# DOUBLE QUANTUM DOT COUPLED TO TWO SUPERCONDUCTORS: TRANSPORT AND SPIN ENTANGLEMENT

Mahn-Soo Choi, Christoph Bruder, and Daniel Loss
*Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland*

**Keywords:** Quantum dot, superconductivity, entanglement, Josephson effect

**Abstract** A double quantum dot coupled to two superconducting leads is studied in the Coulomb blockade regime. The double dot mediates an effective Josephson coupling between the two superconducting leads, which in turn induce an exchange coupling between the spin states of the dots. The Josephson current depends on the spins on the dots, while the spin exchange coupling can be tuned by the superconducting phase difference. This spin-dependent Josephson current can be used to probe directly the correlated spin states (singlet or triplets).

Superconductors and quantum dots have been subjects of intensive studies on their own. In view of transport phenomena on mesoscopic scales, the most interesting properties of them might be the existence of quantum coherence on the macroscopic level and superconducting energy gap in superconductors, and the Coulomb blockade effects and resonant tunneling in quantum dots (usually coupled to non-interacting leads) [1]. The idea of coupling quantum dots to superconducting leads dates back to the studies of an Anderson impurity [2] or a quantum dot coupled to superconductors [3, 4, 5]. Besides a number of experimental[3] and theoretical [4] papers on the spectroscopic properties of a quantum dot coupled to two superconductors, an effective dc Josephson effect through strongly interacting regions between superconducting leads has been analyzed [6, 7, 8, 9]. Furthermore, recent researches have shown that not only the charges but also the spins on quantum dots can be a valuable resource. In particular, in semiconducting nanostructures, it was found that the direct coupling of two quantum dots by a tunnel junction can be used to create entanglement between spins [10], and that



D. V. Averin et al. (eds.), *Macroscopic Quantum Coherence and Quantum Computing*

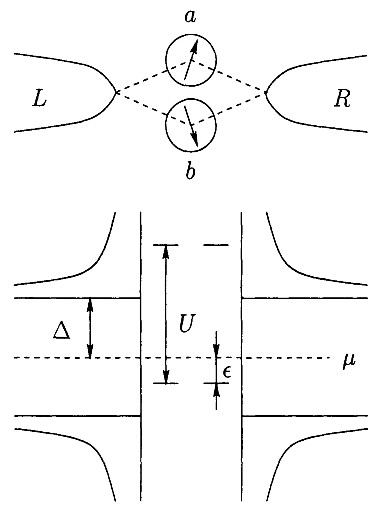


*Figure 1* Sketch of the superconductor-double quantum dot-superconductor (S-DD-S) nanostructure (top), and schematic representation of the quasiparticle energy spectrum in the superconductors and the single-electron levels of the two quantum dots (bottom).

such spin correlations can be observed in charge transport experiments [11].

In this work, we consider a double quantum dot (DD) coupled *in parallel* to two superconducting leads (but not directly with each other). We first investigate two interesting aspects of this particular structure: Josephson current through the strongly interacting region (quantum dots), and effective exchange coupling between the spins on DD due to the superconductors. The Josephson current depends on the spin states whereas the spin exchange coupling can be tuned by the superconducting phase difference between the two superconducting leads. Exploiting such an interplay between the two effects, we also propose an experimental setup to probe spin entanglement on DD.

# 1. MODEL

The double-dot (DD) system we propose is sketched in Fig. 1: Two quantum dots ($a$,$b$), each of which contains one (excess) electron and is connected to two superconducting leads ($L, R$) via tunnel junctions (indicated by dashed lines) [13]. There is no direct coupling between the two dots. The Hamiltonian describing this system consists of three parts, $H_S + H_D + H_T \equiv H_0 + H_T$. The leads are assumed to be conventional singlet superconductors that are described by the BCS Hamiltonian

$$H_S = \sum_{j=L,R} \int_{\Omega_j} \frac{d\mathbf{r}}{\Omega_j} \left[ \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger(\mathbf{r}) h(\mathbf{r}) \psi_\sigma(\mathbf{r}) + \Delta_j(\mathbf{r}) \psi_\uparrow^\dagger(\mathbf{r}) \psi_\downarrow^\dagger(\mathbf{r}) + h.c. \right], \tag{1.1}$$

where $\Omega_j$ is the volume of lead $j$, $h(\mathbf{r}) = (-i\hbar\nabla)^2/2m - \mu$, and $\Delta_j(\mathbf{r}) = \Delta_j e^{-i\phi_j(\mathbf{r})}$ is the pair potential. For simplicity, we assume identical leads with same chemical potential $\mu$, and $\Delta_L = \Delta_R = \Delta$. The two quantum dots are modeled as two localized levels $\epsilon_a$ and $\epsilon_b$ with strong on-site Coulomb repulsion $U$, described by the Hamiltonian

$$H_D = \sum_{n=a,b} \left[ -\epsilon \sum_\sigma d_{n\sigma}^\dagger d_{n\sigma} + U d_{n\uparrow}^\dagger d_{n\uparrow} d_{n\downarrow}^\dagger d_{n\downarrow} \right], \tag{1.2}$$

where we put $\epsilon_a = \epsilon_b = -\epsilon$ ($\epsilon > 0$) for simplicity. $U$ is typically given by the charging energy of the dots, and we have assumed that the level spacing of the dots is $\sim U$ (which is the case for small GaAs dots[1]), so that we need to retain only one energy level in $H_D$. Finally, the DD is coupled *in parallel* (see Fig. 1) to the superconducting leads, described by the tunneling Hamiltonian

$$H_T = \sum_{j,n,\sigma} \left[ t\psi_\sigma^\dagger(\mathbf{r}_{j,n}) d_{n\sigma} + h.c. \right], \tag{1.3}$$

where $\mathbf{r}_{j,n}$ is the point on the lead $j$ closest to the dot $n$. Unless mentioned otherwise, it will be assumed that $\mathbf{r}_{L,a} = \mathbf{r}_{L,b} = \mathbf{r}_L$ and $\mathbf{r}_{R,a} = \mathbf{r}_{R,b} = \mathbf{r}_R$.

Since the low-energy states of the whole system are well separated by the superconducting gap $\Delta$ as well as the strong Coulomb repulsion $U$ ($\Delta, \epsilon \ll U - \epsilon$), it is sufficient to consider an effective Hamiltonian on the reduced Hilbert space consisting of singly occupied levels of the dots and the BCS ground states on the leads. To lowest order in $H_T$, the effective Hamiltonian is

$$H_{eff} = P\, H_T \left[ (E_0 - H_0)^{-1} (1 - P) H_T \right]^3 P, \tag{1.4}$$

where $P$ is the projection operator onto the subspace and $E_0$ is the ground-state energy of the unperturbed Hamiltonian $H_0$. (The 2nd order contribution leads to an irrelevant constant.) The lowest-order expansion (1.4) is valid in the limit $\Gamma \ll \Delta, \epsilon$ where $\Gamma = \pi t^2 N(0)$ and $N(0)$ is the normal-state density of states per spin of the leads at the Fermi energy. Thus, we assume that $\Gamma \ll \Delta, \epsilon \ll U - \epsilon$, and temperatures which are less than $\epsilon$ (but larger than the Kondo temperature).

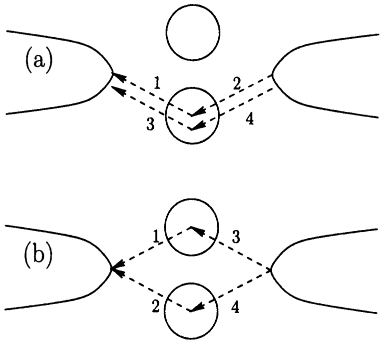


*Figure 2* Two examples of virtual tunneling processes contributing to $H_{eff}$ (1.4). The numbered arrows indicate the direction and the order of occurrence of the charge transfers. Processes of type (a) give a contribution proportional to $J_0$, whereas those of type (b) give contributions proportional to $J$. The two types give significantly different behavior in transport as discussed in the text.

## 2. EFFECTIVE HAMILTONIAN

There are a number of virtual hopping processes that contribute to the effective Hamiltonian (1.4), see Ref. [12] for discussion in more detail. Out of all the possible virtual processes, we show in Fig. 2 the two most physically interesting and significant processes. In a process of type Fig. 2 (a), the two charges of a Cooper pair traverse through the same dot. Such a process, therefore, is not sensitive to spin configuration on DD. In Fig. 2 (b), on the other hand, the two charges take different paths. It turns out that those processes depend strongly on the spin correlation on DD.

Instead of the whole parameter region (see Ref. [12]), here we will focus on two regimes defined by $\zeta \equiv \epsilon/\Delta \gg 1$ and $\zeta \ll 1$. In the limit $\zeta \ll 1$, the most contributions come from the processes of Fig. 2 (b) or the similar and the effective Hamiltonian (1.4) is dominated by a single term

$$H_{eff} \approx J(1+\cos\phi)\left[\mathbf{S}_a \cdot \mathbf{S}_b - \frac{1}{4}\right] \tag{1.5}$$

up to terms of order $\zeta$, where $\phi = \phi_L - \phi_R$ is the superconducting phase difference between the two leads and

$$J \approx 2\Gamma^2/\epsilon\,. \tag{1.6}$$

Equation (1.5) is one of our main results. A remarkable feature of it is that a Heisenberg exchange coupling between the spin on dot $a$ and on dot $b$ is induced by the superconductor. This coupling is antiferromagnetic ($J > 0$) and thus favors a singlet ground state of spin $a$ and

$b$. This in turn is a direct consequence of the assumed singlet nature of the Cooper pairs in the superconductor [14]. As discussed below, an immediate observable consequence of $H_{eff}$ is a *spin-dependent* Josephson current from the left to right superconducting lead (see Fig. 1) which probes the correlated spin state on the DD.

Since the exchange coupling in Eq. (1.5) results from two charges of the same Cooper pair traversing to different dots, it is clear that the dot cannot be separated too far. When the distance between the contact points is given by $\delta r = |\mathbf{r}_{L,a} - \mathbf{r}_{L,b}| = |\mathbf{r}_{R,a} - \mathbf{r}_{R,b}|$, one can show that $J(\delta r) \approx J(0)e^{-2\delta r/\xi} \sin^2(k_F\delta r)/(k_F\delta r)^2$ up to order $1/k_F\xi$. Here $\xi$ is the superconducting coherence length and $k_F$ the Fermi wave vector in the leads. Thus, to have $J(\delta r)$ non-zero, $\delta r$ should not exceed the superconducting coherence length $\xi$.

Another interesting but experimentally less feasible region is the limit $\zeta \gg 1$. In this region, the effective Hamiltonian (1.4) can be reduced to

$$H_{eff} \approx J_0 \cos\phi + 2J_0(1+\cos\phi)\left[\mathbf{S}_a \cdot \mathbf{S}_b - \frac{1}{4}\right] , \tag{1.7}$$

up to order $(\log\zeta)/\zeta$, where

$$J_0 \approx 0.1\frac{\Gamma^2}{\epsilon}\frac{\log\zeta}{\zeta} . \tag{1.8}$$

As can be seen in Fig. 2 (a), the first term in (1.7) has the same origin as that in the single-dot case [2]: Each dot separately constitutes an effective Josephson junction with coupling energy $-J_0/2$ (i.e. $\pi$-junction) between the two superconductors. The two resulting junctions form a dc SQUID, leading to the total Josephson coupling in the first term of (1.7). The Josephson coupling in the second term in (1.7) has a spin-dependence similar to Eq. (1.5) but its origin is quite different (coming from processes other than either of Fig. 2). For the singlet state, it gives an ordinary Josephson junction with coupling $2J_0$ and competes with the first term, whereas it vanishes for the triplet states.

## 3. TO PROBE SPIN ENTANGLEMENT

The result (1.5) allows us to probe directly the correlated spin states on the double dot by measuring the Josephson current. But it would be useful if one could prepare the system in states other than the singlet ground state and compare their different behaviors in a single setup. We propose a dc-SQUID-like structure Fig. 3. We now propose a possible experimental setup to probe the correlations (entanglement) of the spins on the dots, based on the effective model (1.5). According to (1.5) the S-DD-S structure can be regarded as a *spin-dependent* Josephson junction.

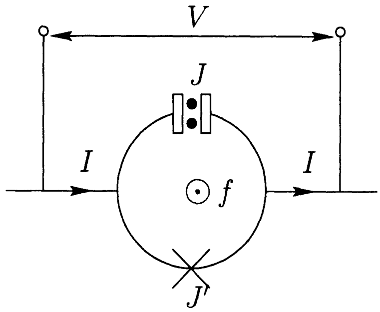


*Figure 3* dc-SQUID-like geometry consisting of the S-DD-S structure (filled dots at the top) connected in parallel with another ordinary Josephson junction (cross at the bottom).

Moreover, this structure can be connected with an ordinary Josephson junction to form a dc-SQUID-like geometry, see Fig. 3. The Hamiltonian of the entire system is then given by

$$\begin{aligned} H &= J[1+\cos(\theta-2\pi f)]\left(\mathbf{S}_a\cdot\mathbf{S}_b-\frac{1}{4}\right) \\ &\quad +\alpha J(1-\cos\theta)\,, \end{aligned} \tag{1.9}$$

where $f=\Phi/\Phi_0$, $\Phi$ is the flux threading the SQUID loop, $\Phi_0=hc/2e$ is the superconducting flux quantum, $\theta$ is the gauge-invariant phase difference across the auxiliary junction ($J'$), and $\alpha=J'/J$ with $J'$ being the Josephson coupling energy of the auxiliary junction[16]. One immediate consequence of (1.9) is that at zero temperature, we can *effectively* turn on and off the spin exchange interaction: For half-integer flux ($f=1/2$), singlet and triplet states are degenerate at $\theta=0$. Even at finite temperatures, where $\theta$ is subject to thermal fluctuations, singlet and triplet states are almost degenerate around $\theta=0$. On the other hand, for integer flux ($f=0$), the energy of the singlet is lower by $J$ than that of the triplets.

This observation allows us to probe directly the spin state on the double dot via a Josephson current across the dc-SQUID-like structure in Fig. 3. The supercurrent through the SQUID-ring is defined as $I_S=(2\pi c/\Phi_0)\partial\langle H\rangle/\partial\theta$, where the brackets refer to a spin expectation value on the DD. Thus, depending on the spin state on the DD we find

$$I_S/I_J=\begin{cases}\sin(\theta-2\pi f)+\alpha\sin\theta & \text{(singlet)}\\ \alpha\sin\theta & \text{(triplets)}\end{cases}, \tag{1.10}$$

where $I_J=2eJ/\hbar$. When the system is biased by a dc current $I$ larger than the spin- and flux-dependent critical current, given by $\max_\theta\{|I_S|\}$, a finite voltage $V$ appears. Then one possible experimental procedure might be as follows: Apply a dc bias current such that $\alpha I_J < I < (\alpha + 1)I_J$. Here, $\alpha I_J$ is the critical current of the triplet states, and $(\alpha + 1)I_J$ the critical current of the singlet state at $f = 0$, see (1.10). Initially prepare the system in an equal mixture of singlet and triplet states by tuning the flux around $f = 1/2$. (With electron $g$-factors $g \sim 0.5$–$20$ the Zeeman splitting on the dots is usually small compared with $k_B T$ and can thus be ignored.) Adopting the discussion of Ref. [17] to our case, we find that in this mixture, one will measure a dc voltage given by $(V_0 + 3V_1)/4$, where $V_0 = \pi\Delta[2e(\alpha+1)]^{-1}\sqrt{(I/I_J)^2 - (\alpha-1)^2}$ is associated with the singlet and $V_1 = \pi\Delta[2e\alpha]^{-1}\sqrt{(I/I_J)^2 - \alpha^2}$ with the triplet states. At a later time $t = 0$, the flux is switched off (i.e. $f = 0$), with $I$ being kept fixed. The ensuing time evolution of the system is characterized by three time scales: the time $\tau_{coh} \sim \max\{1/\Delta, 1/\Gamma\} \sim 1/\Gamma$ it takes to establish coherence in the S-DD-S junction, the spin relaxation time $\tau_{spin}$ on the dot, and the switching time $\tau_{sw}$ to reach $f = 0$. We will assume $\tau_{coh} \ll \tau_{spin}, \tau_{sw}$, which is not unrealistic in view of measured spin decoherence times in GaAs exceeding 100 ns [18]. If $\tau_{sw} < \tau_{spin}$, the voltage is given by $3V_1/4$ for times less than $\tau_{spin}$, i.e. the singlet no longer contributes to the voltage. For $t > \tau_{spin}$ the spins have relaxed to their ground (singlet) state, and the voltage vanishes. One therefore expects steps in the voltage versus time. If $\tau_{spin} < \tau_{sw}$, a broad transition region of the voltage from $(V_0 + 3V_1)/4$ to 0 will occur [19].

To our knowledge, there are no experimental reports on quantum dots coupled to superconductors. However, hybrid systems consisting of superconductors (e.g., Al or Nb) and 2DES (InAs and GaAs) have been investigated by a number of groups [20].

In conclusion, we have investigated a double quantum dot each dot of which is coupled to two superconductor leads. The superconductors induce an effective exchange coupling between spins on the double dot whereas the Josephson current from one lead to the other depends on the spin state. This leads to the possibility to probe the spin states of the dot electrons by measuring a Josephson current.

## Acknowledgment

We would like to thank G. Burkard, C. Strunk, and E. Sukhorukov for discussions and the Swiss NSF for support.

## References

[1] See, e.g., *Mesoscopic Electron Transport*, eds. L. L. Sohn, L. P. Kouwenhoven, G. Schön (Kluwer, Dordrecht, 1997).

[2] L. I. Glazman, K. A. Matveev, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 570 (1989) [JETP Lett. **49**, 659 (1989)]; B. I. Spivak, S. A. Kivelson, Phys. Rev. B **43**, 3740 (1991).

[3] D. C. Ralph, C. T. Black, M. Tinkham, Phys. Rev. Lett. **74**, 3241 (1995).

[4] C. B. Whan, T. P. Orlando, Phys. Rev. B **54**, R5255 (1996); A. Levy Yeyati, J. C. Cuevas, A. Lopez-Davalos, A. Martin-Rodero, Phys. Rev. B **55**, R6137 (1997).

[5] S. Ishizaka, J. Sone, T. Ando, Phys. Rev. B **52**, 8358 (1995); A. V. Rozhkov, D. P. Arovas, Phys. Rev. Lett. **82**, 2788 (1999); A. A. Clerk and V. Ambegaokar, Phys. Rev. B **61**, 9109 (2000).

[6] K. A. Matveev *et al.*, Phys. Rev. Lett. **70**, 2940 (1993).

[7] R. Bauernschmitt, J. Siewert, A. Odintsov, Yu. V. Nazarov, Phys. Rev. B **49**, 4076 (1994).

[8] R. Fazio, F. W. J. Hekking, A. A. Odintsov, Phys. Rev. B **53**, 6653 (1996).

[9] J. Siewert, G. Schön, Phys. Rev. B **54**, 7424 (1996).

[10] D. Loss, D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).

[11] D. Loss, E. V. Sukhorukov, Phys. Rev. Lett. **84**, 1035 (2000).

[12] Mahn-Soo Choi, C. Bruder, Daniel Loss, cond-mat/0001011.

[13] One could also consider atomic impurities embedded between the grains of a granular superconductor.

[14] If, instead we had assumed leads consisting of unconventional superconductors with triplet pairing, we would find a *ferromagnetic* exchange coupling favoring a triplet ground state of spin $a$ and $b$ on the DD. Thus, by probing the spin ground state of the dots (e.g. via its magnetic moment) we would have a means to distinguish singlet from triplet pairing. The magnetization could be made sufficiently large by extending the scheme from two to $N$ dots coupled to the superconductor.

[15] See, e.g., M. Tinkham, *Introduction to Superconductivity*, 2nd ed. (McGraw-Hill, New York, 1996).

[16] Without restriction we can assume $\alpha > 1$, since $J'$ could be adjusted accordingly by replacing the $J'$-junction by another dc SQUID and flux through it.

[17] V. Ambegaokar, B. I. Halperin, Phys. Rev. Lett. **22**, 1364 (1969).

[18] J. M. Kikkawa, D. D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998).

[19] Another experimental setup would be to use an rf-SQUID geometry, i.e., to embed the S-DD-S structure into a superconducting ring [15].

However, to operate such a device, ac fields are necessary, and the sensitivity is not as good as for the dc-SQUID geometry.

[20] B. J. van Wees, H. Takayanagi, in [1]; H. Takayanagi, E. Toyoda, T. Akazaki, Superlattices and Microstructures **25**, 993 (1999); A. Chrestin *et al.*, ibid., 711 (1999).