Decoherence-Driven Quantum Transport

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We propose a new mechanism to generate a dc current of particles at zero bias based on a noble interplay between coherence and decoherence. We show that a dc current arises if the transport process in one direction is maintained coherent while the process in the opposite direction is incoherent. We provide possible implementations of the idea using an atomic Michelson interferometer and a ring interferometer.

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The rates of emission and absorption between two quantum states are equal as governed by the principle of detailed balance. Under special conditions, however, the detailed balance can be broken, and one of the processes can even be completely suppressed. One relevant example is the so-called lasing without inversion (LWI) in quantum optics [1]. LWI is achieved in an ensemble of atoms that have a pair of nearly degenerate ground-state levels, say, a and b (Fig. 1). The atoms are prepared in a coherent superposition of a and b. This coherent superposition can be realized by applying a microwave resonant with the ground-state splitting, $E_b - E_a$. The excitation probability to an upper level c, $T_{ab\to c}$, undergoes two-path interference and, hence, can be modulated by changing the relative phase of the levels a and b in the coherent superposition. On the contrary, the decaying probability from state c to both a and b, $T_{c\to ab}$, is just the summation of the probabilities of two spontaneous decays since it is an *incoherent* process. When the phase of the microwave field is adjusted to make $T_{ab\to c}$ smaller than $T_{c\to ab}$, the lasing operation is possible without population inversion.

A dc current of particles can be achieved by applying an external dc bias, e.g., an electrical potential difference for charges, density difference for masses, temperature difference for heat, and so on. In these examples, the external bias breaks directly the detailed balance between the currents in opposite directions. It is also possible to obtain a dc current by breaking the detailed balance *indirectly*: In rectification, a dc current is generated by an external ac voltage. In "ratchets," the directed motion of particles can be caused from even random fluctuations [2]. Quantum mechanics provides still another way to generate dc current by applying ac fields, e.g., the (*mesoscopic*) photovoltaic effect [3], quantum pumps [4], and a quantum version of classical ratchet [2,5].

In this Letter, we propose a new mechanism to generate dc currents at zero bias by generalizing the concept of LWI to the transport problem. The basic idea is simple: A dc current will arise if the transport process in one direction is coherent while the process in the opposite direction is incoherent. One can easily check the idea by noting that the transmission probability of the coherent transport varies with the relative phases of multiple paths, which do not affect the incoherent one. The important question is then how to realize such *spatially* anisotropic, coherent, or incoherent transport processes. It is clearly distinguished from the original idea of LWI, since the asymmetry in coherence/decoherence of our concern addresses spatial directions rather than the excitation and relaxation of energy.

The scheme of our implementation is quite general, but, for definiteness, here we take two specific examples: one based on a Michelson interferometer and the other on a ring interferometer.

Let us first consider the scheme based upon the Michelson interferometer; see Fig. 2. We have an atomic Michelson interferometer [6] and two reservoirs, 1 and 2, of two-level atoms at the ends of the two input/output channels of the interferometer. The atoms from a reservoir enter the interferometer, experience scattering and/or interference, and are either reflected back to the original reservoir or transmitted to the other. In addition, we have an important component, the microcavity (C) between reservoir 2 and the atomic beam splitter (BS) [6]. The cavity is set resonant to the level splitting Δ of the two-level atoms, so that the atoms entering the cavity in the ground state come out of the cavity in the excited state.

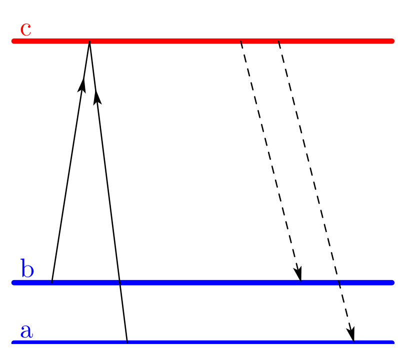

FIG. 1 (color online). The schematic diagram of the energy levels of an atom for LWI operation. a, b, and c represent the energy levels.

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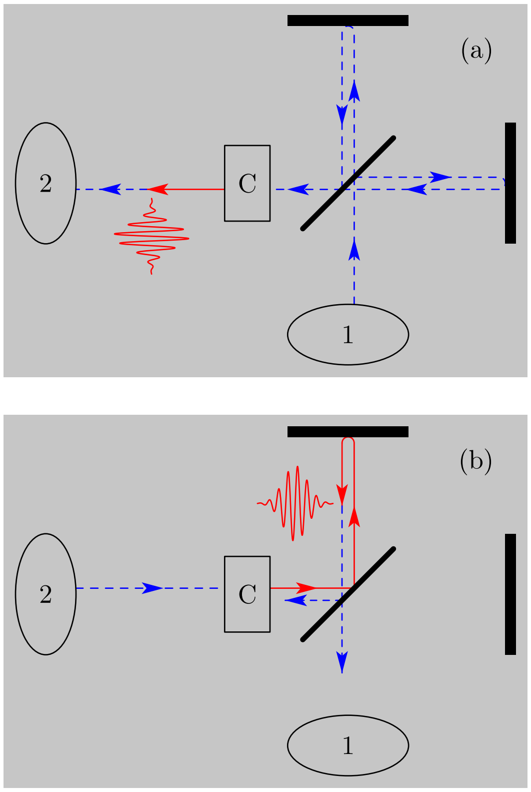

FIG. 2 (color online). The scheme based on the Michelson interferometer. 1 and 2 represent reservoirs. The (blue) dashed and the (red) solid lines represent the trajectories of the ground state and excited atoms, respectively. The horizontal and the vertical thick lines are mirrors, and the titled thick lines in the middle are an atomic beam splitter. The box with "C" is the microcavity. (a) The coherent process: An atom from reservoir 1 undergoes a constructive interference and reaches reservoir 2 with unit probability. The cavity does not affect the transmission of this atom. (b) The incoherent process: An atom from reservoir 2 is excited at the cavity and spontaneously emits a photon within the vertical path. The atom is then transmitted to either reservoir with equal probability 0.5.

Therefore, when entering the interferometer, the atoms from reservoir 2 are in the excited state, while those from reservoir 1 remain in the ground state. This difference in the energy state between the atoms entering the interferometer can cause a significant difference in the coherence of their center-of-mass (c.m.) motions in the interferometer.

To see this, let $L_\tau = v\tau$, where v is the velocity of the atoms and τ is the lifetime of the excited energy level. Provided that $L_\tau < 2L$, with L being the lengths of the arms (i.e., the paths from the atomic BS to the mirrors) of the interferometer (the lengths of the arms are assumed to be equal), an excited atom in the interferometer will relax back to its ground state emitting a photon; see Fig. 2(b). In the ideal case, the photon enables us to locate the atom definitely on one of the two arms of the interferometer. The excited atoms, thus, never experience an interference through the Michelson interferometer. In this sense, the c.m. motion of the atoms from reservoir 2 is *incoherent*. Furthermore, starting from the just located arm (whichever it is), the atom is transmitted to reservoir 1 with probability 0.5 and reflected back to reservoir 2 with 0.5 (we consider a 50:50 BS); see Fig. 2(b).

On the other hand, the atoms from reservoir 1 (ground-state atoms) do not allow such relaxation and will experience coherent interference as long as $L_\phi \gg 2L$, where L_ϕ is the coherence length of the c.m. motion of *ground-state* atoms; see Fig. 2(a). Because of the constructive interference, an atom from reservoir 1 is perfectly transmitted to reservoir 2; see Fig. 2(a). Comparing these two transport processes, one can see that 50% of the incoming atoms contribute to the net dc current. Namely, when the currents from reservoirs 1 and 2 are equal, $I_1 = I_2 = I$, the *net* current from 1 to 2 is given by

$$I_{12} = I_1 - 0.5 I_2 = 0.5 I. \tag{1}$$

The current expressed in Eq. (1) is the maximum current attainable in our scheme assuming idealistic situations. Certain imperfections in reality will diminish the current. First, only a fraction P_{ex} of the atoms from reservoir 2 may be excited by the cavity. Second, an excited atom entering the interferometer may not necessarily relax to the ground state *inside* the interferometer. The probability P_τ for such an event to occur is given by $P_\tau \approx \int_0^{2L/v} dt e^{-t/\tau}$, ignoring the distance between the BS and the cavity. Third, even if the excited atom relaxes inside the interferometer and a photon is emitted, the atom cannot contribute to the net current unless the photon gives enough information about which path of the interferometer the atom takes. For example, if the wavelength λ of the photon is comparable to or larger than the size of the interferometer $L(\lambda \gtrsim L)$, then one cannot get enough which-path information, and the c.m. motion still remains coherent. Fourth, while the decoherence due to photon emission is of a particular type and of our primary concern, in general, the c.m. motion is subject to additional decoherence of the usual type due to the "environment" even when the atom keeps its internal state (either ground or excited). Unlike the former, however, the latter is spatially isotropic and tends to reduce the net current.

The simple inspections in Eq. (1) and the effects of the imperfections mentioned above can be treated more rigorously based on the scattering theory [7]. Important ingredients to be included in the formalism are the time dependence and the wave-packet description. It is because the decoherence process due to photon emission should locate the atom within the interferometer and the subsequent scattering process of the located atom is separated in time from that of the incoming atom. Another important element is an effective description of decoherence. To provide a unified description of both types of decoherence (see above), we adopt the framework of the unitary representation [8]. A 50:50 BS is described by the unitary scattering matrix (for both the wave entering and leaving the interferometer):

$$S = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}, \tag{2}$$

ignoring the weak energy dependence in the range of interest. The state vector of the atom that has come from reservoir 2 is given *inside* the interferometer by [7,8]

$$|\Psi(t)\rangle = \int \frac{dk}{\sqrt{2\pi}} \phi(k) e^{-i\omega(k)t}\Big[(\alpha|g\rangle + \beta e^{-i\Delta t/\hbar}|e\rangle) \otimes |0\rangle \otimes \frac{ie^{+ikx_1}|e_1\rangle + e^{+ikx_2}|e_2\rangle}{\sqrt{2}} + \gamma|g\rangle \otimes \frac{ie^{+ikx_1}|\nu_1\rangle \otimes |e_1\rangle + e^{+ikx_2}|\nu_2\rangle \otimes |e_2\rangle}{\sqrt{2}}\Big], \tag{3}$$

where $\phi(k)$ is the envelope function of the wave packet, $\omega(k)$ is the dispersion of the c.m. motion, and $x_1(x_2)$ is the position from the BS along the vertical (horizontal) arm of the interferometer. $|g\rangle$ $(|e\rangle)$ is the internal ground (excited) state of the atom. The photon emitted from the vertical (horizontal) arm is represented by $|\nu_1\rangle$ $(|\nu_2\rangle)$, whereas $|0\rangle$ is the vacuum state. $\langle\nu_1|\nu_2\rangle = 0$ implies that the photon provides sufficient which-path information and one can locate perfectly the atom on one of the two arms. The environment that couples to the c.m. motion and causes the decoherence of the usual type has the state $|e_1\rangle$ $(|e_2\rangle)$ when the atom takes the vertical (horizontal) arm [9,10]. $\langle e_1|e_2\rangle = 0$ means that the c.m. motion is completely incoherent even when the atom keeps the same internal state. The coefficients α, β, and γ are related to the probabilities $P_{\rm ex}$ and P_τ by $1 - P_{\rm ex} = |\alpha|^2$, $P_{\rm ex}(1 - P_\tau) = |\beta|^2$, and $P_{\rm ex}P_\tau = |\gamma|^2$. In Eq. (3), we have assumed that the atom keeps the same shape of the c.m. wave packet before and after the emission of a photon. This is valid when the packet size is already small compared with L and (to avoid the recoil of the atom when emitting a photon) the c.m. momentum $\hbar k$ of the atom is sufficiently larger than that of the photon h/λ. The atom scatters again off the BS to get out of the interferometer and then has the state vector

$$|\Psi(t)\rangle = i\int \frac{dk}{\sqrt{2\pi}} \phi(k) e^{i2kL - i\omega(k)t}\Big[\frac{e^{-ikx_1}}{2}(\alpha|g\rangle + \beta e^{-i\Delta t/\hbar}|e\rangle) \otimes |0\rangle \otimes (|e_1\rangle + |e_2\rangle) + \frac{ie^{-ikx_2}}{2}(\alpha|g\rangle + \beta e^{-i\Delta t/\hbar}|e\rangle) \otimes |e_1\rangle - |e_2\rangle) + \frac{\gamma e^{-ikx_1}}{2}|g\rangle \otimes (|\nu_1\rangle \otimes |e_1\rangle + |\nu_2\rangle \otimes |e_2\rangle) + \frac{i\gamma e^{-ikx_2}}{2}|g\rangle \otimes (\nu_1\rangle \otimes |e_2\rangle - |\nu_2\rangle \otimes |e_2\rangle)\Big]. \tag{4}$$

Therefore, the probability that an atom from reservoir 2 reaches reservoir 1 is given by $P(1 \leftarrow 2) = (|\alpha|^2 + |\beta|^2)(1 + \langle e_1|e_2\rangle)/2 + |\gamma|^2(1 + {\rm Re}\langle e_1|e_2\rangle\langle\nu_1|\nu_2\rangle)/2$, which leads to the net current

$$I_{12} = 0.5P_{\rm ex}P_\tau\,{\rm Re}\{(1 - \langle\nu_1|\nu_2\rangle)\langle e_1|e_2\rangle\}I. \tag{5}$$

Equation (5) shows a sharp contrast between the roles of the two types of decoherence. The decoherence that is due to photon emission and described in effect by $|\nu_j\rangle$ enhances the current, while the usual decoherence process (described by $|e_j\rangle$) due to the coupling to the environment suppresses the current.

At this point, it will be interesting to address the question: Does this spontaneous dc current violate the second law of thermodynamics? Consider four atoms, two from each reservoir. One will end up with (on average) three atoms in reservoir 1 but one in reservoir 2, which corresponds to the decrease of the entropy by log(3/2). However, the increase in entropy induced by the decoherence is enough to compensate this decrease and give a net increase in *total* entropy. To see this, note that the complete decoherence makes the off-diagonal components of the density matrix zeros, which gives rise to the increase of the entropy by log2. Therefore, the net increase in total entropy is $(1/2)\log 2 – (1/4)\log(3/2) \approx \log 1.3$ per atom.

Now we turn to the second example, i.e., the scheme based on the ring geometry; see Fig. 3. This scheme is interesting in the light that the coherent electron transport through the ring has been extensively investigated in condensed matter physics. The basic principle is exactly the same as in the first scheme: The coherent propagation [blue dashed line in Fig. 3(a)] of the atoms from reservoir 1 enables them to reach reservoir 2 with unit probability. The atoms from reservoir 2, on the other hand, are excited before entering the interferometer and experience decoherence within the interferometer; Fig. 3(b). This incoherent propagation reduces the probability for the atoms to reach reservoir 1. Overall, we have a net dc current from reservoir 1 to 2. To estimate the amount of the current, we calculate the transmission probability for the incoherent process using the same formalism as in the Michelson interferometer, except that we replace the scattering matrix in Eq. (2) for the BS with that for the three-terminal junction [11]:

$$J = \begin{pmatrix} -(a+b) & \sqrt{\epsilon} & \sqrt{\epsilon} \\ \sqrt{\epsilon} & a & b \\ \sqrt{\epsilon} & b & a \end{pmatrix}, \tag{6}$$

with $a = (\sqrt{1-2\epsilon} - 1)/2$ and $b = (\sqrt{1-2\epsilon} + 1)/2$. ϵ is the coupling parameter: $\epsilon = 1/2$ gives no reflection when the waves enter the ring from the reservoirs. For $\epsilon = 0$, the

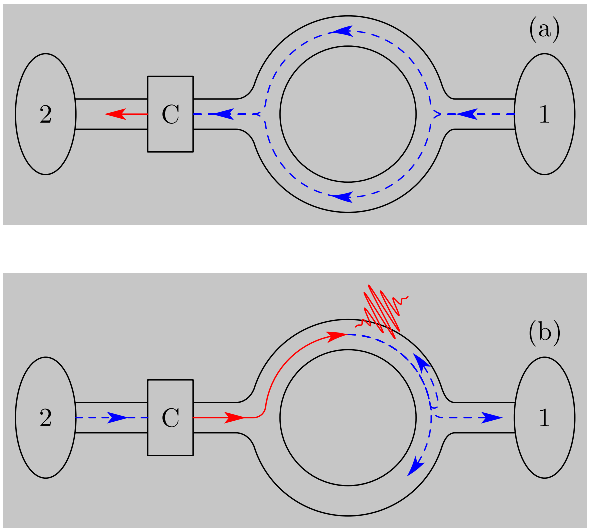

FIG. 3 (color online). The scheme based on a ring geometry. (a) The coherent process: An atom from reservoir 1 experiences a coherent constructive interference and reaches reservoir 2 with unit probability. The cavity does not affect the transmission of this atom. (b) The incoherent process: An atom from reservoir 2 is first excited at the cavity and spontaneously emits a photon passing through the upper arm. The wave starting from the upper arm is scattered at the right junction.

ring and the lead are completely decoupled. For simplicity, let us assume that $\epsilon = 1/2$.

Now suppose the atom emits a photon, say, in the lower arm. The atom (now in the ground state) starts coherent propagation. Simple inspection shows that the final transmission probability is 0.5, and there is no reflection. Because of the conservation of the number of the particles, the remaining half should be trapped in the ring.

Let us consider this striking result more carefully. In Fig. 3(b), the wave starting from the upper arm scatters on the right junction. $\sqrt{1/2}$ of the wave amplitude (probability $1/2$) escapes from the ring, $-1/2$ is reflected back to the upper arm, and $1/2$ is transmitted to the lower. The phase accumulation during the passage of either arm is ignored for the moment. The two transmitted and reflected waves are scattered again on the left junction. Since these two waves are out of phase with the same magnitude, they interfere destructively in the lead attached to reservoir 2. Repeating similar analysis shows that 50% of the wave is trapped in the ring when the spontaneous decay takes place in the ring. In reality, due to various sources of decoherence, the atom should finally escape from the ring to the right or the left reservoir with the same probability. Roughly, 75% of the incoming wave is transmitted and 25% reflected. Compared with the coherent case, 25% of the transmission probability decreases, which results in the net dc current. The trapping probability depends on the geometry of the ring such as ϵ and the lengths of the arms L [12].

In conclusion, we have proposed a new mechanism to generate dc current using a noble interplay between coherence and decoherence. Two specific schemes of implementation have been presented based on the Michelson and the ring interferometers. A coherent superposition of states has more information (or, equivalently, less entropy) than incoherent ones. In some sense, this extra information has been exploited to generate a dc current. Thus, it will be interesting to compare our work with another striking proposal by Scully *et al.* [13], a quantum heat engine operating from a single heat bath prepared in a certain coherent superposition and with a greater efficiency than a classical Carnot engine. The idea presented here may hopefully shed light on deeper understanding of the nature of decoherence and the subtle boundary between classical and quantum physics.

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[1] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997).

[2] R. D. Astumian and P. Hänggi, Phys. Today **55**, No. 11, 33 (2002).

[3] V. I. Fal'ko and D. E. Khmel'nitskii, Sov. Phys. JETP **68**, 186 (1989); B. Spivak *et al.*, Phys. Rev. B **51**, 13 226 (1995).

[4] B. L. Altshuler and L. I. Glazman, Science **283**, 1864 (1999); P. W. Brouwer, Phys. Rev. B **58**, R10 135 (1998).

[5] H. Linke *et al.*, Science **286**, 2314 (1999).

[6] R. Dumke *et al.*, Phys. Rev. Lett. **89**, 220402 (2002); D. Cassettari *et al.*, Phys. Rev. Lett. **85**, 5483 (2000); D. Muller *et al.*, Phys. Rev. A **63**, 041602(R) (2001); W. Hänsel *et al.*, Phys. Rev. Lett. **86**, 608 (2001).

[7] J. R. Taylor, *Scattering Theory: The Quantum Theory on Nonrelativistic Collisions* (Wiley, New York, 1972).

[8] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambride University Press, Cambridge, England, 2000); J. Preskill, Quantum Information and Computation (http://www.theory.caltech.edu/people/preskill/ph229/).

[9] A. Stern, Y. Aharonov, and Y. Imry, Phys. Rev. A **41**, 3436 (1990).

[10] Effective descriptions of decoherence in terms of voltage probes or dephasing stubs are also common in mesoscopic physics; M. Moskalets and M. Büttiker, Phys. Rev. B **64**, 201305 (2001); J. N. H. J. Cremers and P. W. Brouwer, Phys. Rev. B **65**, 115333 (2002); C. W. Beenakker and B. Michaelis (unpublished).

[11] B. Shapiro, Phys. Rev. Lett. **50**, 747 (1983); M. Büttiker, Y. Imry, and M. Y. Azbel, Phys. Rev. A **30**, 1982 (1984).

[12] More precisely, taking into account the finite packet size $P_{\rm trap} = \int dk|\phi(k)|^2(1 - |t|^2 - |r|^2)$, where $t = \sqrt{\epsilon}/[1 - \exp(2ikL)(1 - 2\epsilon)]$ and $r = t\sqrt{1 - 2\epsilon}$.

[13] M. O. Scully *et al.*, Science **299**, 862 (2003).