius Josephson ladder.

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- Matt. Desjardins (ENS Paris, France)