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Geometric quantum computation on solid-state qubits

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Abstract

Geometric quantum computation is a scheme to use non-Abelian holonomic operations rather than the conventional dynamic operations to manipulate quantum states for quantum information processing. Here we propose a geometric quantum computation scheme which can be realized with current technology on nanoscale Josephson junction networks, known as a promising candidate for solid-state quantum computers.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The elementary units of quantum information processing are quantum two-state systems, called quantum bits or 'qubits'. Unlike a classical bit, a qubit can be in any superposition $\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle$ (with α and β arbitrary complex numbers satisfying the normalization condition $|\alpha|^2 + |\beta|^2 = 1$) of the computational basis states $|\bar{0}\rangle$ and $|\bar{1}\rangle$. A qubit needs not only to preserve quantum coherence for a sufficiently long time but also to allow an adequate degree of controllability. Among a number of ideas proposed so far to realize qubits, the ones based on solid-state devices have attracted interest due to the scalability for massive information processing, which could make a quantum computer of practical value [1].

Another crucial element in quantum information processing is the ability to perform quantum operations on qubits in a controllable way and with sufficient accuracy. In most proposed schemes such quantum operations are unitary, and have conventionally been achieved based on quantum dynamics governed by the Schrödinger equation.

Recently, it has been proposed that controllable quantum operations can be achieved by a novel geometric principle as well [2, 3]. When a quantum system undergoes an adiabatic cyclic evolution, it acquires a nontrivial geometric operation called a holonomy. Holonomy is determined entirely by the geometry of the cyclic path in the parameter space, independent of any detail of the dynamics. If the eigenspace of the Hamiltonian in question is nondegenerate, the holonomy reduces to a simple phase factor, a Berry phase, otherwise it becomes a non-Abelian unitary operation, i.e. a nontrivial rotation in the eigenspace. It has been shown that universal quantum computation is possible by means of holonomies only [2, 3].

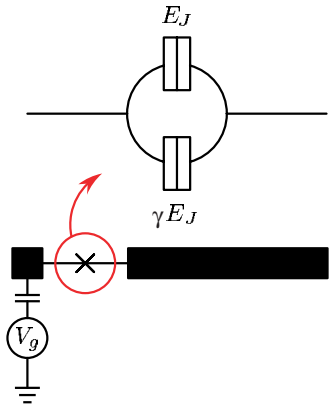

Figure 1. A schematic of a 'Josephson charge qubit'. The cross denotes a *tunable* Josephson junction, which consists of two parallel tunnel junctions (upper panel). The strength of the effective Josephson coupling is tuned by flux threading the loop. The gate voltage V_g controls the induced charge on the box.

Further, holonomic quantum computation schemes have intrinsic tolerance to certain types of computational errors [4, 5].

A critical requirement for holonomies is that the eigenspace should be preserved throughout the adiabatic change of parameters, which is typically fulfilled by symmetry [6]. It is nontrivial to devise a physical system with a proper eigenspace which will serve as a computational space. Recently a scheme for geometric quantum computation with nuclear magnetic resonance was proposed and demonstrated experimentally [7]. A similar scheme was proposed on superconducting nanocircuits [8]. In these schemes, however, only the Abelian Berry phase was geometrically available, and additional dynamic manipulations were required for universal quantum computation. A scheme based solely on holonomies has been proposed for quantum optical systems [9]. However, it relies on nonlinear optics, which may make this scheme less practical. More recently, another holonomic quantum computation scheme has been proposed for trapped ions [10]. Supposedly, it is the only experimentally feasible scheme proposed so far for holonomic computation. Here we propose a scheme for holonomic quantum computation on nanoscale Josephson networks, known to be a promising candidate for solid-state quantum computers [11–13]. It relies on tunable Josephson junctions and capacitive coupling, which are already viable with current technology.

2. Josephson charge qubits

A 'Josephson (charge) qubit' [11, 12] can be realized by a small superconducting grain (a Cooper pair box) of size $\sim$100 nm, coupled to a large superconducting charge reservoir or another Cooper pair box via a Josephson junction (see figure 1). The computational bases are encoded in two consecutive charge states $|\bar{0}\rangle \rightarrow |0\rangle$ and $|\bar{1}\rangle \rightarrow |1\rangle$ with $|n\rangle$ denoting a state with n *excess* Cooper pairs on the box. States with more (or fewer) Cooper pairs are suppressed due to the strong Coulomb repulsion (the gate-induced charge $2en_g$ is tuned close to $1/2 \bmod 1$), characterized by the charging energy $E_C = (2e)^2/2C$ (with C the total capacitance of the box). Excitation of quasiparticles is also ignored assuming a sufficiently low temperature. The tunnelling of Cooper pairs across the junction, characterized by the Josephson coupling energy E_J ($\ll E_C$), allows coherent superposition of the charge states $|0\rangle$ and $|1\rangle$. A *tunable*

junction is attained by two parallel junctions making up a SQUID (superconducting quantum interference device) with a magnetic flux Φ threading through the loop (see upper panel of figure 1) [14]. Namely, in the two-state approximation, the Hamiltonian is written in terms of the Pauli matrices σ^z and $\sigma^{\pm} = \frac{1}{2}(\sigma^x \pm \mathrm{i}\sigma^y)$ as [8, 11]

$$H = -\tfrac{1}{2}h(n_{\mathrm{g}})\sigma^z - \tfrac{1}{2}[J(\phi)\sigma^+ + J^*(\phi)\sigma^-] \tag{1}$$

where $h(n_{\mathrm{g}}) = E_{\mathrm{C}}(2n_{\mathrm{g}} - 1)$, $J(\phi)$ is the effective Josephson coupling of the tunable junction (i.e. SQUID), and $\phi = \pi\Phi/\Phi_0$ (we assume that $|\phi| \leqslant \pi/2$) with $\Phi_0 = hc/2e$ the superconducting flux quantum. Given Josephson energies E_{J} and γE_{J} ($\gamma > 0$) of the two parallel junctions on a SQUID loop, the magnetic flux gives rise to a phase shift $\alpha(\phi)$ as well as an amplitude modulation $A(\phi)$ of the effective Josephson coupling $J(\phi) = 2E_{\mathrm{J}}A(\phi)\mathrm{e}^{-\mathrm{i}\alpha(\phi)}$. $A(\phi)$ and $\alpha(\phi)$ are given by [8]

$$A(\phi) = \sqrt{(1-\gamma)^2/4 + \gamma\cos^2\phi} \tag{2}$$

and

$$\tan\alpha(\phi) = \frac{1-\gamma}{1+\gamma}\tan\phi \tag{3}$$

respectively. It is worth noticing that for identical junctions ($\gamma = 1$), (i) there is no phase modulation [$\alpha(\phi) = 0$] and (ii) the effective Josephson coupling can be turned off completely [$J(\phi) = 0$] at $\phi = \pi/2$. In what follows, some tunable junctions have $\gamma = 1$ and others $\gamma \neq 1$ depending on their roles in the system.

Below we will demonstrate that one can obtain the three unitary operations $U_Z(\varphi) = \exp(\mathrm{i}\varphi|\bar{1}\rangle\langle\bar{1}|)$ (phase shift), $U_X(\varphi) = \exp(\mathrm{i}\varphi\sigma^x)$ (rotation around x axis) and $U_{CZ}(\varphi) = \exp(+\mathrm{i}\varphi|\bar{1}\bar{0}\rangle\langle\bar{1}\bar{0}|)$ (controlled phase shift) on an arbitrary qubit or pair of qubits, using *geometric* manipulations only. It is known that these unitary operations form a universal set of gate operations for quantum computation [10, 15]. Since the charge degrees of freedom is used in the present scheme, the state preparation and the state readout, which are other important procedures required for quantum computation, can be achieved using the same methods used in dynamical schemes [11].

3. Elementary gates

Before demonstrating the geometric implementation of elementary gates, we suggest a prototype Hamiltonian which reveals the proper symmetry for the geometric manipulations in question. All the Hamiltonians considered in this paper share the following common structure:

$$H = \epsilon|\hat{0}\rangle\langle\hat{0}| - \tfrac{1}{2}\sum_{i=1}^{N}(\Omega_i|\hat{i}\rangle\langle\hat{0}| + \Omega_i^*|\hat{0}\rangle\langle\hat{i}|). \tag{4}$$

Here Ω_i is the transition amplitude from the state $|\hat{0}\rangle$ to $|\hat{i}\rangle$ and ϵ is the energy of the state $|\hat{0}\rangle$ measured from the degenerate energy of the states $|\hat{i}\rangle$ ($i = 1, \ldots, N$). For our consideration below, one may regard the state vector $|\hat{i}\rangle$ ($i = 0, 1, \ldots, N$) as representing, for example, an excess Cooper pair on the ith superconducting box (see figure 2(a)). $|\hat{i}\rangle$ may also represent electronic levels in atoms, as discussed in [10].

As one changes the parameters Ω_i, the Hamiltonian in equation (4) preserves the $(N-1)$-dimensional degenerate subspace. This can be clearly seen by defining $|\hat{\Omega}\rangle \equiv \Omega^{-1}\sum_{i=1}^{N}\Omega_i|\hat{i}\rangle$ with $\Omega \equiv \sqrt{|\Omega_1|^2 + \cdots + |\Omega_N|^2}$, and rewriting H as

$$H = \epsilon|\hat{0}\rangle\langle\hat{0}| - \tfrac{1}{2}\Omega(|\hat{\Omega}\rangle\langle\hat{0}| + |\hat{0}\rangle\langle\hat{\Omega}|). \tag{5}$$

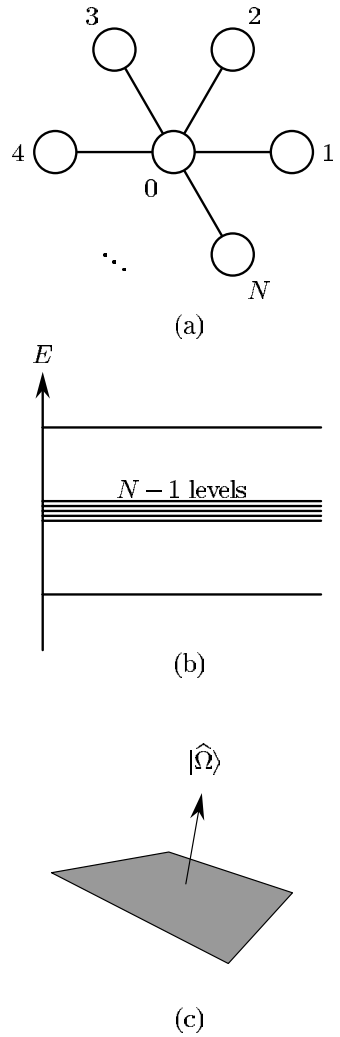

Figure 2. (a) A schematic representation of the prototype model, equation (4), which has a proper degenerate eigenspace for geometric quantum computation. (b) The degenerate level structure of the model. (c) The structure of the corresponding Hilbert space, in which the degenerate subspace is always perpendicular to $|\Omega\rangle$.

The Hamiltonian in equation (5) corresponds to a particle in a (biased) double-well potential in the tight-binding approximation. Therefore, it has two eigenstates $|\lambda_\pm\rangle = \frac{1}{\sqrt{2}}(|\hat{\Omega}\rangle \mp |\hat{0}\rangle)$ with energies $\lambda_\pm = \frac{1}{2}(\epsilon \pm \sqrt{\Omega^2 + \epsilon^2})$. The other $N-1$ levels out of $N+1$ form a degenerate subspace $\mathcal{E}_{N-1}$ with energy 0, which we will use later for a computational subspace (see figure 2(b)). Notice that the degenerate eigenspace $\mathcal{E}_{N-1}$ is always perpendicular both to $|\hat{0}\rangle$ and $|\hat{\Omega}\rangle$; as the parameters Ω_i change, $|\hat{\Omega}\rangle$ rotates in the Hilbert space, and the eigenspace $\mathcal{E}_{N-1}$ is attached *rigidly*, perpendicular to $|\hat{\Omega}\rangle$ (see figure 2(c)).

We first show how to get the unitary operation U_Z geometrically. We consider three Cooper pair boxes coupled in series as shown in figure 3. The first (solid) box encodes the computational bases while the second and third (empty) boxes serve as 'ancilla' qubits.

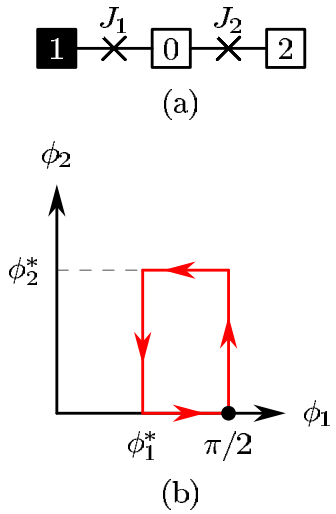

Figure 3. Configuration for the phase shift operation U_Z. Throughout the work, computational bases are encoded in the solid boxes and empty boxes serve as ancilla qubits.

The Hamiltonian is given by (see equation (1))

$$H = -\tfrac{1}{2}\sum_{n=1}^{2}(J_n\sigma_n^+\sigma_0^- + \text{h.c.}) - \tfrac{1}{2}h\sigma_0^z \tag{6}$$

with h.c. meaning the Hermitian conjugate. Comparing equation (6) with the prototype Hamiltonian in (4), the following correspondences are noticed: $|\hat{0}\rangle \to |100\rangle_{012}$ ($|100\rangle_{123}$ is short for $|1\rangle_0|0\rangle_1|0\rangle_2$), $|\hat{1}\rangle \to |010\rangle_{012}$, $|\hat{2}\rangle \to |001\rangle_{012}$, $|\hat{3}\rangle \to |000\rangle_{012}$, $\Omega_1 \to J_1$, $\Omega_2 \to J_2$, and $\Omega_3 = 0$. From this (or direct diagonalization of the Hamiltonian equation (6), of course), one can see that the two states

$$|\lambda_1\rangle = J_2^*|010\rangle_{012} - J_1^*|001\rangle_{012} \tag{7}$$

(not normalized) and

$$|\lambda_2\rangle = |000\rangle_{012} \tag{8}$$

form a degenerate subspace with energy $-h/2$, which is preserved during the change of J_1 and J_2 (equivalently ϕ_1 and ϕ_2). Since the computational basis is only encoded in the 'true' qubit (box 1), the total wavefunction $|\Psi\rangle$ of the logical block should be initially prepared in a separable state with respect to the true qubit and the ancilla qubits, $|\Psi\rangle = |\psi\rangle_1 \otimes |\psi'\rangle_{02}$. In other words, one should be able to turn off at will the *tunable junction* 1, which should therefore be made of identical parallel junctions ($\gamma_1 = 1$) (see equations (2) and (3)). After a cyclic evolution of the parameters ϕ_1 and ϕ_2 along a closed loop starting and ending at the point with $\phi_1 = \pi/2$ (i.e. $J_1 = 0$), the state $|\lambda_1\rangle$ acquires the Berry phase [16]. For example, along the loop depicted in figure 3(b), the Berry phase is given by

$$\varphi_B = \frac{1-\gamma_2^2}{4}\int_0^{\phi_2^*} d\phi_2\left(\frac{1}{\cos^2\phi_1^* + A_2^2(\phi_2)} - \frac{1}{A_2^2(\phi_2)}\right). \tag{9}$$

(The Berry phase vanishes if $\gamma_2 = 1$ as expected [16].) The state $|\lambda_2\rangle$ remains unchanged. Therefore, the cyclic evolution amounts to $U_Z(\varphi_B)$.

Here it should be emphasized that although the *Abelian* Berry phase is used, the degenerate structure is crucial. The dynamic phases acquired by $|\lambda_1\rangle$ and $|\lambda_2\rangle$ are the same and result in a

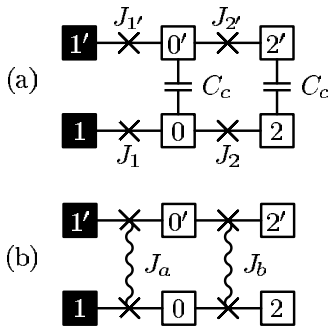

Figure 4. (a) Configuration for the two-qubit gate U_{CZ}. (b) Effective 'joint tunnel' coupling (crossed wiggly lines) resulting from the capacitive coupling in (a).

trivial global phase. In recently proposed schemes [7, 8], which have no degenerate structure, at least one dynamic manipulation was unavoidable to remove the dynamically accumulated phases. Another point to be stressed is that the phase shift $\alpha_2(\phi_2)$ in the effective Josephson coupling $J_2(\phi_2)$ is indispensable for the Berry phase.

The two-qubit gate operation U_{CZ} can be realized geometrically using capacitive coupling. As shown in figure 4(a), the ancilla qubits on different three-box systems are coupled in parallel via capacitors with capacitance C_c. It is known [17–20] that for C_c sufficiently larger than the self-capacitance C of each box, the states $|01\rangle_{00'}$ and $|10\rangle_{00'}$ are strongly favoured over the states $|00\rangle_{00'}$ and $|11\rangle_{00'}$, and the same for boxes 2 and 2′ (recall that $n_g \approx 1/2$ for each box). This effectively leads to 'joint tunnelling' (see figure 4(b)): tunnelling of a charge from box 1 to 0 should be accompanied by tunnelling of another charge from 0′ to 1′. The joint-tunnelling amplitudes are given by $J_a \approx 4J_1J_{1'}^*/E_C$ and $J_b \approx 4J_2J_{2'}^*/E_C$ [17]. Then the total Hamiltonian has the form

$$H = -\tfrac{1}{2}[J_a(\sigma_1^+\sigma_0^-)(\sigma_{1'}^-\sigma_{0'}^+) + \text{h.c.}] - \tfrac{1}{2}[J_b(\sigma_2^+\sigma_0^-)(\sigma_{2'}^-\sigma_{0'}^+) + \text{h.c.}] - \tfrac{1}{2}h[\sigma_0^z - \sigma_{0'}^z]. \tag{10}$$

In analogy with equations (4) and (6), the above Hamiltonian has an eigenspace containing the four degenerate states

$$|\lambda_{00}\rangle = |00\rangle_{11'} \otimes |0101\rangle_{00'22'}, \tag{11}$$

$$|\lambda_{01}\rangle = |01\rangle_{11'} \otimes |0101\rangle_{00'22'}, \tag{12}$$

$$|\lambda_{11}\rangle = |11\rangle_{11'} \otimes |0101\rangle_{00'22'}, \tag{13}$$

and

$$|\lambda_{10}\rangle = J_b^*|10\rangle_{11'} \otimes |0101\rangle_{00'22'} - J_a^*|01\rangle_{11'} \otimes |0110\rangle_{00'22'} \tag{14}$$

(not normalized) with energy $-h$. As in the previous case (see discussions below equation (8)), it is assumed that the tunable junctions J_1 and $J_{1'}$ are composed of identical parallel junctions ($\gamma_1 = \gamma_{1'} = 1$), while $\gamma_2 \neq 1$ and $\gamma_{2'} \neq 1$. One can fix $\phi_{1'} = \phi_{2'} = 0$ and change the parameters ϕ_1 and ϕ_2 along a closed loop starting from the point with $\phi_1 = \pi/2$ ($J_1 = J_a = 0$). Upon this cyclic evolution, the state $|\lambda_{10}\rangle$ acquires the Berry phase in equation (9) while the other three states remain unchanged, leading to the two-qubit gate operation $U_{CZ}(\varphi_B)$. We mention that in this implementation of U_{CZ}, the capacitive coupling is merely an example and can be replaced by any other coupling that effectively results in a sufficiently strong Ising-type interaction of the form $J_{ij}\sigma_i^z\sigma_j^z$.

In the demonstrations of implementing U_Z and U_{CZ} above, we have encoded the bases $|\bar{0}\rangle$ and $|\bar{1}\rangle$ in a single Cooper pair box for simplicity. To realize U_X, we need to encode the

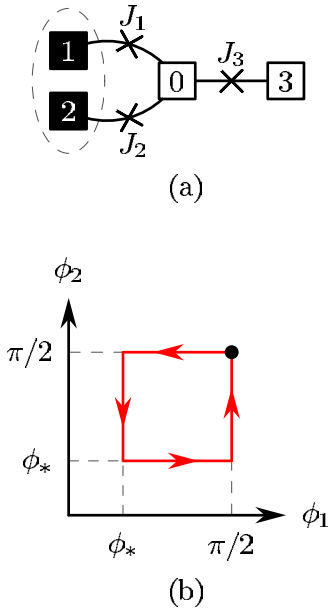

Figure 5. (a) Configuration for a rotation around the x-axis, U_X. The computational basis is encoded over the two (solid) boxes, not in a single box. (b) An adiabatic path in the parameter space to achieve U_X (ϕ_3 is kept constant all along the path).

basis states over two Cooper pair boxes, e.g. box 1 and 2 in figure 5(a): $|\bar{0}\rangle \rightarrow |01\rangle_{12}$ and $|\bar{1}\rangle \rightarrow |10\rangle_{12}$. It is straightforward to generalize the above implementations of U_Z and U_{CZ} in this two-box encoding scheme. Now we turn to the remaining single-qubit gate operation U_X. The Hamiltonian is given by

$$H = -\tfrac{1}{2}\sum_{n=1,2,3}(J_n\sigma_n^+\sigma_0^- + \text{h.c.}) - \tfrac{1}{2}h\sigma_0^z. \tag{15}$$

The degenerate subspace is defined by the two eigenstates

$$|\lambda_1\rangle = (J_2^*|10\rangle_{12} - J_1^*|01\rangle_{12}) \otimes |00\rangle_{03} \tag{16}$$

(not normalized) and

$$|\lambda_2\rangle = \frac{J_3^*}{|J_1|^2 + |J_2|^2}(J_1|10\rangle_{12} + J_2|01\rangle_{12}) \otimes |00\rangle_{03} - |00\rangle_{12} \otimes |01\rangle_{03} \tag{17}$$

(not normalized) both with energy $-h/2$. In this case, it is required that $\gamma_1 = \gamma_2 = 1$ but $\gamma_3 \neq 1$ (see discussions below equation (8)). As an example, we take a closed loop shown in figure 5(b) (one may choose any path starting and ending at $\phi_1 = \phi_2 = \pi/2$, i.e., $J_1 = J_2 = 0$) with ϕ_3 fixed. The adiabatic theory of holonomies [6] ensures that from this adiabatic cycle, a state $|\psi\rangle$ initially belonging to the eigenspace undergoes a change to $|\psi'\rangle = U|\psi\rangle$. The unitary operator U is given by $U = U_Z^\dagger(\varphi')U_X(\varphi)U_Z(\varphi')$ with $\varphi' = \alpha_3(\phi_3)/2 - \pi/4$ and

$$\varphi = 2\int_{\cos\phi_*}^{1} \frac{\mathrm{d}x\ \cos\phi_*}{(x^2 + \cos^2\phi_*)\sqrt{1 + (x^2 + \cos^2\phi_*)/A_3^2(\phi_3)}}. \tag{18}$$

Removing the first and last factors of U_Z (if necessary) with additional phase shift operations, one can achieve U_X.

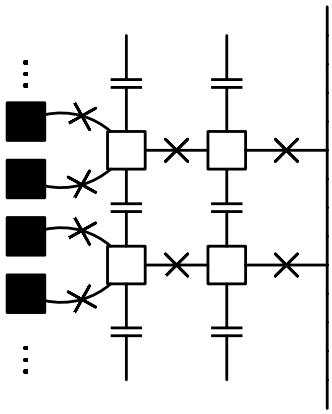

Figure 6. A computational network for universal holonomic quantum computation.

Finally, a quantum network can be constructed as in figure 6 to perform all the unitary operations discussed above (and hence universal quantum computation) geometrically. It is noted that the coupling capacitance C_c is not tunable, but it suffices to have a control over each tunable junction (i.e. SQUID) and gate voltage.

4. Discussion

So far we have discussed fundamental requirements for geometric manipulations in idealized systems. In this section, we discuss several situations one may come up with when attempting an experimental realization of the present scheme.

First of all, the present scheme is based on the adiabatic theorem [21]. Ideally the change in the control parameters should be infinitely slow. At a finite rate of change, there can be a transition out of the computational subspace. However, typically such a Landau–Zener-type transition occurs with an exponentially small probability $P \sim \exp(-\pi\Delta/\eta)$, where $\hbar\Delta$ is the smallest energy gap between the computational subspace and the nearby energy levels and η is the adiabaticity parameter (i.e. $\frac{\partial}{\partial t}H(t) \sim \eta\hbar\Delta$) [22–26]. In Josephson networks the level distance is of the order of the Josephson coupling energy, $\Delta \sim E_J/\hbar \sim (80\ \text{ps})^{-1}$. For an operation time $\tau_{op} = 1/\eta \gtrsim 3/\Delta$, $P \lesssim 10^{-4}$. (For comparison, in a recent experiment concerning dynamic quantum computation on Josephson qubits [12], the switching time was no shorter than $0.25\hbar/E_J \sim 20$ ps.)

In the previous section, on each superconducting box we neglected higher charge states other than the two lowest. In dynamic quantum computation schemes, the existence of those higher levels may cause the quantum leakage errors, i.e. it leads to a nonzero probability of leakage out of the computational space, and more severely to renormalization of the energy levels in the computational space (which therefore reduces the gate fidelity) [27]. In Josephson qubits, however, the coupling to the higher charge states is only through the Josephson tunnelling of Cooper pairs, which can easily be included in our considerations (see equations (4), (6), (10), and (15)). Those higher charge states form well-separated energy levels, and do not alter the degenerate structure of the subspace in question, at least up to the order of $(E_J/E_C)^2$ (in the dynamic scheme the quantum leakage error occurs already at the order of $(E_J/E_C)^2$, see [27]). The leakage to the higher charge states out of the computational subspace can therefore be considered within the framework of Landau–Zener

tunnelling, which has already been discussed above. The renormalization of (degenerate) energy of the computational space is irrelevant in our geometric scheme since it does not rely on the dynamical time-evolution operator but only on the purely geometric means.

In reality there are fluctuations of the (reduced) flux ϕ (tuning the junctions) and the gate-induced charge $2en_g$ (resulting from the fluctuations of random charges in the substrate or gate voltage itself). One consequence of these fluctuations is the Landau–Zener-type transitions out of the computational subspace. A recent experiment on Josephson charge qubits [30] suggests that fluctuations of n_g as well as ϕ are dominated by low-frequency fluctuations. Therefore, the Landau–Zener-type transitions might be small. The fluctuations of n_g can cause another type of error: while the eigenspace is by construction robust against the low-frequency fluctuations of ϕ, the random charge fluctuations lift the degeneracy of the computational subspace. The wavefunction of the system then acquires dynamically accumulated phase factors $\exp(-i\delta E\tau_{op}/\hbar)$, where δE is the small level spacing caused by the fluctuations of n_g. Such dynamical phases can be ignored for sufficiently small fluctuations and sufficiently short—yet long enough for adiabaticity—operation times $(1/\Delta \ll \tau_{op} \ll 1/\delta E)$.

Another common source of decoherence in Josephson charge qubits is the quasi-particle tunnelling [28]. In particular, since the computational eigenspace is not the lowest energy state (equation (4) and figure 2(b)), it gives rise to the relaxation out of the eigenspace to lower energy states (this effect cannot be described by Landau–Zener-type transitions). At sufficiently low temperatures compared with the superconducting gap Δ_S, the quasi-particle tunnelling rate Γ_{qp} is exponentially small [28], $\Gamma_{qp} \sim \exp[-(2\Delta_S + E_C)/k_BT]$. For example, in the experiment on a Cooper pair box [12], $\Gamma_{qp} \sim (6\ \text{ns})^{-1} \sim 10^{-2}E_J/\hbar$ at temperature 30 mK even through the probe junction was biased by a voltage $eV \sim 2\Delta_S + E_C$ (without the voltage bias Γ_{qp} should be even smaller). In such a situation, one can therefore conclude that the effect of quasi-particles is negligible.

For a brief comparison of the present scheme with the conventional Josephson charge qubit [11, 12], we estimate the fidelity for a single phase shift operation U_Z. The fidelity in this case is given by

$$\text{fidelity} \simeq \sqrt{(1-P)[1-\sin^4(\delta\phi/2)]}, \tag{19}$$

where P is the probability of Lanau–Zener-type transitions or quasiparticle tunnellings occurring and $\delta\phi$ is the error in phase shift due to the background charge fluctuations, i.e. $\delta\phi = \delta E\tau_{op}/\hbar$ (see above). Taking $\Delta = (80\ \text{ps})^{-1}$, $\tau_{op} = 3/\Delta$, $\Delta_S = 5\Delta$, $k_BT = \Delta/10$ and $\delta E/\hbar = \Delta/10$ (see [12]), one estimates fidelity $\simeq$0.998. In the dynamic scheme the fidelity is also given by the same form as equation (19). The differences are that P is mainly responsible for the quasiparticle tunnellings and that the phase error $\delta\phi$ comes from the finite ramping time of the gate pulse. Taking the parameters from a recent experiment [12], we see that the fidelity takes the same value (to three decimal places).

Lastly, in the ideal case some tunable junctions (e.g. J_1 in equation (6), see discussions below equation (8)) need to be turned off completely. In reality, the Josephson energies of the two parallel junctions on a SQUID loop (figure 1) may not be identical (i.e. $\gamma \neq 1$ in equation (1) and figure 1). Then, a tunable junction (i.e. a SQUID) cannot be turned off completely [8]. This makes it nontrivial to prepare an initial state which should be a product state of the 'true' qubit and the ancilla qubits in a logical block (see, for example, $|\lambda_1\rangle$ below equation (6)). In practice, such a difficulty can be overcome by means of fast relaxation processes with the gate voltages of the ancilla qubits adjusted far off the resonance in the initial state preparation stage. This process also allows for preparation of the 'true' qubit in a definite initial state [11].

5. Conclusion

We have proposed a scheme based on geometric means to implement quantum computation on solid-state devices. The main advantage of a geometric computation scheme is its intrinsically fault-tolerant feature [4, 5]. However, it is usually nontrivial to find a physical system whose Hamiltonian has a particular degenerate structure for geometric computation. The scheme discussed in this work provides a generic way to construct such a system from arbitrary quantum two-state systems as long as couplings satisfy certain requirements specified above. Such requirements are rather easy to fulfil on solid-state devices. A drawback of this scheme is that it requires more resources (four Cooper pair boxes for each qubit). Considering quantum error-correcting codes, however, it may not be a major disadvantage. Moreover, since the current scheme is based on adiabatic evolution, it does not require sharp pulses of flux and gate voltage. With current technology, it is still challenging to obtain sufficiently sharp pulses of flux and gate voltages (in [12], the rising and falling times were of the order of $\hbar/E_J$). Finite rising and falling times of pulses can result in a significant error in dynamic computation schemes [29].

Acknowledgments

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