Quantum-based mechanical force realization in piconewton range

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Mechanical force realization based on flux quantization in the piconewton range is proposed. By controlling the number of flux quantum in a superconducting annulus, a force can be created as integer multiples of a constant step. For a 50 nm thick Nb annulus with inner and outer radii of 5 and 10 μm, respectively, and field gradient of 10 T/m, the force step is estimated to be 184 fN. The stability against thermal fluctuations is also addressed. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2591398]

Due to remarkable improvement in its sensitivity, force measurement has become a useful and essential probe for leading-edge nano-/bioresearches.[1] Its application goes well beyond morphological nanoimaging, and now more areas are requiring higher accuracy in ultrasmall force measurement such as DNA unzipping,[2] Casimir force testification,[3] and so on. During DNA unzipping or protein stretching, for example, accurate measurement of force (usually in the piconewton range) and length can provide its free energy landscape and help understand its biological mechanism. In calibration of such a sensitive force sensor, with a spring constant of $\sim 10^{-3}$ N/m or less, a thermal noise spectrum method[4] among many others (for reviews, see Ref. 5) has been widely used due to its load-free and nondestructive scheme. However, considering its typical uncertainty in the 10%–20% range,[5] a new method is needed which can provide better precision and verify the thermal method, and this necessity is present for general ultrasmall force measurements. On the other hand, from a standardization motive, international system traceable force realization has not been established even at the subnewton level, which can be the primary standard for force-to-force comparison, while a conventional method using deadweight becomes no longer valid below 10 μN. At the micronewton level, an electrostatic force realization has been proposed recently by Pratt *et al.*[1] In this letter, we present a concept of controllable piconewton force realization utilizing a macroscopic quantum phenomenon: magnetic flux quantization in a superconducting annulus. The realized force is static, stepwise by a quantum-mechanically determined fixed magnitude, and straightforwardly extensible to the subpiconewton level, with accuracy of a few percent.

Figure 1 shows a schematic of quantum-based force realization. A tens-micron-sized superconducting annulus (with inner and outer radii and thickness a, b, and d, respectively) is mounted on a microcantilever with an ultrasmall spring constant ($k=10^{-4}-10^{-5}$ N/m).[6] An external magnetic field $\mathbf{B}_{\text{ext}}=B_{\text{ext}}\hat{\mathbf{z}}$ is applied perpendicular to the annulus. Below a superconducting transition temperature T_c, magnetic fluxoid Φ through the annulus is quantized in units of the flux quantum $\Phi_0 \equiv h/2e \approx 2.07\times 10^{-15}$ Wb, and the resultant magnetic moment m has a stepwise component depending on the number n of trapped flux quanta. The interaction between the quantized magnetic flux and the calibrated magnetic field gradient, dB/dz, exerts a stepwise force on the cantilever. The step size is determined by fundamental constants (such as electron charge) and the geometry of the annulus. Through a predesigned procedure of magnetic field and temperature, the number of trapped flux quanta can be controlled, and the discrete force steps are measured by monitoring the displacement of the cantilever using an optical interferometer.

The total magnetic moment m has two contributions, $m(n,B_{\text{ext}})=m_1(n)+m_2(B_{\text{ext}})$: $m_1(n)$ from the quantized flux and $m_2(B_{\text{ext}})$ induced by the external field.[7] Experimentally, the magnetic field at the annulus from the z-gradient magnet or other background sources is not simple to null (relatively small values $\sim$0.13 Oe of B_{ext} already allow one flux quantum) and would generate a nonzero force offset even in the zero-flux-quantum state. Therefore, we need a set of specially designed procedures to extract selectively the contribution from the quantized flux. To this end, we first note that m_1 depends only on n while m_2 is linearly proportional to B_{ext}, $m_2=\chi B_{\text{ext}}$. As demonstrated in Fig. 2, the difference $m(n,B_{\text{ext}})-m(n',B_{\text{ext}})$ between different flux states (blue thin lines) is constant at any fixed B_{ext}.

Based on this observation, we suggest a force-realization and cantilever-calibration procedure as follows (see the thick dotted line in Fig. 2 for an illustration): (0) Measure a zero-reference position of a cantilever in $n=n_i$ state at $B_{\text{ext}}=B_i$ and $T \ll T_c$. (1) Increase the external field up to a target value B_f, for which $n=n_f=n_i+\Delta n$ is a global minimum of Gibbs free energy, keeping $T\ll T_c$. Note that the system will still stay at the $n=n_i$ state due to the free energy barriers. (2) Increase T above T_c and decrease it back to $T \ll T_c$, which would send

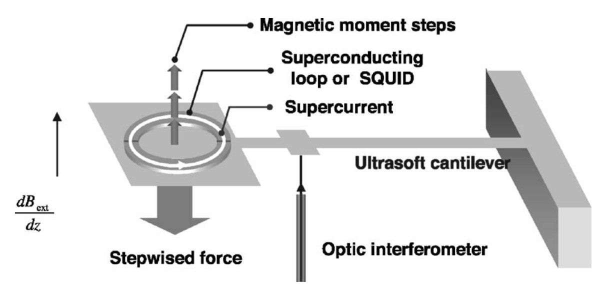

FIG. 1. Schematic of force realization based on flux quantization in a superconducting annulus.

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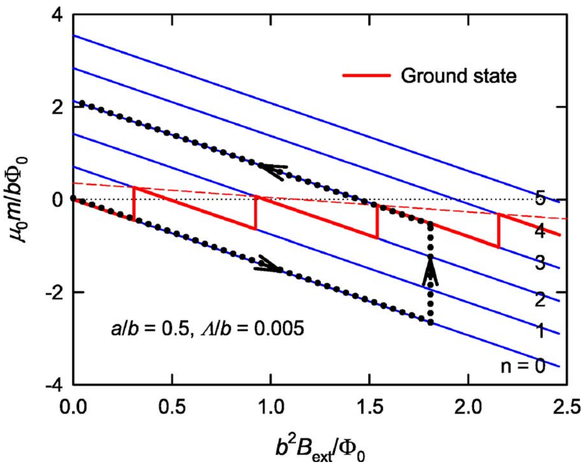

FIG. 2. (Color online) Total magnetic moment as a function of B_{ext} for n trapped fluxoids (solid blue lines); $a/b=0.5$ and $\Lambda/b=0.005$. The thick (red) line is for the ground-state configuration. The dashed (red) line is the guide for the eyes connecting the peaks of the ground-state magnetic moment. The thick dotted (black) line indicates the experimental procedure.

the system to the global minimum state, $n=n_f$. (3) Decrease the uniform external field back to B_i. (4) Measure the displacement of the cantilever. The net change in magnetic moment, $\Delta n \times m_Q$, is only related to flux quanta and so is the change in a force. From Δn, dB/dz, and the annulus dimensions,[8] the change in force can be evaluated. This force can be applied to other micromechanical systems or used in determining the spring constant of the cantilever itself in combination with its displacement. We stress that the starting and target values, B_i and B_f, of B_{ext} do not need to be zero, which is a great benefit for the case of nonzero background field.

To estimate values of m, let us consider a Nb annulus with $a=5$ μm, $b=10$ μm, and $d=50$ nm. The Nb material was chosen because its T_c of 9.25 K (Ref. 9) is readily accessible in cryogenics and its thin-film fabrication is well established to yield good quality thin-film circuits such as a superconducting quantum interference device (SQUID). Its London penetration depth $\lambda_L \approx 50$ nm and coherence length $\xi \approx 39$ nm.[9] These parameters fall on the case of $a/b=0.5$ and $\Lambda/b=0.005$ ($\Lambda \equiv \lambda_L^2/d$ is the effective penetration depth) in Fig. 2. Then, a magnetic moment step m_Q, by adding a single flux quantum, is numerically estimated as $1.116 m_b$, where $m_b \equiv \Phi_0 b/\mu_0 \sim 1.65 \times 10^{-14}$ A m^2. With $dB/dz \simeq 10$ T/m in typical commercial superconducting or permanent gradient magnets, the corresponding force step, $F_Q = m_Q dB/dz$, is estimated to be 184 fN, which causes $\sim$2 nm static displacement of a cantilever with $k=10^{-4}$ N/m. In practice, a maximum possible force is limited by the critical current of the superconducting material. For the Nb annulus with the above geometry, the force limit is roughly estimated to be $\sim$40 pN from the approximate equation $F_{max}=I_c \pi[(a+b)/2]^2(dB/dz)$ and the critical current density of Nb, $j_c=9\times 10^{10}$ A/m^2 at $T=6$ K.[10]

For precise force realization, one has to be able to accurately calibrate the magnetic field gradient dB/dz.[11] Experimentally, it is tricky to extract the precise value of dB/dz at the local position of the annulus, unless the field gradient is strictly uniform and well precalibrated in microscale.

Here we argue that we can calibrate it *in situ* by means of the characteristic properties of the magnetic moment of

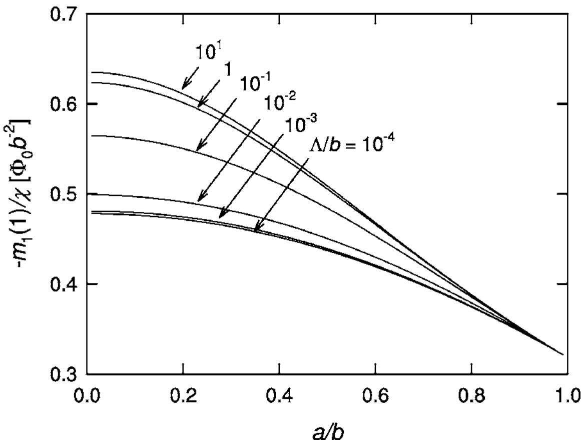

FIG. 3. $m_1(1)/\chi$ as a function of a/b for different values of Λ/b.

the superconducting annulus. As mentioned above, the magnetic moment can be written into the form $m=m_1(n)+\chi B_{ext}$. By expanding B_{ext} around B_0 at the equilibrium position, force $F(z)$ at position z is given by

$$F(z) = [m_1(1)n + \chi B_0]\frac{dB}{dz} + \chi\left(\frac{dB}{dz}\right)^2 z. \quad (1)$$

Then it follows that

$$\frac{\Delta\omega_0}{\omega_0} = \frac{1}{2}\frac{\Delta k}{k} = \frac{\chi}{2k}\left(\frac{dB}{dz}\right)^2, \quad (2)$$

where ω_0 (k) is the natural vibration frequency (Hook's coefficient) of the cantilever and $\Delta\omega_0$ (Δk) is the shift in ω_0 (k) due to the field gradient. Let $\Delta z = z_{eq}(n) - z_{eq}(n=0)$, i.e., the shift in the equilibrium position z_{eq} for nonzero flux n. Since the force $F(\Delta z)$ at $z=\Delta z$ in Eq. (1) should be at equilibrium with $(k+\Delta k)z_{eq}$, i.e., $m_1(1)n dB/dz = k\Delta z$, one arrives at the relation

$$\frac{dB}{dz} = \frac{2n}{\Delta z}\frac{m_1(1)}{\chi}\frac{\Delta\omega_0}{\omega_0}. \quad (3)$$

We now note that in Eq. (3), the field gradient dB/dz has been expressed only in known quantities that can be determined to high precision. The relative frequency shift $\Delta\omega_0/\omega_0$ is typically probed with high precision. As shown in Fig. 3, the ratio $m_1(1)/\chi$ in Eq. (3) does not depend strongly on the geometry and can be easily determined accurately.[8] We therefore conclude that Eq. (3) allows us to calibrate the field gradient *in situ* very precisely. For $k=10^{-4}$ N/m and other parameters adopted above for a force step estimation, $\Delta\omega_0/\omega_0$ is estimated to be 2×10^{-3} from Eq. (2) and becomes larger for lower k. The relative shift is much larger than the sharpness of the resonance peak or $1/Q$, with a typical quality factor $Q=10^4-10^5$ for single-crystalline Si cantilevers,[6] and can be measured to high precision.

The static displacement Δz of the cantilever can be precisely measured using, for example, an optic interferometer known to have rms noise less than 0.01 nm in the 1 kHz bandwidth at low frequencies.[12] The thermal vibration amplitude, Δz_{th}, itself may not be small in comparison with a static displacement Δz_Q by a single flux quantum; $\Delta z_{th}/\Delta z_Q = \sqrt{k k_B T}/F_Q \sim 0.41$ for $k \sim 10^{-4}$ N/m and $T=4.2$ K. However, for high Q factor, the statistical error in *static* displacement Δz due to thermal vibration can be made small by averaging (or using equivalent methods) over a sufficiently long time, $\tau_m \gg 2\pi/\omega_0 \sim$ a few milliseconds; the error in $\tau_m^{-1}\int_0^{\tau_m} dt z_{\rm eq}(n,t)$ decreases as $1/\sqrt{\omega_0 \tau_m}$. In addition, for large multiples of F_Q the relative error due to interferometer and thermal noises gets proportionally smaller. It is also important to suppress the low-frequency laboratory vibration noise well enough, since the scheme requires a relatively soft cantilever compared with typical atomic force microscopy experiments.

The superconducting annulus can be replaced by a SQUID with leads. Then, instead of temperature-field cycling, a bias current through SQUID can be used to permit additional flux quanta, with a merit that the whole procedure is done at a fixed temperature. Moreover, each entrance of the quantum can be monitored in real time from the current-voltage characteristics.

Finally, we briefly remark on the effects of thermal fluctuations. In principle, the fluxoids trapped in the annulus may undergo relaxation processes and their number changes. At our working temperature ($\sim$4 K), the relaxation is dominated by the thermal activation and governed by the law $\gamma_0 \exp(-\Delta U/k_B T)$, where ΔU is the energy barrier against the relevant process. For a wide annulus with $b-a \gg \Lambda$, the relaxation comes from crossing of the vortices across the annulus of the superconducting thin film. Naively, the energy barrier can be estimated by the condensation energy cost inside the vortex core, $\Delta U \sim H_c^2 \xi^2 d/8\pi$, where H_c is the thermodynamic critical field. For Nb, $H_c \sim 0.1$ T and $\Delta U \sim 10^5$ K.[9] It implies that the effects of the flux relaxation can be safely ignored in our scheme using a wide enough annulus.

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