Ballistic spin currents in mesoscopic metal/In(Ga)As/metal junctions

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(Received 14 July 2004; revised manuscript received 7 February 2005; published 18 April 2005)

We investigate ballistic spin transport through a two-dimensional mesoscopic metal/semiconductor/metal double junction in the presence of spin-orbit interactions. It is shown that finite transverse and/or longitudinal spin currents can flow in the presence of the Rashba and Dresselhaus terms.

DOI: 10.1103/PhysRevB.71.153306 PACS number(s): 73.63.−b, 72.10.−d, 71.70.Ej

I. INTRODUCTION

Since the advent of "spintronics" to utilize an electron's spin, rather than its charge, for information processing and storage,[1] there has been growing interest in generating spin currents.[2–14] Though injecting spin-polarized carriers electrically still remains a challenge,[2] there have been proposed various all-semiconductor devices based on ferromagnetic semiconductors[3] or spin-orbit (SO) interactions.[4] In particular, the latter enables us to manipulate the spin by controlling the orbital motion of electric carriers, say, by applying an electric field. Moreover, it has been suggested that the SO coupling gives rise to dissipationless spin currents perpendicular to the external electric field, which is known as the intrinsic-spin Hall effect.[4–7]

Theoretically, the existence of the spin Hall current has been highly controversial. Sinova *et al.*[5] predicted a finite-spin Hall current and universal-spin Hall conductivity in a clean, infinite two-dimensional electron system (2DES).[6] Different groups have provided mutually contradicting arguments on the effect of impurity scattering in an infinite 2DES.[7,8] Recently, it was claimed that vanishing bulk-spin conductivity is an intrinsic property of clean 2DES.[9] The spin Hall effect in (semi)finite-size systems was also studied: It was argued that a finite-spin Hall current flows in the vicinity of the contacts, while the spin current vanishes in an infinite system.[10] Numerical studies have also reported finite-spin conductances in four-terminal samples.[11] Another important issue has been raised regarding how the predicted nonequilibrium-spin current is related to (or is distinguished from) the background-spin current, which exists even in equilibrium.[12,13]

A recent experiment[14] reports a finite-spin accumulation, possibly due to the spin current, in the very clean samples. It implies that the spin current, while it may vanish in the bulk limit, can be nonzero in finite or semifinite systems.

In this paper we study ballistic spin transport through a *clean, mesoscopic* double-junction system consisting of a semiconductor stripe sandwiched by two normal-metal leads (see Fig. 1). We use coherent scattering theory and show that in the presence of SO couplings, both longitudinal and transverse spin currents can flow in a semiconductor.

II. MODEL AND SCATTERING THEORY

We consider a two-dimensional electron system of a semiconductor (S) between two normal (N)-metal leads. We choose such a coordinate system that the x axis (y axis) is perpendicular (parallel) to the N/S interfaces, and the z axis is perpendicular to the two-dimensional (2D) plane (Fig. 1). The length (width) of the semiconductor is $L(W)$; we will consider the limit $W\to\infty$. Within the effective-mass approximation,[15] the Hamiltonian reads as

$$\mathcal{H} = -\frac{\hbar^2}{2}\boldsymbol{\nabla}\cdot\frac{1}{m(x)}\boldsymbol{\nabla} + V(x,y) + \mathcal{H}_R(x) + \mathcal{H}_D(x). \quad (1)$$

The position-dependent effective mass $m(x)$ has values of m_e and $m_e^*\equiv\epsilon_m m_e$ in the normal metals and in the semiconductor $(-L/2<x<L/2)$, respectively. The confinement potential has a potential barrier of height V_0 inside the semiconductor,

$$V(x,y) = V_0[\Theta(x+L/2) - \Theta(x-L/2)] + V(y), \quad (2)$$

where $\Theta(x)$ is the Heaviside step function and $V(y)$ accounts for the finite width W. The potential barrier height V_0 is lower than the Fermi energy E_F in the normal metals, so that $E_F^*\equiv E_F - V_0>0$. The Rashba[16] and Dresselhaus[17] SO coupling terms are given by

$$\mathcal{H}_R = \frac{\alpha}{\hbar}(\sigma_x p_y - \sigma_y p_x) \quad \text{and} \quad \mathcal{H}_D = \frac{\beta}{\hbar}(\sigma_y p_y - \sigma_x p_x), \quad (3)$$

respectively, inside the semiconductors, while they vanish in the normal-metal sides. In Eq. (3), $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ are the Pauli matrices.

The Rashba term $\mathcal{H}_R$ arises when the confining potential of the quantum well lacks inversion symmetry, while the Dresselhaus term $\mathcal{H}_D$ is due to the bulk-inversion

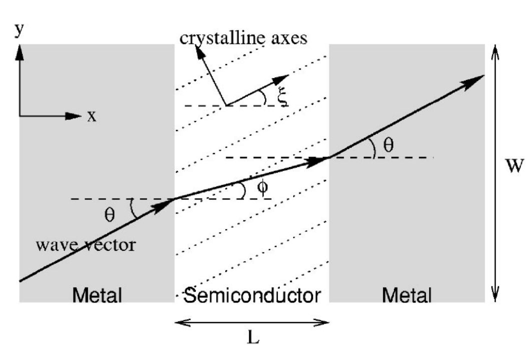

FIG. 1. A schematic of the system.

asymmetry. In some semiconductor heterostructures (e.g., InAs quantum wells) $\mathcal{H}_R$ dominates,[18] and in others (e.g., GaAs) $\mathcal{H}_D$ is comparable to $\mathcal{H}_R$.[19] The coupling constants may range around $\alpha \sim 0.1$ eV Å and $\beta \sim 0.09$ eV Å, respectively.

Inside the semiconductor, the electrons feel a fictitious, in-plane magnetic field in the direction $\hat{\mathbf{n}}_{\mathbf{k}} = \hat{\mathbf{x}} \cos \varphi_{\mathbf{k}} + \hat{\mathbf{y}} \sin \varphi_{\mathbf{k}}$, where $\varphi_{\mathbf{k}} = \arg[(\beta k_x - \alpha k_y) + i(\alpha k_x - \beta k_y)]$. Accordingly, the eigenstates with spins parallel ($\mu = +$) and antiparallel ($\mu = -$) to $\hat{\mathbf{n}}_{\mathbf{k}}$ for a given wave vector $\mathbf{k} = k(\hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi)$ are written in the spinor form,

$$\Psi^{\mu}_{\mathbf{k}}(\mathbf{r}) = \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{2}} \begin{bmatrix} \mu e^{-i\varphi_{\mathbf{k}}/2} \\ e^{+i\varphi_{\mathbf{k}}/2} \end{bmatrix}. \tag{4}$$

The eigenenergies are $E_\mu(\mathbf{k}) = (\hbar^2/2m_e^*)[k^2 - 2\mu k_{so}(\phi)k]$, where $k_{so}(\phi) \equiv (m_e^*/\hbar^2)\sqrt{\alpha^2 + \beta^2 - 2\alpha\beta \sin 2\phi}$. From the continuity equation for the charge density, one can get the expression for the charge-current density associated with a given wave function $\Psi(\mathbf{r})$,[20]

$$\mathbf{j}_c = e \, \mathrm{Re}[\Psi^\dagger(\mathbf{r})\mathbf{v}\Psi(\mathbf{r})], \tag{5}$$

where $\mathbf{v}$ is the velocity operator defined by

$$\mathbf{v} = \frac{\mathbf{p}}{m_e^*} - \frac{\alpha}{\hbar}(\sigma_y \hat{\mathbf{x}} - \sigma_x \hat{\mathbf{y}}) - \frac{\beta}{\hbar}(\sigma_x \hat{\mathbf{x}} - \sigma_y \hat{\mathbf{y}}). \tag{6}$$

In the same manner, we define the spin-current density,[13]

$$\mathbf{j}_s(\hat{\mathbf{n}}) = \frac{\hbar}{2}\Psi^\dagger(\mathbf{r})\frac{\mathbf{v}(\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}) + (\hat{\mathbf{n}}\cdot\boldsymbol{\sigma})\mathbf{v}}{2}\Psi(\mathbf{r}), \tag{7}$$

according to the continuity equation,

$$\partial_t Q_s + \boldsymbol{\nabla} \cdot \mathbf{j}_s = S_s, \tag{8}$$

for the spin density (with respect to the spin direction $\hat{\mathbf{n}}$),

$$Q_s(\hat{\mathbf{n}}) \equiv \frac{\hbar}{2}[\Psi^\dagger(\mathbf{r})(\hat{\mathbf{n}}\cdot\boldsymbol{\sigma})\Psi(\mathbf{r})], \tag{9}$$

and the spin source,

$$S_s(\hat{\mathbf{n}}) = \frac{\hbar}{2}\,\mathrm{Re}\left[\Psi^\dagger(\mathbf{r})\frac{i}{\hbar}[\mathcal{H}, \hat{\mathbf{n}}\cdot\boldsymbol{\sigma}]\Psi(\mathbf{r})\right]. \tag{10}$$

The spin-source term arises in Eq. (8) because the spin-orbit couplings break the spin conservation.

Before going further, it will be useful to understand the origin of the spin current in physical terms. As illustrated in Fig. 2, for $\alpha, \beta \neq 0$ the Fermi contours,

$$k_F^{\mu}(\phi) = \mu k_{so}(\phi) + \sqrt{k_{so}^2(\phi) + k_F^{*2}}, \tag{11}$$

with $k_F^* \equiv \sqrt{2m^* E_F^*}/\hbar$, are no longer isotropic,[26] and the group velocities $\mathbf{v}^{\mu}(\mathbf{k}) = \Psi_{\mathbf{k}}^{\mu\dagger}\mathbf{v}\Psi_{\mathbf{k}}^{\mu}$ of the eigenstates in Eq. (4) are not parallel to the wave vector $\mathbf{k}$.[20–22] Nevertheless, Eq. (11) reveals an important symmetry property of the group velocities, $|\mathbf{v}^{+}(\mathbf{k}_F^{+})| = |\mathbf{v}^{-}(\mathbf{k}_F^{-})|$. It means that the two eigenstates with opposite spin orientations make the same contributions to the charge transport along the $\hat{\mathbf{k}}$ direction (and opposite contributions along the perpendicular direction). The spin transport with $\hat{\mathbf{n}} = \hat{\mathbf{n}}_{\hat{\mathbf{k}}}$ is to the contrary: two eigenstates contribute the opposite (same) spin currents along (perpendicular to) $\hat{\mathbf{k}}$. This implies that the net-spin current is perpendicular to the charge current. Particularly interesting are the cases of $\alpha = \pm\beta$, where all the spin orientations $\pm\hat{\mathbf{n}}_{\mathbf{k}}$ for the different wave vectors are parallel or antiparallel to each other ($\varphi_{\mathbf{k}} = \pi/4$) [see Fig. 2(b)]. It results from the conservation of $(\sigma_x \pm \sigma_y)/\sqrt{2}$, and the spin state becomes independent of the wave vector.[22,23]

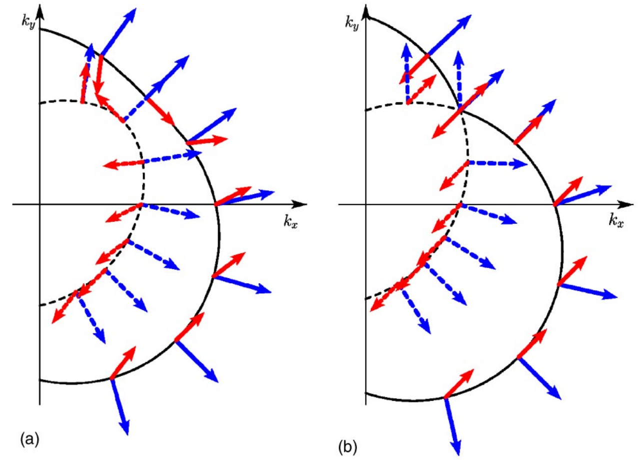

FIG. 2. (Color online) Relative configurations of group velocities and spin quantization axes on Fermi contours for (a) $2\alpha = \beta = 0.25\hbar v_F^*$ and (b) $\alpha = \beta = 0.25\hbar v_F^*$ with $v_F^* = \hbar k_F^*/m^*$. Legend: thin solid-dashed curve: Fermi contour for $\mu = \pm$; thick solid-dashed tall (blue) arrows: group velocities for $\mu = \pm$; thick solid-dashed short (red) arrows: spin quantization axes for $\mu = \pm$. The configuration is symmetric under inversion.

Now we study charge and spin transport in N/S/N junctions. A coherent scattering theory at the N/S interfaces was already developed in the previous studies,[20,21] considering the Rashba SO effect and appropriate boundary conditions. It is straightforward to extend the scattering theory to incorporate the Dresselhaus effect. We use the transfer-matrix formalism to calculate the conductance through the semiconductor (see Refs. 20 and 24).

We consider the electrons incident from the left lead. The wave vector of the incident electron is at angle θ with the normal to the interface (see Fig. 1). Contrary to the Rashba effect, the Dresselhaus effect is not invariant under rotations, causing anisotropic transport.[22] Hence the relative orientation, ξ, of the crystal symmetry axes and the interface (Fig. 1) strongly affects the spin current. Below we will calculate the charge conductance $G_\nu^{(c)}(\theta) \equiv I_\nu^{(c)}(\theta)/V$ ($\nu = x, y$) in the ν direction for a definite incident angle θ as well as the angle-averaged quantity $G_\nu^{(c)} = \int_{-\pi/2}^{\pi/2} d\theta G_\nu^{(c)}(\theta)$, where V is the voltage difference between two contacts and $I_\nu^{(c)}$ is the corresponding charge-current density in Eq. (5). Also calculated are the analogously defined spin conductances $G_\nu^{(s,\hat{\mathbf{n}})}(\theta)$ and $G_\nu^{(s,\hat{\mathbf{n}})}$, polarized in the direction $\hat{\mathbf{n}}$.[27]

The typical values for the parameters used below are $E_F = 4.2$ eV, $\epsilon_m = 0.063$, $\beta = 0.1$ eV Å, $L = 200$ nm, and $W = 1$ μm. α ranges from -2β to $+2\beta$, and E_F^* ranges from 0 to 20 meV. We assume sufficiently low temperatures ($k_B T \ll E_F^*$).

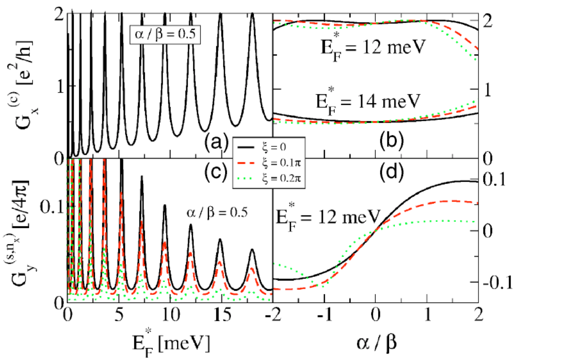

FIG. 3. (Color online) The charge conductance $G_x^{(c)}(\theta=0)$ [(a) and (b)] and the spin conductance $G_y^{(s,\hat{\mathbf{n}}_{\hat{\mathbf{x}}})}(\theta=0)$ [(c) and (d)] for the normal incidence as functions of E_F^* [(a) and (c)] and α/β [(b) and (d)]. In (a) three curves overlap almost completely.

III. NORMAL INCIDENCE

Owing to the symmetry $|\mathbf{v}^+(\mathbf{k}_F^+)|=|\mathbf{v}^-(\mathbf{k}_F^-)|$ [see the discussion below Eq. (10)] for normal incidence ($\theta=0$), the charge current is purely longitudinal; i.e., $G_y^{(c)}(\theta=0)=0$. For a single transverse mode, we obtain the longitudinal charge conductance,

$$G_x^{(c)}(\theta=0)=\frac{e^2}{h}\frac{32\kappa^2}{|(1+\kappa)^2-(1-\kappa)^2e^{2i\Delta kL}|^2}, \tag{12}$$

where $\Delta k\equiv\sqrt{k_{so}^2(-\xi)+k_F^{*2}}$, $\kappa\equiv\Delta k/\epsilon_m k_F$, and $k_F\equiv\sqrt{2m_eE_F}/\hbar$. On the other hand, the spin current has only a transverse component and is polarized entirely in the xy plane; i.e., $G_x^{(s,\hat{\mathbf{n}})}(\theta=0)=0$ for any $\hat{\mathbf{n}}$ and $G_y^{(s,\hat{\mathbf{z}})}(\theta=0)=0$. The $\hat{\mathbf{n}}_{\hat{\mathbf{x}}}$-polarized spin conductance $G_y^{(s,\hat{\mathbf{n}}_{\hat{\mathbf{x}}})}(\theta=0)$ is given by

$$G_y^{(s,\hat{\mathbf{n}}_{\hat{\mathbf{x}}})}(\theta=0)=\frac{e}{4\pi}\frac{L}{W}\frac{32(m_e^{*2}/\hbar^4)\alpha\beta\cos 2\xi}{\epsilon_m k_F k_{so}(-\xi)} \times\frac{(1+\kappa^2)-(1-\kappa^2)\dfrac{\sin 2\Delta kL}{2\Delta kL}}{|(1+\kappa)^2-(1-\kappa)^2e^{2i\Delta kL}|^2}. \tag{13}$$

$G_x^{(c)}(\theta=0)$ and $G_y^{(s,\hat{\mathbf{n}}_{\hat{\mathbf{x}}})}(\theta=0)$ are plotted in Fig. 3 as functions of E_F^* and α/β for different crystal orientations ξ. The peaks in $G_x^{(c)}(\theta=0)$ and $G_y^{(s,\hat{\mathbf{n}}_{\hat{\mathbf{x}}})}(\theta=0)$ as a function of E_F^* come from the Fabry-Perot interference, which gives rise to resonances for $\Delta kL=n\pi$ ($n=0,1,2,\ldots$). Unlike the (longitudinal) charge current, the spin current is very sensitive to α, β, and ξ, as seen from the factor $\alpha\beta\cos 2\xi$ in Eq. (13). Note that $G_y^{(s,\hat{\mathbf{n}}_{\hat{\mathbf{x}}})}$ has no contribution from background-spin currents; within our scattering formalism, we count only the contributions from electrons between the two Fermi levels of the two metal leads.[13]

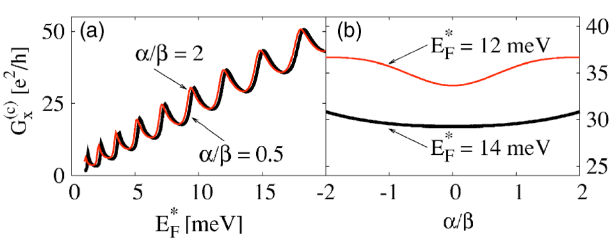

FIG. 4. (Color online) Angle-averaged charge conductance $G_x^{(c)}$ as a function of (a) E_F^* and (b) α/β with $\xi=0$.

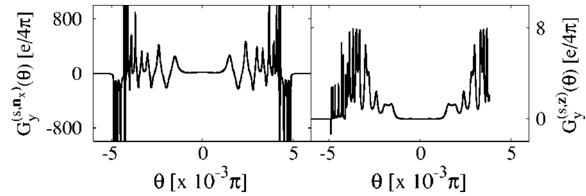

FIG. 5. Angle dependences of the spin conductances $G_y^{(s,\hat{\mathbf{n}}_{\hat{\mathbf{x}}})}(\theta)$ and $G_y^{(s,\hat{\mathbf{z}})}(\theta)$ for $E_F^*=14$ meV, $\alpha/\beta=0.5$, and $\xi=0$.

IV. ANGLE-AVERAGED CONDUCTANCES

For true one-dimensional (1D) leads ($k_FW\ll 1$), where only a single transverse mode is allowed, one has only to consider normal incidence ($\theta=0$) or at a certain fixed θ.[25] In the opposite limit ($k_FW\to\infty$), where many transverse modes contribute to the transport, we should add up all the contributions from θ in the range $(-\pi/2,\pi/2)$. It is tedious to find the scattering states for nonzero incidence angle θ and more convenient to work numerically.

Apparently, the main contribution to the longitudinal charge current comes from the normal incidence. Consequently, as shown in Fig. 4, the θ-averaged longitudinal conductance $G_x^{(c)}$ is rather similar to the normal incidence case $G_x^{(c)}(\theta=0)$.

This is not the case for the spin transport. Figure 5 shows the θ dependence of the spin conductances polarized in the $\hat{\mathbf{n}}_{\hat{\mathbf{x}}}$ and $\hat{\mathbf{z}}$, respectively. Again, the peaks correspond to the Fabry-Perot-type resonances. When summing up, the contributions to the $\hat{\mathbf{n}}_{\hat{\mathbf{x}}}$-polarized spin current from different angles are mostly canceled with each other, and hence the angle-averaged spin conductance $G_y^{(s,\hat{\mathbf{n}}_{\hat{\mathbf{x}}})}$ becomes small compared with the charge conductance $G_x^{(c)}$. On the other hand, the $\hat{\mathbf{z}}$-polarized spin current is not subject to such cancellations, and remains relatively large (still smaller than the longitudinal charge current) especially for $\xi=0$ [see Fig. 6(a)]. This is reminiscent of the intrinsic-spin Hall effect.[6] However, in our

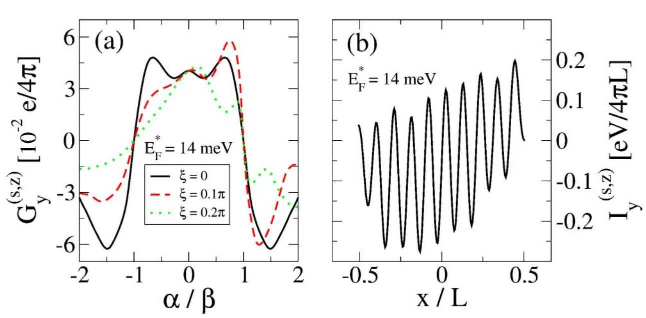

FIG. 6. (Color online) (a) Transverse spin conductance $G_y^{(s,\hat{\mathbf{z}})}$ as a function of α/β. (b) Spatial dependence of the spin current density $I_y^{(s,\hat{\mathbf{z}})}$ with $\alpha=0.1$ eV Å and $\beta=0$.

case $G_y^{(s,\hat{\mathbf{z}})}$ depends on α, β, ξ, the potential barrier, and the channel length, showing no universal characteristics; for instance, $G_y^{(s,\hat{\mathbf{z}})}$ increases almost monotonically with E_F^*. Our result is different from that of Mishchenko *et al.*[10] as well: The spin current is finite through the semiconductor region and oscillates with position, even alternating its sign [Fig. 6(b)]. This feature is due to contributions from the coherent standing waves. In the presence of impurity scattering the coherent oscillation inside the sample should die away and the spin current will be manifested only near the contacts.[10] Finally we remark that in the presence of both SO couplings the angle-averaged longitudinal spin conductance $G_x^{(s,\hat{\mathbf{n}})}$ is finite, even if much smaller than $G_x^{(c)}$. It reflects that spin is not conserved for an oblique incidence, because the spin-quantization directions are not consistent with the boundary conditions.

V. CONCLUSION

Ballistic spin currents with different spin polarizations through mesoscopic metal/2DES/metal junctions have been investigated in the presence of spin-orbit interactions. Using the coherent scattering theory we showed that longitudinal and/or transverse spin currents can flow through a clean 2DES. The spin coherence can induce spin current and polarization, with properties that are different from the ones in the diffusive limit.

ACKNOWLEDGMENTS

M.L. thanks W. Belzig, C. Bruder, and J. Schliemann for helpful discussions. This work was supported by the SK-Fund, the SKORE-A, and the eSSC at Postech. M.-S.C. acknowledges the support from KIAS.

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[26] The Rashba and Dresselhaus terms in Eq. (3) are equivalent up to a unitary transformation. However, in the presence of both effects, the anisotropy due to the Dresselhaus term manifests itself.

[27] In 2D, the conductance and conductivity have the same dimension. In particular, the spin conductance $G_y^{(s,\mathbf{z})}$ is identical to the spin Hall conductivity σ_{sH} in, e.g., Refs. 5 and 10.