#### 김예림(고려대),장재권(고려대),양승진(고려대)



#### Content

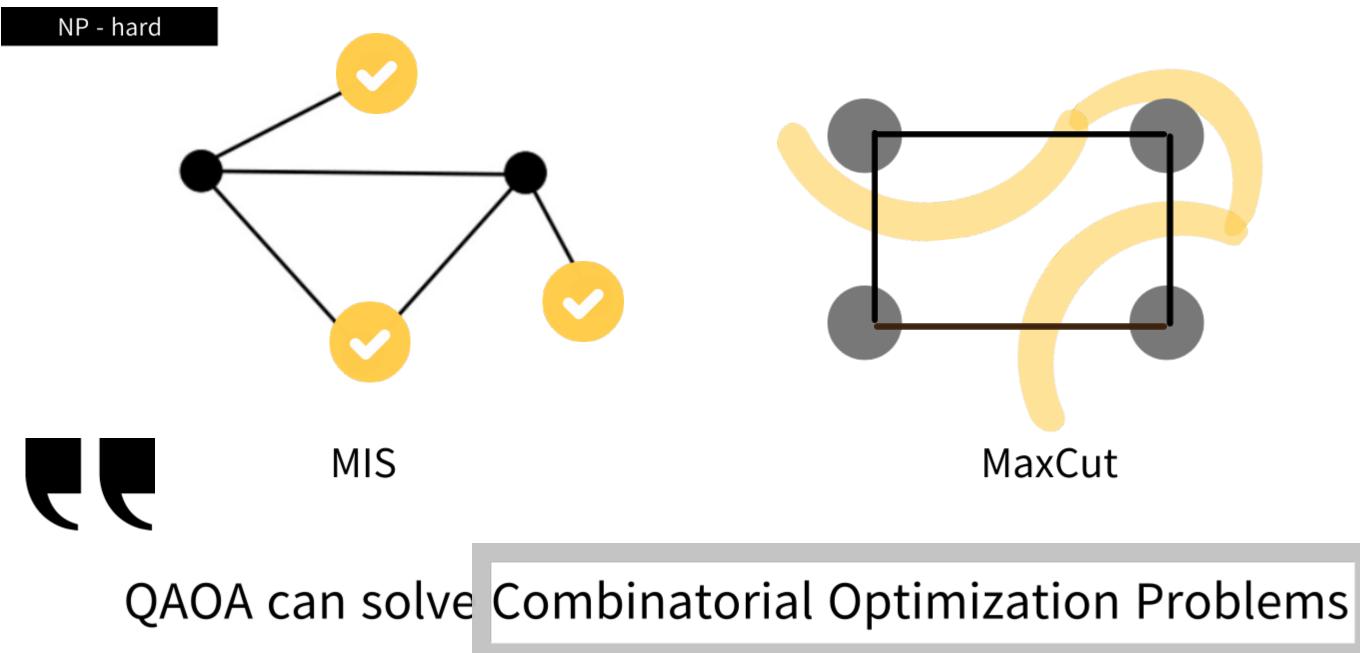




## 1. Research Object



### Background



Cost Hamiltonian : Ising Model Hamiltonian

))

### Background

#### QAOA는 근본적으로 Ising model Hamiltonian 문제를 다루는 VQA 인데 이 문제들을 굳이 Gate-based quatum simulation 로 풀 필요가 있을까?

#### QAOA Ansatz의 motivation인 Adiabatic Theorem을 이용하여 즉 QA(Quantum Anealing)으로 풀면 되지 않을까?

### **Research Object**



QAOA 가 푸는 문제들을 굳이 Gate-based quatum simulation 을 해야 할 필요가 있을까?

애초에 Adiabatic Quantum Optimization 즉 QA(Quantum Anealing)으로 풀면 되지 않을까?

Combinatorial Problem(MaxCut)을 QA와 QAOA를 통해 풀어보고 수행능력을 비 교해고자 한다.



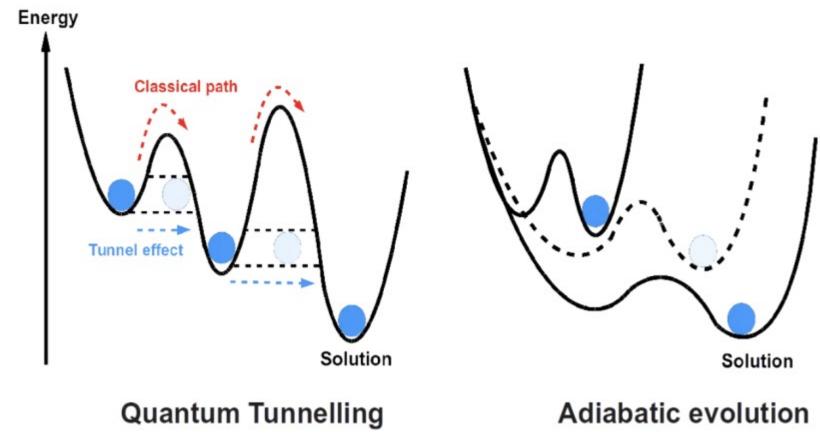
QA와 QAOA의 차이점 분석 후, 각각의 개선방향 및 발전 방향성 또한 제시하고자 한다.

## 2. Theory

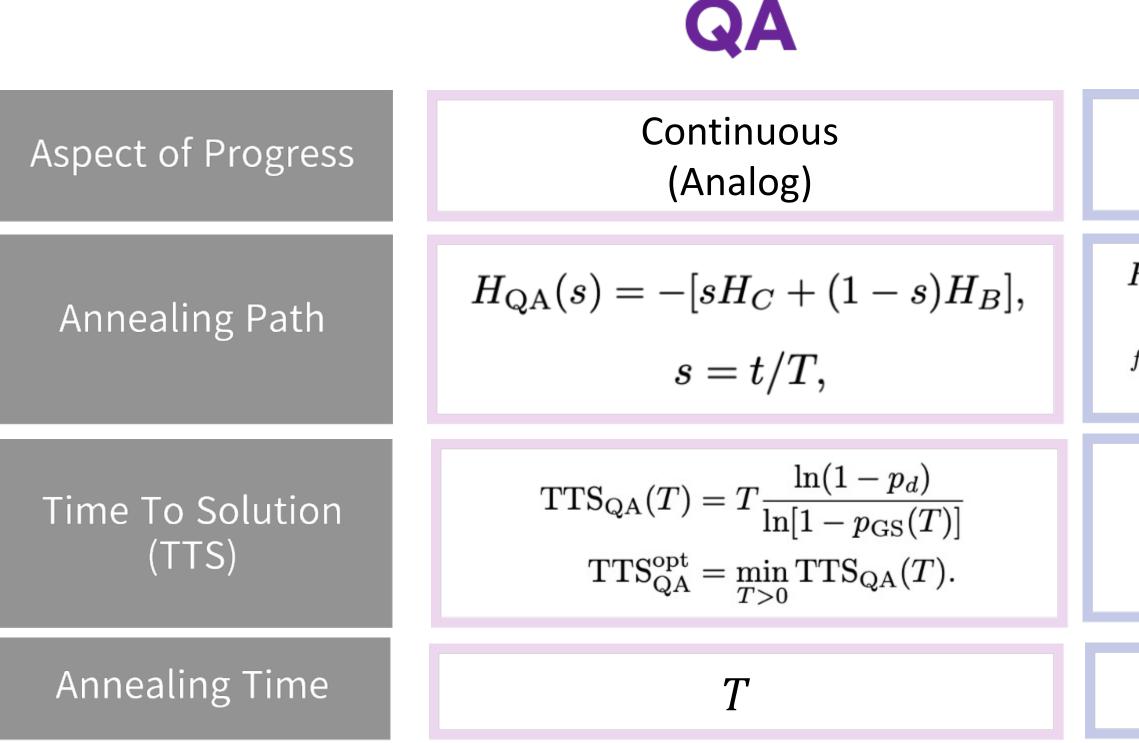
## What is QA?

#### Quantum Annealing

Quantum computation which uses quantum fluctuations(quantum tunneling), in order to search for the ground state of a user programmed Hamiltonian.







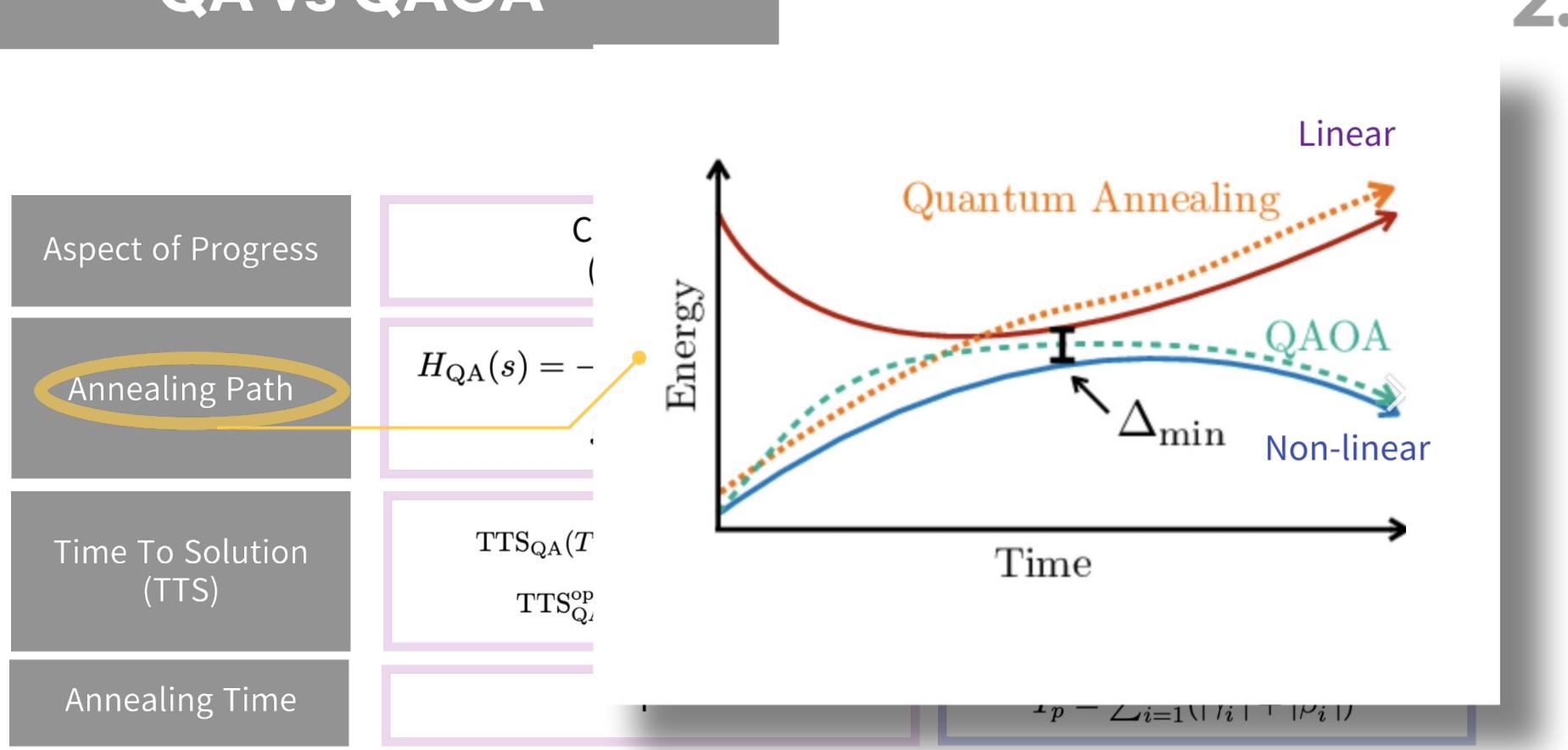


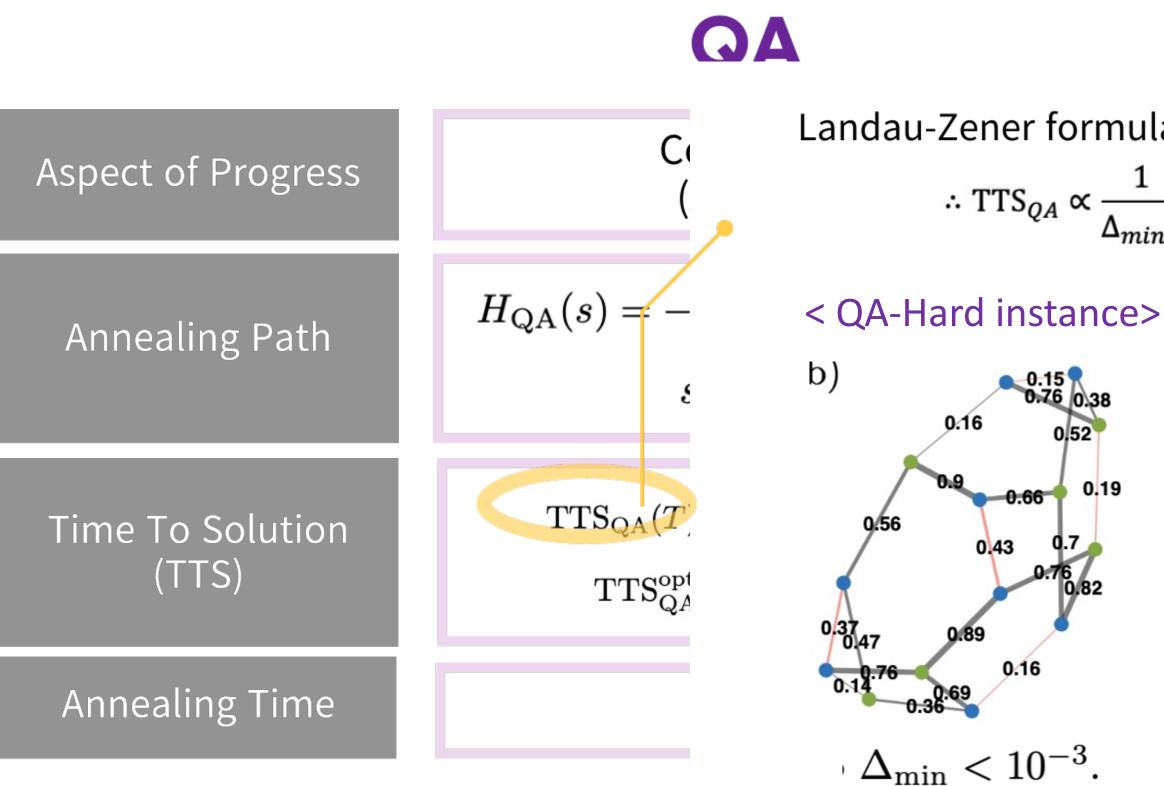
Discrete (Digital)

$$H_{\text{QAOA}}(t) = -[f(t)H_C + (1 - f(t))H_B],$$
  
$$f\left(t_i = \sum_{j=1}^i (|\gamma_j^*| + |\beta_j^*|) - \frac{1}{2}(|\gamma_i^*| + |\beta_i^*|)\right) = \frac{\gamma_i^*}{|\gamma_i^*| + |\beta_i^*|}$$

$$TTS_{QAOA}(p) = T_p \frac{\ln(1 - p_d)}{\ln[1 - p_{GS}(p)]}$$
$$TTS_{QAOA}^{opt} = \min_{p>0} TTS_{QAOA}(p),$$

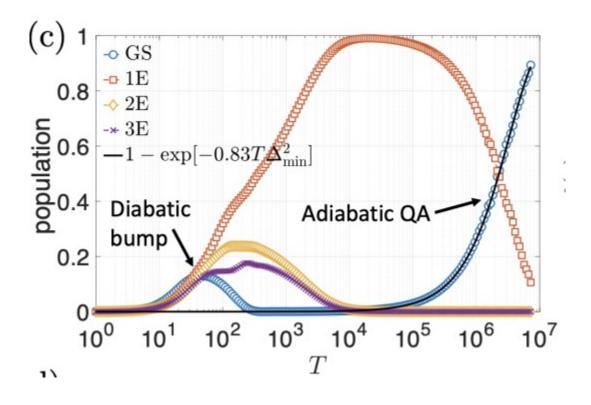
$$T_p = \sum_{i=1}^p (|\gamma_i^*| + |\beta_i^*|)$$

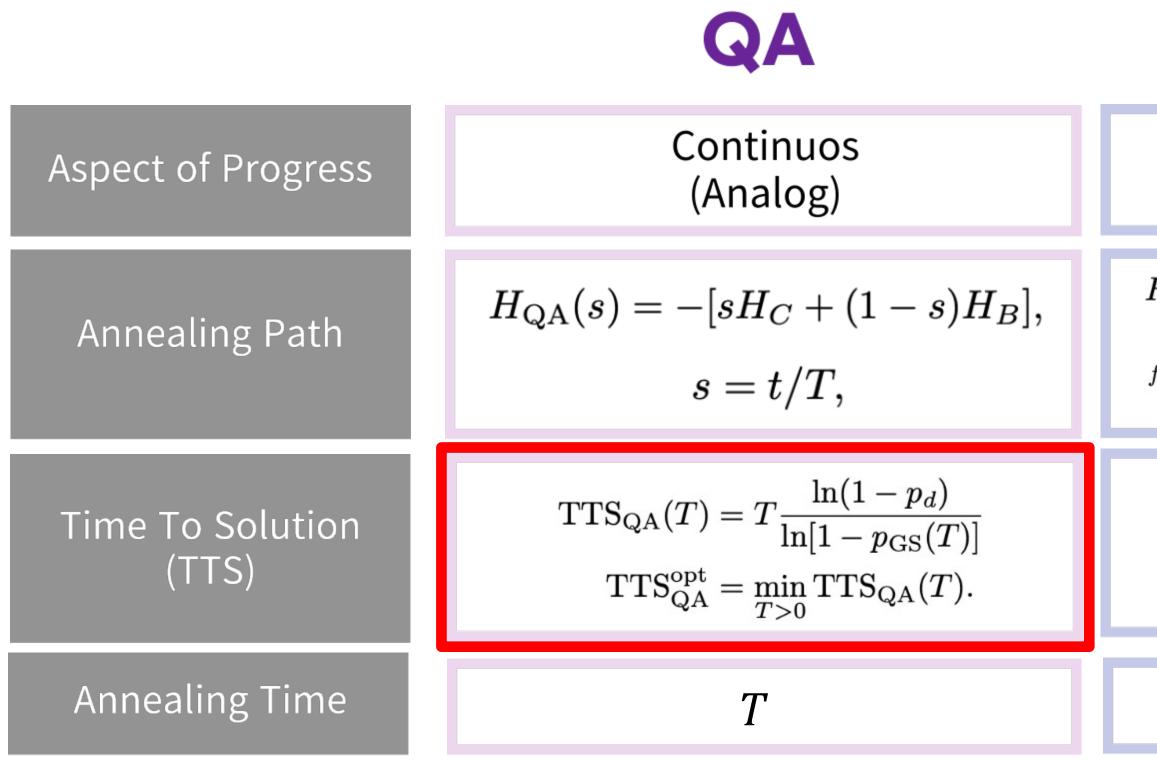




#### QAOA

Landau-Zener formula  $\ln (1 - p_{GS}(T)) \propto T \Delta_{min}^2$  $\therefore \text{TTS}_{QA} \propto \frac{1}{\Delta_{min}^2} \text{ (independent of T)}$ 







Discrete (Digital)

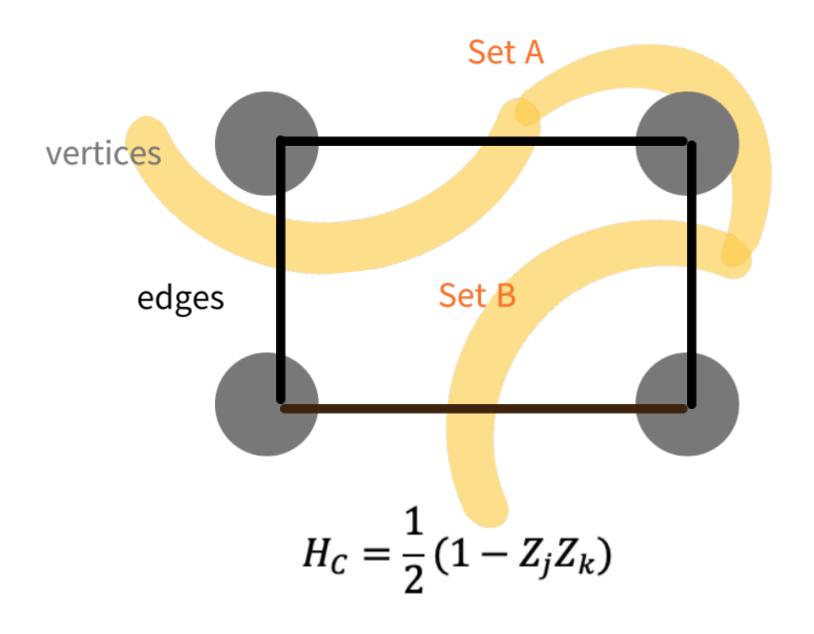
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$$TTS_{QAOA}(p) = T_p \frac{\ln(1 - p_d)}{\ln[1 - p_{GS}(p)]}$$
$$TTS_{QAOA}^{opt} = \min_{p>0} TTS_{QAOA}(p),$$

$$T_p = \sum_{i=1}^p (|\gamma_i^*| + |\beta_i^*|)$$

### Max Cut

#### 대표적인 Combinotorial Optimization Problem 2개의 part로 vertex를 나누면서 가장 많이 egdge를 지나가야하는 문제

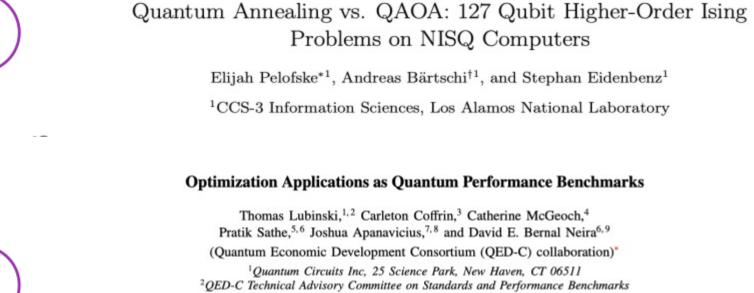






## 3. Method

#### References



Thomas Lubinski,<sup>1,2</sup> Carleton Coffrin,<sup>3</sup> Catherine McGeoch,<sup>4</sup> Pratik Sathe,<sup>5,6</sup> Joshua Apanavicius,<sup>7,8</sup> and David E. Bernal Neira<sup>6,9</sup> (Quantum Economic Development Consortium (QED-C) collaboration)\* <sup>1</sup>Quantum Circuits Inc, 25 Science Park, New Haven, CT 06511 <sup>2</sup>QED-C Technical Advisory Committee on Standards and Performance Benchmarks <sup>3</sup>Advanced Network Science Initiative, Los Alamos National Laboratory, USA <sup>4</sup>D-Wave Systems, Burnaby, British Columbia, Canada, V5G 4M9, Canada <sup>5</sup>Department of Physics and Astronomy, University of California at Los Angeles, USA <sup>6</sup>Research Institute of Advanced Computer Science, Universities Space Research Association, Mountain View, CA, USA <sup>8</sup>Indiana University Department of Physics, Bloomington, Indiana 47405, USA <sup>9</sup>Quantum Artificial Intelligence Laboratory, NASA Ames Research Center, Mountain View, CA, USA

#### Quantum Approximate Optimization Algorithm: Performance, Mechanism, and Implementation on Near-Term Devices

Leo Zhou,<sup>1,\*</sup> Sheng-Tao Wang,<sup>1,†</sup> Soonwon Choi,<sup>1,2</sup> Hannes Pichler,<sup>3,1</sup> and Mikhail D. Lukin<sup>1</sup>

<sup>1</sup>Department of Physics, Harvard University, Cambridge, MA 02138, USA <sup>2</sup>Department of Physics, University of California Berkeley, Berkeley, CA 94720, USA <sup>3</sup>ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138, USA (Dated: November 12, 2019)



유일하게 QA의 한계점이 드러난 문제 제기 Depth가 늘어난다면 QAOA의 더 나은 수행능력 예상





#### References

#### Quantum Annealing vs. QAOA: 127 Qubit Higher-Order Ising Problems on NISQ Computers

Elijah Pelofske<sup>\*1</sup>, Andreas Bärtschi<sup>†1</sup>, and Stephan Eidenbenz<sup>1</sup>

<sup>1</sup>CCS-3 Information Sciences, Los Alamos National Laboratory

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#### Measuring Performance

#### 1. Result Fidelity

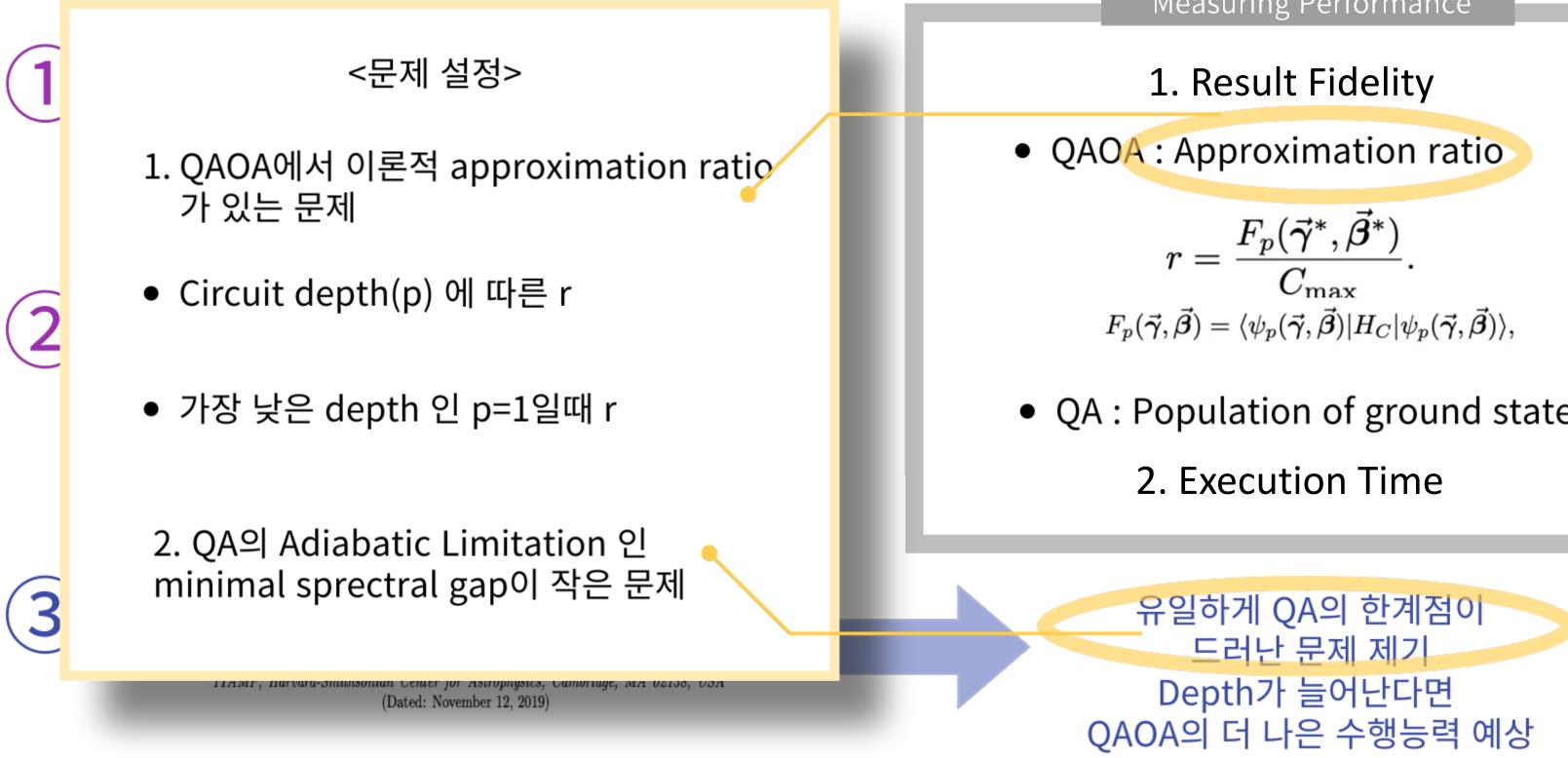
• QAOA : Approximation ratio

$$r = rac{F_p(ec{m{\gamma}}^*, ec{m{eta}}^*)}{C_{ ext{max}}}. 
onumber \ F_p(ec{m{\gamma}}, ec{m{eta}}) = \langle \psi_p(ec{m{\gamma}}, ec{m{eta}}) | H_C | \psi_p(ec{m{\gamma}}, ec{m{eta}}) 
angle,$$

QA : Population of ground state
 2. Execution Time

유일하게 QA의 한계점이 드러난 문제 제기 Depth가 늘어난다면 QAOA의 더 나은 수행능력 예상

### References

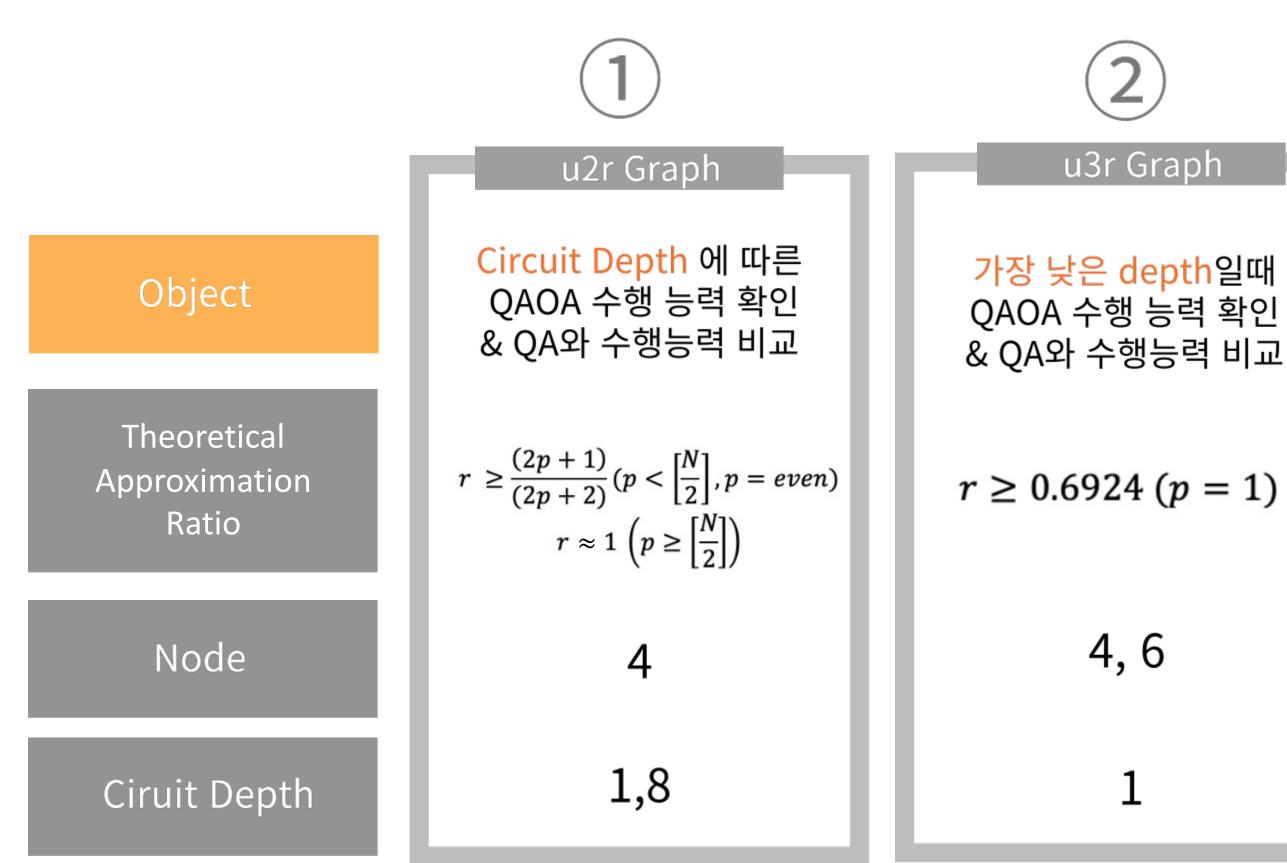


#### Measuring Performance

$$r = rac{F_p(ec{m{\gamma}}^*, ec{m{eta}}^*)}{C_{ ext{max}}}. 
onumber \ F_p(ec{m{\gamma}}, ec{m{eta}}) = \langle \psi_p(ec{m{\gamma}}, ec{m{eta}}) | H_C | \psi_p(ec{m{\gamma}}, ec{m{eta}}) 
angle,$$

• QA : Population of ground state

### Problems





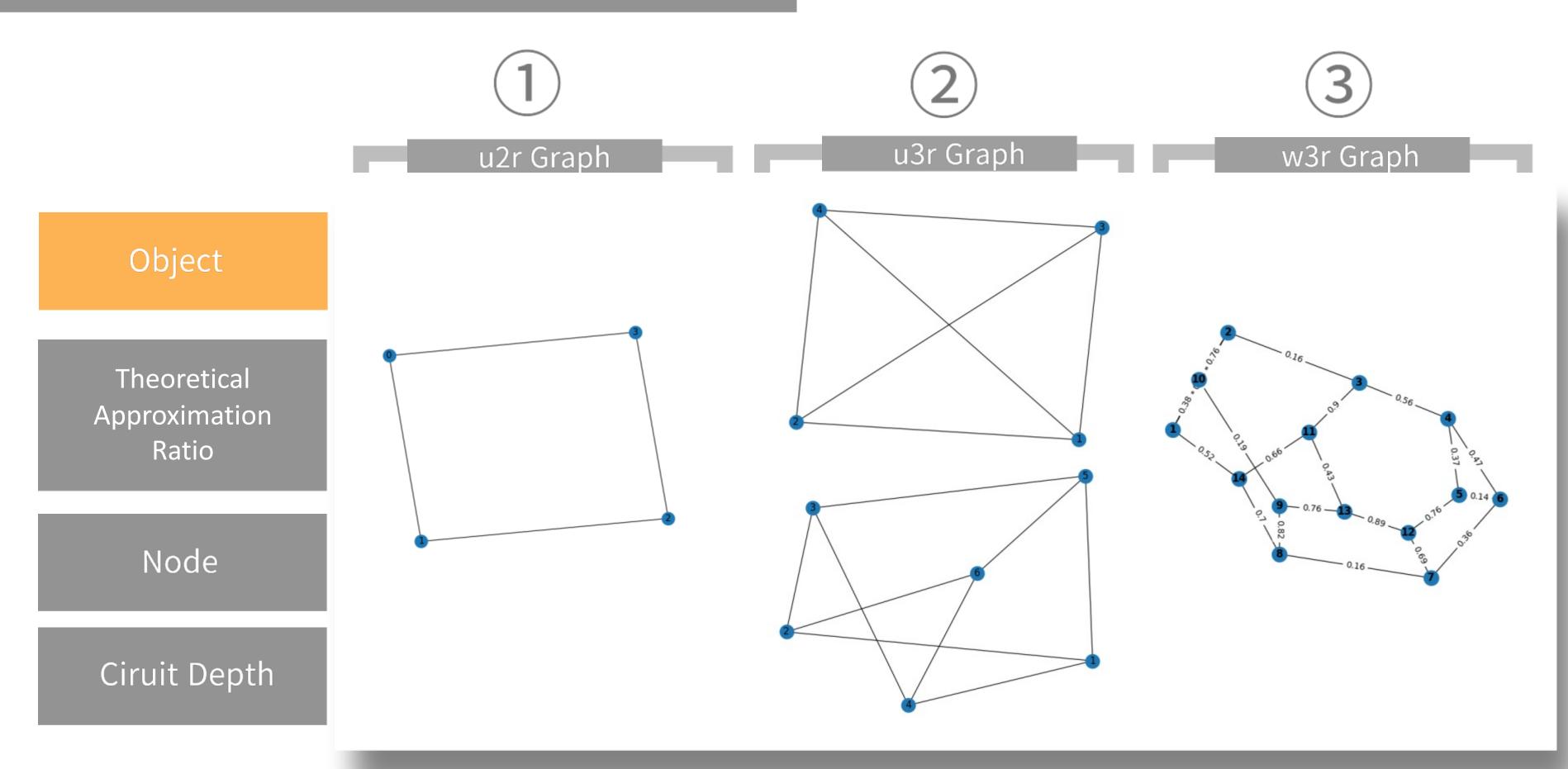


w3r Graph

#### Minimal spectral gap<sup>0</sup> 작은 문제에서의 QA 수행 능력 확인 & QAOA와 수행능력 비교

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### Problems





#### Parameters



#### Device

#### Quantum Simulator/Sampler

#### Classical Optimizer

#### CPU

pennylane의 default.qubit device # of samples : 100

Adagrad Optimizer step size : 0.5 step number : 100 3.



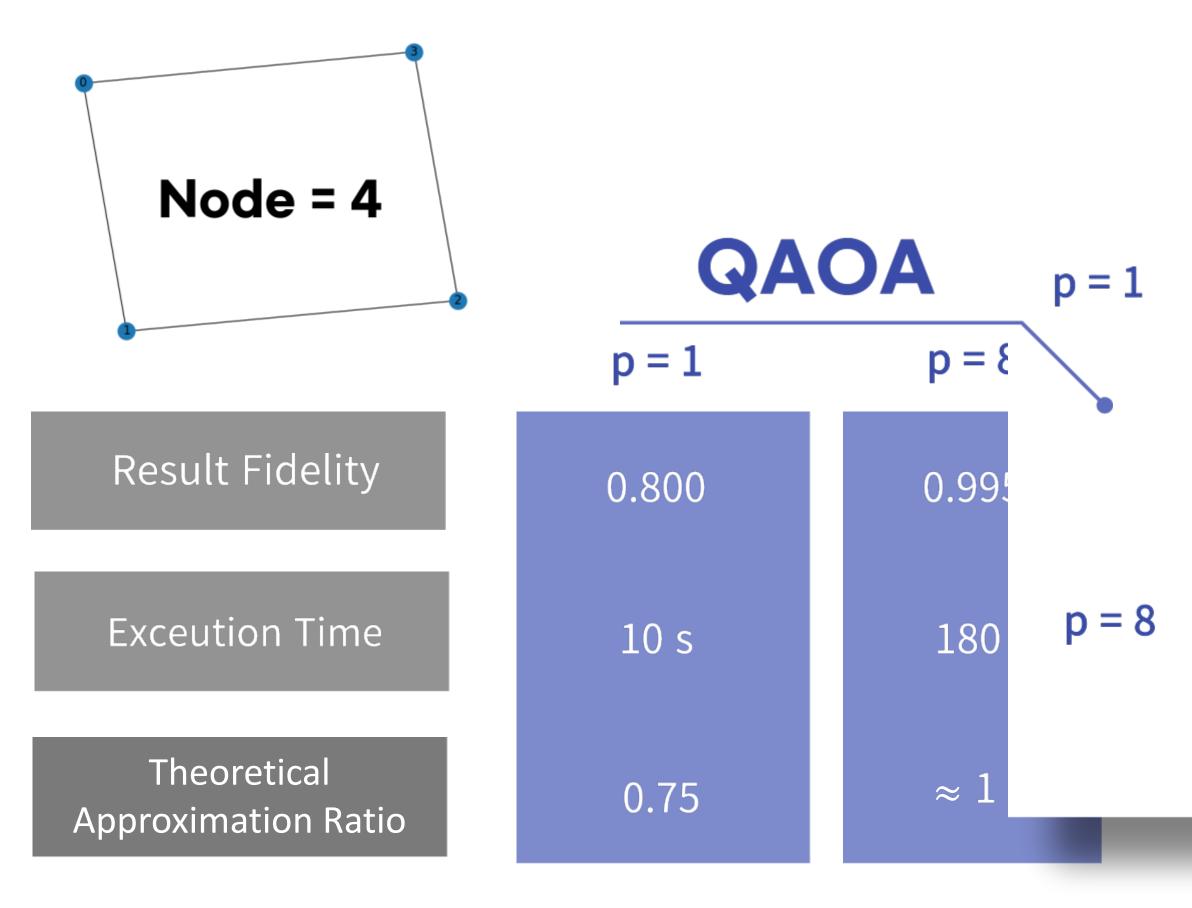
#### CPU, QPU (D-Wave Cloud)

sample\_qubo, smaple\_ising Chainstrength : 8, 2

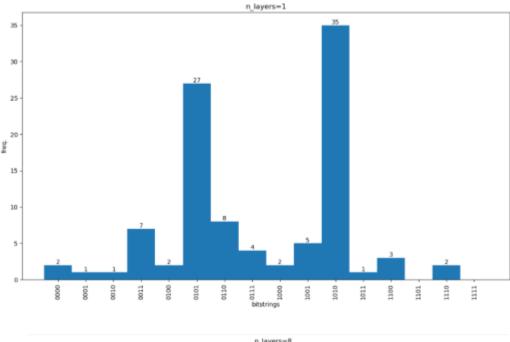
Х

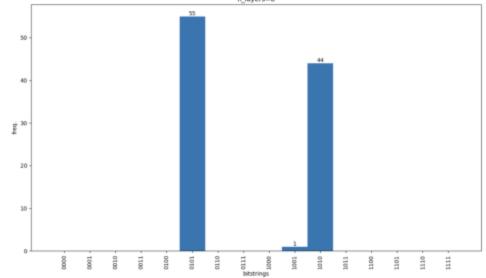
## 4. Results

### u2r Graph

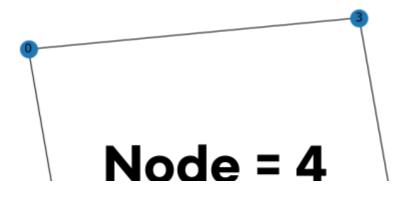








## u2r Graph



Set 0	Set 1	Energy	Cut Size
[1, 3]	[2, 4]	-4.0	4
[2, 4]	[1, 3]	-4.0	4
Set Ø	Set 1	Energy	Cut Size
[2, 4]	[1, 3]	-4.0	4
[1, 3]	[2, 4]	-4.0	4
Set 0	Set 1	Energy	Cut Size
[1, 3]	[2, 4]	-4.0	4
[2, 4]	[1, 3]	-4.0	4
Set Ø	Set 1	Energy	Cut Size
[1, 3]	[2, 4]	-4.0	4
[2, 4]	[1, 3]	-4.0	4
Set Ø	Set 1	Energy	Cut Size
[2, 4]	[1, 3]	-4.0	4
[1, 3]	[2, 4]	-4.0	4
Set Ø	Set 1	Energy	Cut Size
[1, 3]	[2, 4]	-4.0	4
[2, 4]	[1, 3]	-4.0	4
Set 0	Set 1	Energy	Cut Size
[2, 4]	[1, 3]	-4.0	4
[1, 3]	[2, 4]	-4.0	4

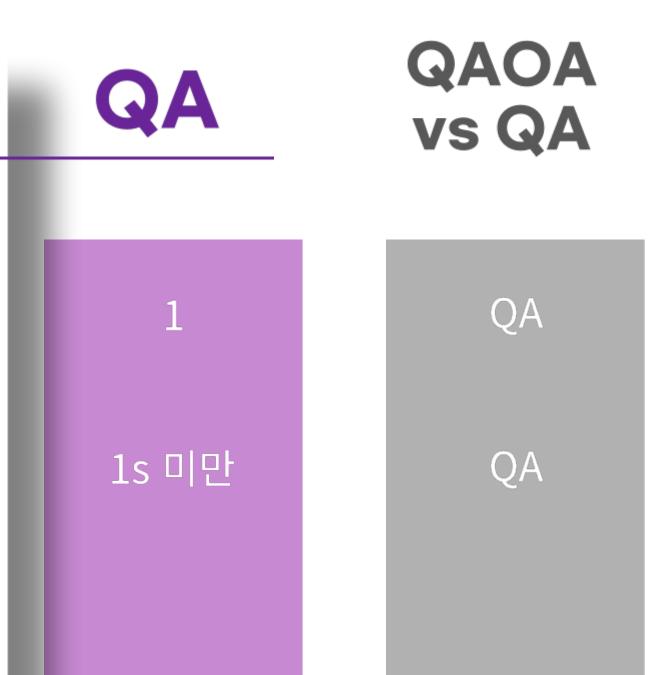
<100번 중 답 나온 횟수>

X 100

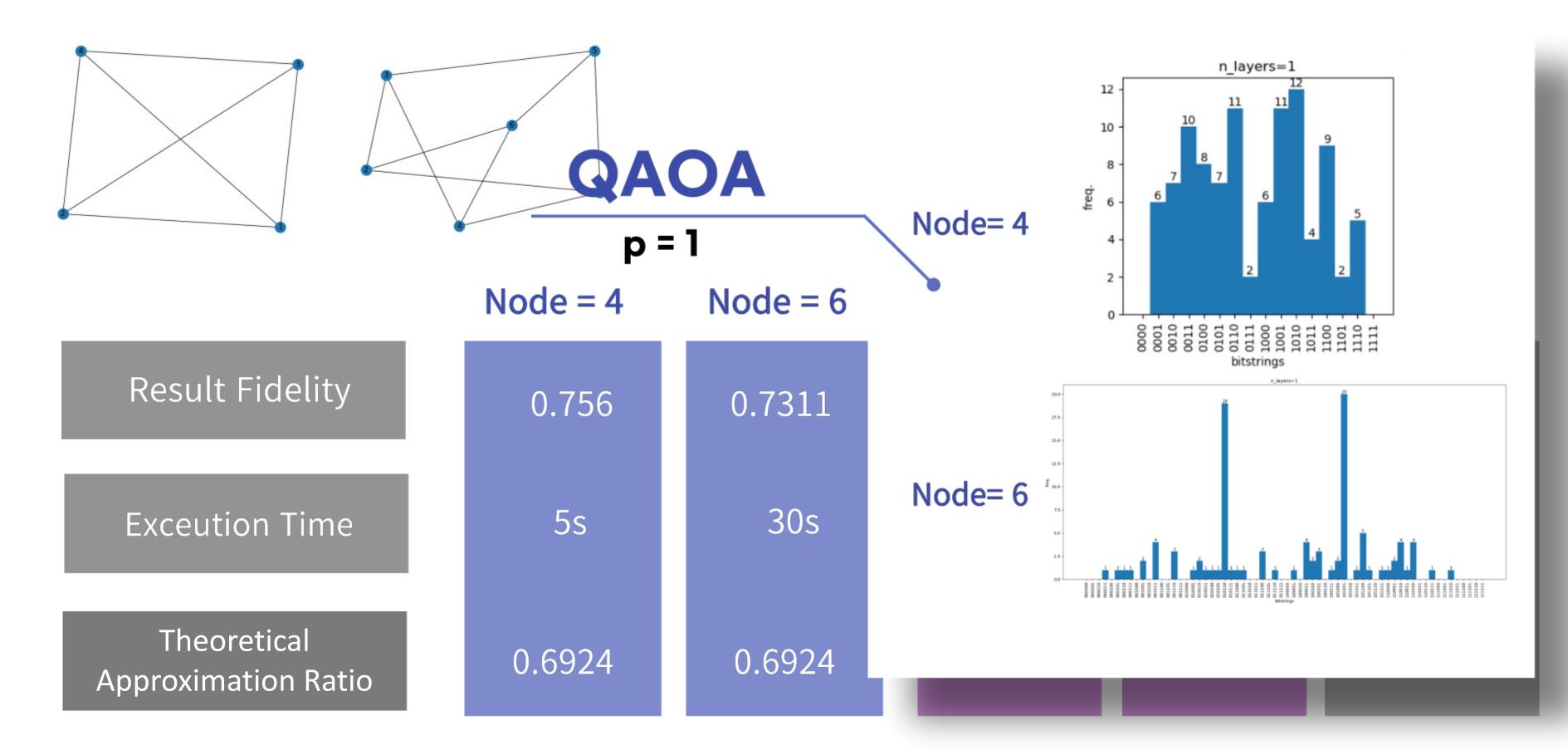
Sol1 |1010) : 100/100

*So*12 |0101) : 100/100





## u3r Graph





## u3r Graph

Set 0	Set 1	Energy	Cut Size
[1, 2]	[0, 3]	-2.0	4
[0, 3]	[1, 2]	-2.0	4
[0, 2]	[1, 3]	-2.0	4
[2, 3]	[0, 1]	-2.0	4
[0, 1]	[2, 3]	-2.0	4
[1, 3]	[0, 2]	-2.0	4
Set 0	Set 1	Energy	Cut Size
[0, 1]	[2, 3]	-2.0	4
[2, 3]	[0, 1]	-2.0	4
[1, 3]	[0, 2]	-2.0	4
[1, 2]	[0, 3]	-2.0	4
[0, 3]	[1, 2]	-2.0	4
[0, 2]	[1, 3]	-2.0	4
[0, 2, 3]	[1]	0.0	3
Set 0	Set 1	Energy	Cut Size
[1, 3]	[0, 2]	-2.0	4
[0, 2]	[1, 3]	-2.0	4
[0, 3]	[1, 2]	-2.0	4
[2, 3]	[0, 1]	-2.0	4
[0, 1]	[2, 3]	-2.0	4
[1, 2]	[0, 3]	-2.0	4
Set 0	Set 1	Energy	Cut Size
[1, 3]	[0, 2]	-2.0	4
[0, 2]	[1, 3]	-2.0	4
[0, 3]	[1, 2]	-2.0	4
[2, 3]	[0, 1]	-2.0	4
[0, 1]	[2, 3]	-2.0	4
[1, 2]	[0, 3]	-2.0	4
[1, 2, 3]	[0]	0.0	3
Set 0	Set 1	Energy	Cut Size
[0, 1]	[2, 3]	-2.0	4
[2, 3]	[0, 1]	-2.0	4
[1, 2]	[0, 3]	-2.0	4
[1, 3]	[0, 2]	-2.0	4
[0, 2]	[1, 3]	-2.0	4
[0, 3]	[1, 2]	-2.0	4

#### Node = 4

Sol1 |010110) : 100/100 Sol2 |101001) : 100/100

Sol1 |0011> : 100/100 Sol2 |1100> : 100/100 Sol3 |0110> : 100/100 Sol4 |1001> : 100/100 Sol5 |0101> : 100/100 Sol6 |1010> : 100/100

#### Node = 6

Set Ø	Set 1	Energy	Cut	Size
	2,5] 3,4]	-9.0 -9.0		9 9
Set 0	Set 1	Energy	Cut	Size
	2, 5] 3, 4]	-9.0 -9.0		9 9
Set Ø	Set 1	Energy	Cut	Size
3,4] 2,5]	2, 5] 3, 4]	-9.0 -9.0		9 9
Set Ø	 Set 1	Energy	Cut	Size
	3, 4] 2, 5]	-9.0 -9.0		9 9
Set 0	Set 1	Energy	Cut	Size
	2,5] 3,4]	-9.0 -9.0		9 9
Set Ø	Set 1	Energy	Cut	Size
	2,5] 3,4]	-9.0 -9.0		9 9
Set Ø	Set 1	Energy	Cut	Size
	2, 5] 3, 4]	-9.0 -9.0		9 9





미만

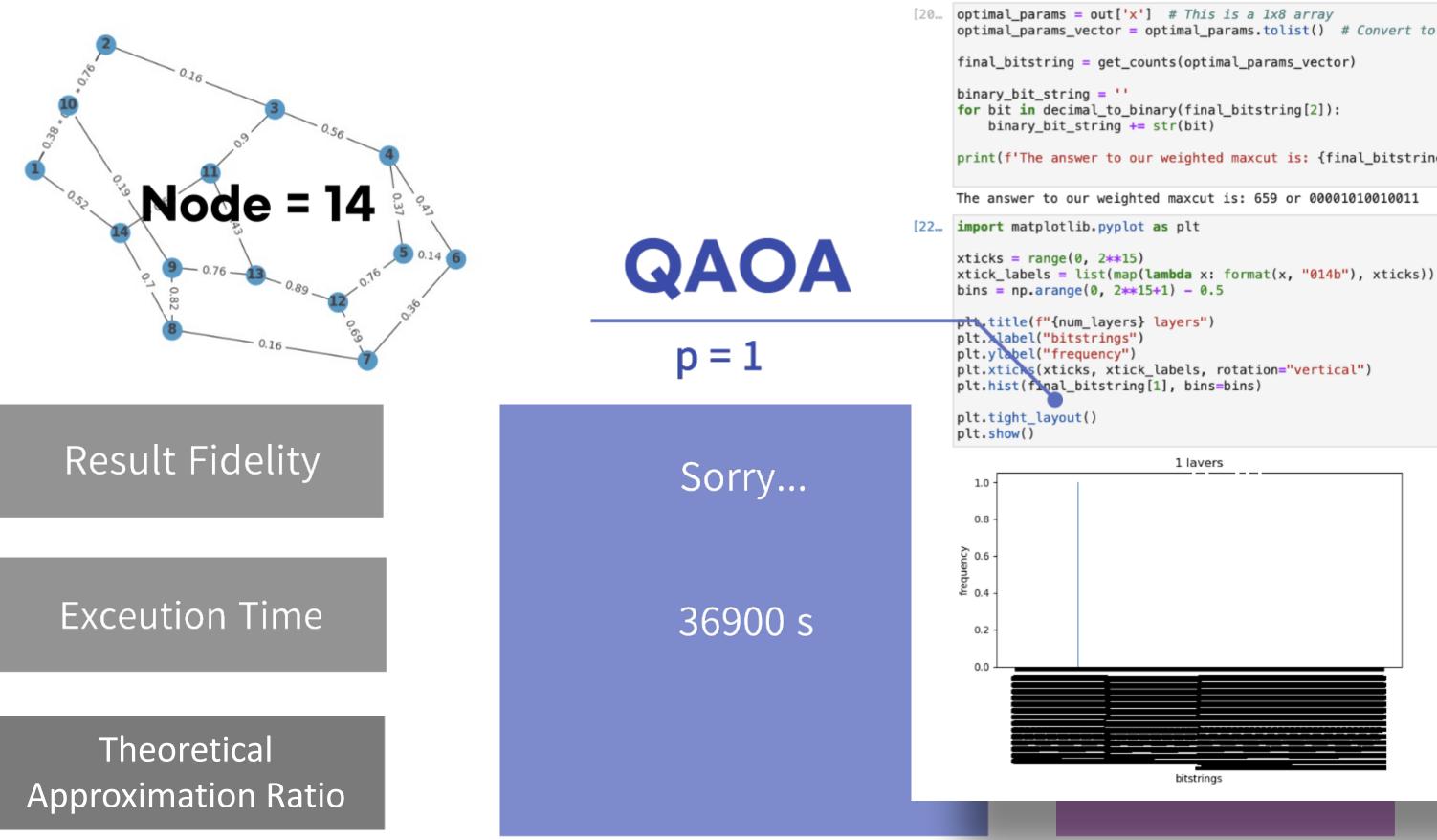
#### QAOA vs QA

#### de = 4 Node = 6

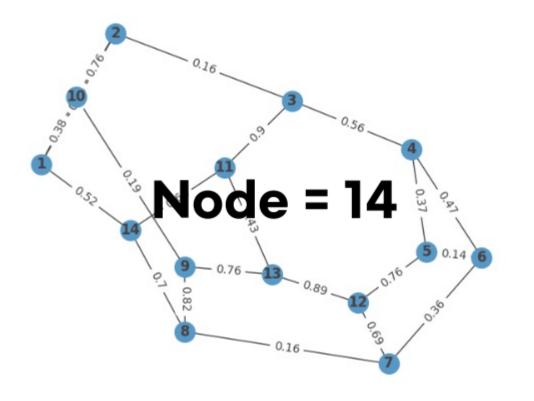
1 1s 미만



QA



```
optimal_params_vector = optimal_params.tolist() # Convert to a Python list if needed
print(f'The answer to our weighted maxcut is: {final_bitstring[2]} or {binary_bit_string}')
```



#### Result Fidelity

Exceution Time

Theoretical Approximation Ratio **QAOA** p = 1

Sorry...

36900 s





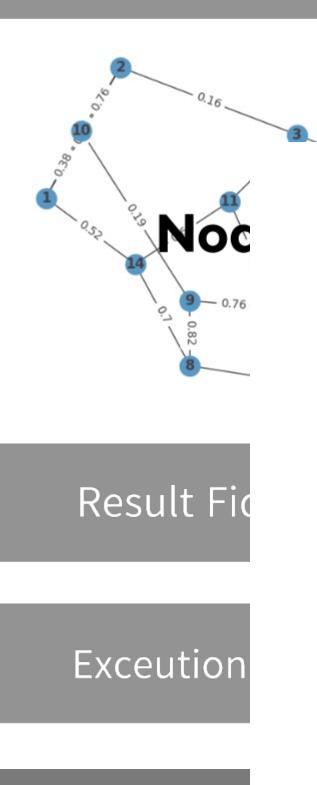
#### QAOA vs QA

0.28

**1.8** s



QA



Theoreti Approximatic *So*12 |00100100110010〉: 56/100 둘중 하나라도 나온 것 : 83/100 둘다 나온 것 : 28/100

*Sol*1 |11011011001101) : 55/100





#### QAOA vs QA

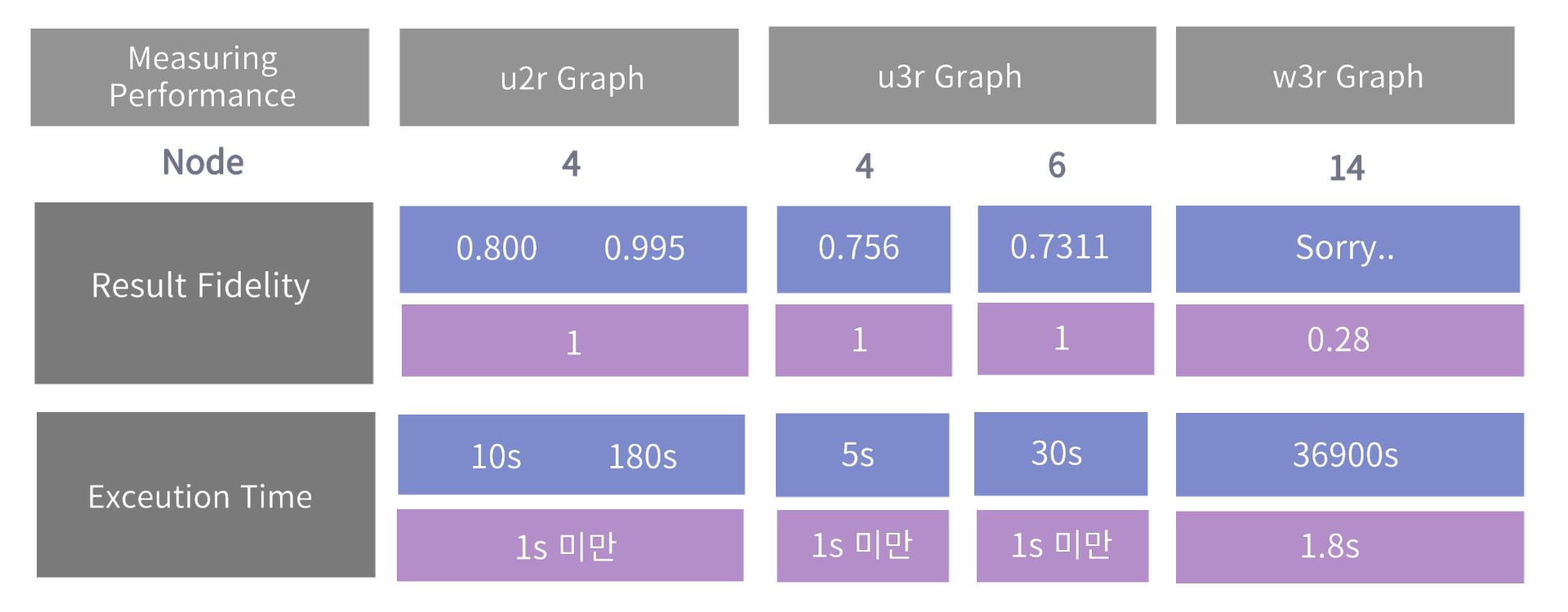
#### 0.28

**1.8** s

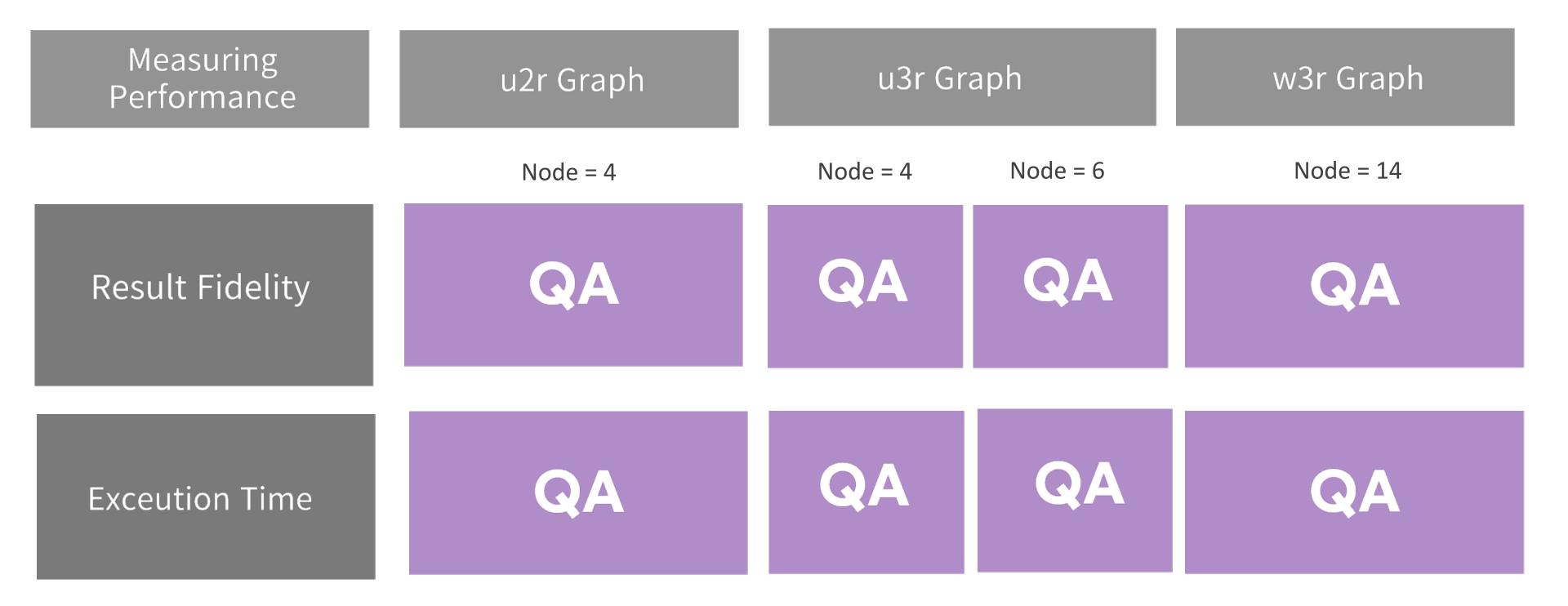


QA

## 5. Analysis



5.



5.

1	Quantum Annealing vs. QAOA: 127 Qubit Higher-Order Ising Problems on NISQ Computers Elijah Pelofske <sup>*1</sup> , Andreas Bärtschi <sup>†1</sup> , and Stephan Eidenbenz <sup>1</sup> <sup>1</sup> CCS-3 Information Sciences, Los Alamos National Laboratory
	<b>Optimization Applications as Quantum Performance Benchmarks</b>
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<sup>1</sup>Department of Physics, Harvard University, Cambridge, MA 02138, USA <sup>2</sup>Department of Physics, University of California Berkeley, Berkeley, CA 94720, USA <sup>3</sup>ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138, USA (Dated: November 12, 2019) 5.

Quantum	Annealing vs.	QAOA:	127	Qubit	Higher-Order	Ising
Problems on NISQ Computers						

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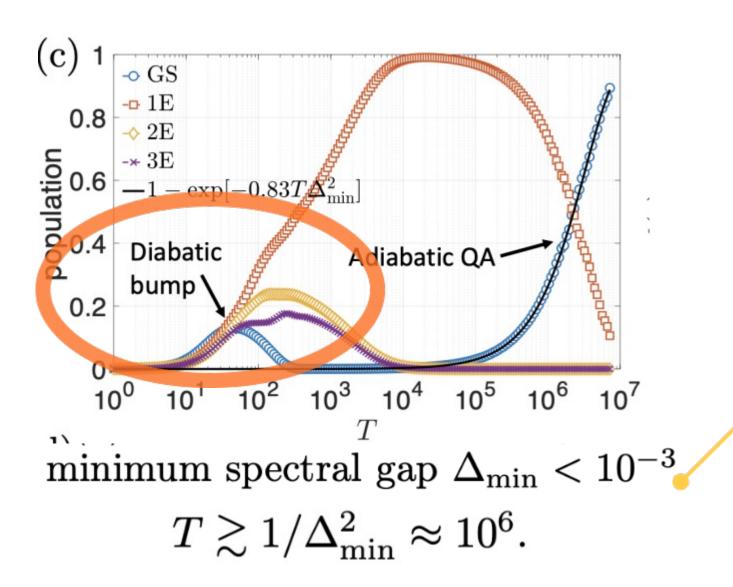


3

5.

# < 논문 1,2 > 수행한 문제들 모두에서 QA가 QAOA보다 더 나은performance를 보여 주었다.

 < 논문3 >
 minimal spectral gap이 작은 w3r (node 14 이상)문제는 QA의 한계점을 보여준다.



0.28 정도의 population을 보일 수 있었던 이유는 Diabatic bump 때문이라 생각됨 5.

# < 논문 1,2 > 수행한 문제들 모두에서 QA가 QAOA보다 더 나은performance를 보여 주었다.

 < 논문3 >
 minimal spectral gap이 작은 w3r (node 14 이상)문제는 QA의 한계점을 보여준다.

## 6. Conclusions

### Conclusions

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그렇다면 아예 QAOA가 QA의 수행능력보다 더 나은 점을 기대해볼 수 있는 건 없을까?

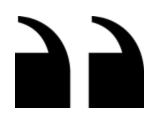
QA -Hard Instance

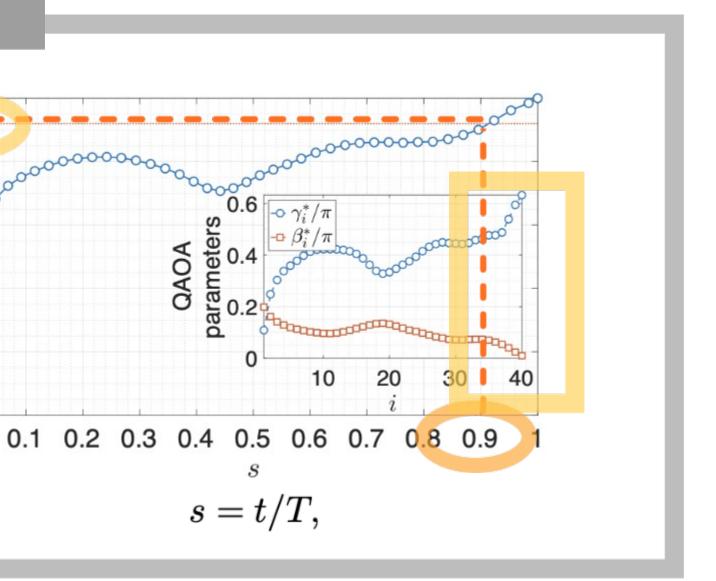
0

(f)

0.92 🥙 QAOA Annealing Path 0.8  $H_{\text{QAOA}}(t) = -[f(t)H_C + (1 - f(t))H_B],$ 0.6 (s) 0.4  $f\left(t_{i} = \sum_{j=1}^{i} (|\gamma_{j}^{*}| + |\beta_{j}^{*}|) - \frac{1}{2}(|\gamma_{i}^{*}| + |\beta_{i}^{*}|)\right) = \frac{\gamma_{i}^{*}}{|\gamma_{i}^{*}| + |\beta_{i}^{*}|}$ 0.2

circuit depth = 40 일 때 f (s) = 0.92 -> 거의 Cost Hamiltonian만 살아남음



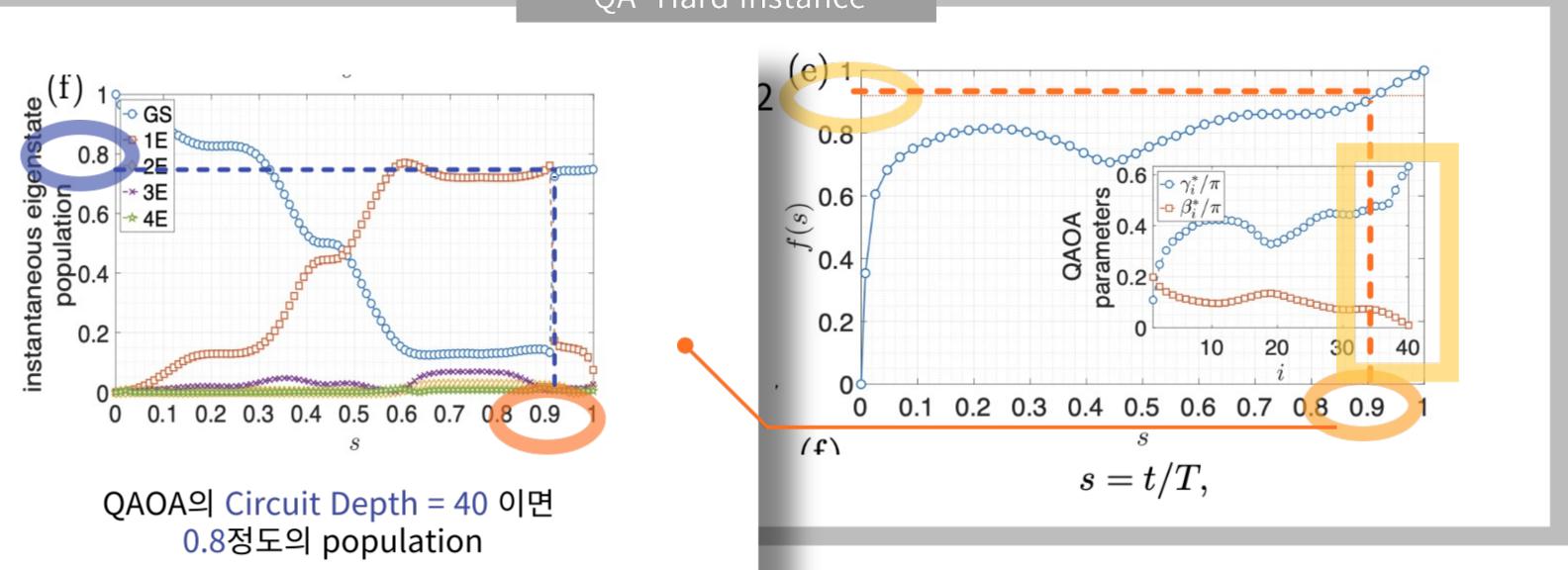


### Conclusions

77

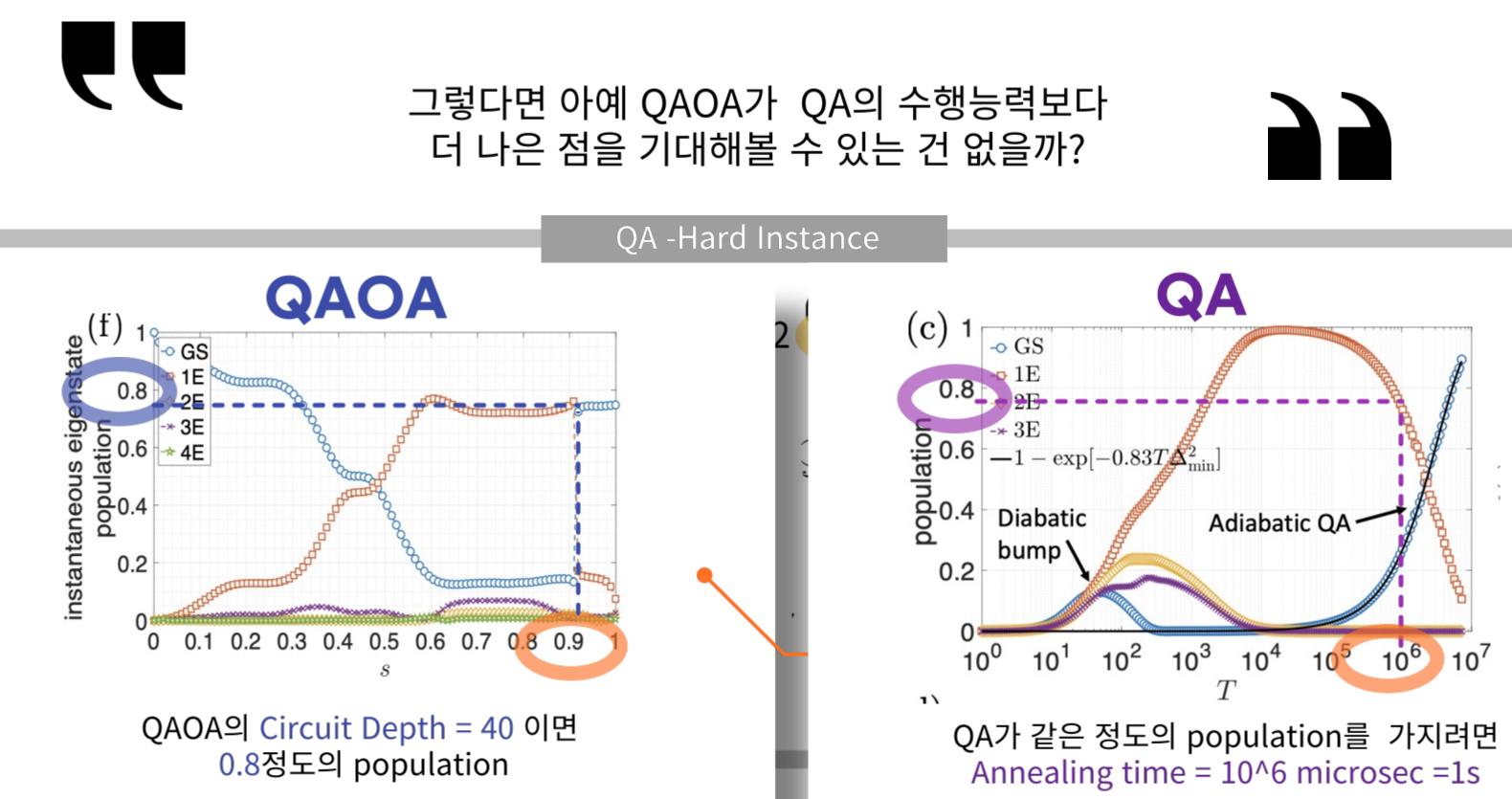
#### 그렇다면 아예 QAOA가 QA의 수행능력보다 더 나은 점을 기대해볼 수 있는 건 없을까?

QA -Hard Instance





### Conclusions





### **Further Research.**





## Circuit Depth = 40 인 연산을 1초 안에 구현할 것이냐

VS



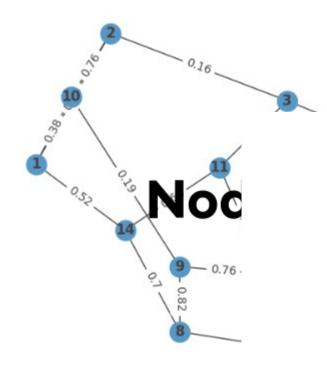
#### 1초동안 Adiabatic Process를 유지할 수 있느냐

## Reference

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Exceution

Theoreti Approximatic Sol1 |11011011001101) : 55/100 Sol2 |00100100110010) : 56/100 둘 중 하나라도 나온 것 : 83/100 둘다 나온 것 : 28/100





#### QAOA vs QA

#### 0.28

**1.8** s



QA