

Dissipative dynamics of quantum vortices in superconducting arrays

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We consider a two-dimensional array of ultrasmall superconducting grains, weakly coupled by Josephson junctions with large charging energy. We start from an effective action based on a microscopic tunneling Hamiltonian, which includes quasiparticle degrees of freedom, and study the resulting dissipative dynamics of quantum vortices. The equation of motion for a single vortex is deduced, and compared with a commonly adopted phenomenological model. [S0163-1829(98)03505-X]

Vortex dynamics plays an essential role in understanding the transport properties of superconducting systems in external magnetic fields. For instance, it is closely related to the Hall resistivity¹ and low-temperature magnetic relaxation,² which has drawn much attention in the properties of vortices in two-dimensional (2D) and highly anisotropic three-dimensional superconductors.³ In addition, the dynamics of vortices has been studied extensively in superconducting arrays as well,⁴ for which the recent advancement in fabrication techniques allows one to control the physical quantities determining vortex dynamics, such as the vortex potential, the effective vortex mass, and viscosity. The superconducting arrays, therefore, provide a convenient model system, on which various theoretical predictions can be compared with experimental results. Further, they may also shed light on physics of the high-temperature ceramic superconductors, particularly in the polycrystalline form, which behave in many respects like random arrangements of weak links.

When the dimensions of the superconducting grains and the capacitances involved are small, the associated charging energy is non-negligible, and quantum dynamics of the phase comes into play at the macroscopic level.⁵ In such an array of ultrasmall junctions, the vortices, which are defined on plaquettes of the lattice, should be taken for quantum-mechanical objects. Here it is generally accepted that a vortex on a superconducting array is a rather well-defined point-like object with a finite effective mass,^{6,7} and feels frictional force as well as nondissipative transverse force in its motion, although there has been long-standing controversy as to the actual determination of the latter.^{1,8} On the other hand, a vortex is a macroscopic object by nature, which raises the question regarding how to macroscopically quantize the proposed classical equation of motion for vortices in the presence of frictional force. It is a commonly adopted recipe for quantum dynamics of vortices to assume a frictional force linearly proportional to the vortex velocity and then phenomenologically quantize the resulting equation of motion according to the Caldeira-Leggett procedure.⁹ However, it is not obvious that the friction should depend linearly on the vortex velocity, and the possibility of nonlinear behavior may not be excluded in advance. Indeed recent numerical simulations of the dynamics of an array containing one

single vortex appears to suggest the friction to be a nonlinear function of the vortex velocity.¹⁰ This makes it necessary to investigate the quantum dynamics of vortices based on a model closer to the first principles, and desirable to obtain the effective action for vortices without phenomenological presumption.

This paper presents an attempt toward such a goal: We start from an effective action based on a microscopic model for a 2D array of Josephson junctions consisting of ultrasmall grains. In particular, we consider the case that the charging energy is non-negligible but still smaller than the Josephson coupling energy, so that vortices are well defined, and show how the tunneling of quasiparticles introduces dissipation into the system. Using the dual form of the effective action, which describes the system of dissipative quantum vortices, we obtain the semiclassical equation of motion for a single vortex. We show that the damping on the vortex is in general nonlinear in the vortex velocity and nonlocal in time. However, it turns out that the nonlinear contribution in most cases becomes negligibly small except at very short length scales, and the frictional force in practice can be considered to be linear in the vortex velocity, thus recovering the commonly adopted phenomenological model.

An array of Josephson junctions can be described by the microscopic tunneling Hamiltonian¹¹

$$H = \sum_i H_i + \sum_{\langle i,j \rangle} H_{T,ij} + \sum_{\langle i,j \rangle} H_{C,ij}, \quad (1)$$

where H_i represents the microscopic Hamiltonian, e.g., the BCS reduced Hamiltonian, for the i th grain. The coupling between neighboring islands results from the transfer of electrons through the insulating barrier, described by $H_{T,ij}$, and from the Coulomb interaction $H_{C,ij}$. $H_{T,ij}$ is characterized by the Josephson coupling energy E_J whereas $H_{C,ij}$ is characterized by the junction capacitance C , self-capacitance C_0 , and the charging energy $E_C \equiv e^2/2C$. Integrating out the quasiparticle degrees of freedom leads to a macroscopic model, where the i th grain is characterized by the number n_i of the superconducting electrons (Cooper pairs) and the phase ϕ_i of its superconducting order parameter.¹² The resulting

Ambegaokar-Eckern-Schön (AES) model gives the partition function in the form of a functional integral

$$Z = \sum_{\{n_i(\tau)\}} \int_0^{2\pi} \mathcal{D}[\phi] \exp[-(S_0 + S_D)], \quad (2)$$

where the Euclidean action is given by

$$S_0 = \int_0^\beta d\tau \left[-i \sum_i n_i \dot{\phi}_i + \frac{1}{2K} \sum_{i,j} n_i \tilde{C}_{ij}^{-1} n_j - K \sum_{\langle i,j \rangle} \cos(\phi_{ij} - A_{ij}) \right], \quad (3)$$

$$S_D = \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\langle i,j \rangle} \alpha(\tau - \tau') \left\{ 1 - \cos \left[\frac{\phi_{ij}(\tau) - \phi_{ij}(\tau')}{2} \right] \right\} \quad (4)$$

with $\phi_{ij} \equiv \phi_i - \phi_j$. Here we have rescaled the (imaginary) time in units of $1/\omega_p$, where $\omega_p \equiv \sqrt{8E_C E_J / \hbar^2}$ is the junction plasma frequency, and temperature in units of $\hbar \omega_p$. Further, we have introduced $K \equiv \sqrt{E_J / 8E_C}$ and the dimensionless capacitance matrix

$$\tilde{C}_{ij} = (C_0 / C + 4) \delta_{i,j} - \delta_{i,j-\hat{x}} - \delta_{i,j-\hat{y}} - \delta_{i,j+\hat{x}} - \delta_{i,j+\hat{y}}.$$

The bond angle A_{ij} is given by the line integral of the vector potential due to the external magnetic field: $A_{ij} = (2\pi / \Phi_0) \int_i^j \mathbf{A} \cdot d\mathbf{l}$, so that the plaquette sum gives the flux per plaquette in units of the flux quantum ($\Phi_0 \equiv 2\pi \hbar c / 2e$) or gauge-invariant (magnetic) frustration, $\sum_p A_{ij} = 2\pi f \tilde{\tau}$, where $\tilde{\tau}$ denotes the position of the plaquette. The quasiparticle degrees of freedom are effectively included through the damping kernel $\alpha(\tau)$ in the dissipative part S_D given by Eq. (4). In case that the grains are ideal BCS superconductors with energy gap Δ and normal-state resistance R_N , the damping kernel is given by

$$\alpha(\tau) = \frac{\Delta^2}{R_N} K_1^2(\Delta |\tau|)$$

in the low-temperature limit ($T \rightarrow 0$), where K_1 is the modified Bessel function and we rescaled the gap energy Δ and normal-state resistance R_N by $\Delta / \hbar \omega_p \rightarrow \Delta$ and $R_N / R_0 \rightarrow R_N$, respectively. It should be stressed here that the damping term entirely originates from the intergrain quasiparticle tunneling and includes neither the effects of the Cooper pair decay into the pool of normal electrons nor those of Ohmic shunt between grains.

A variety of properties of Josephson-junction systems have been successfully described by the effective action in Eqs. (3) and (4). We thus employ the AES model as a good starting point for an effective model for the dissipative dynamics of quantum vortices. For this purpose, it is convenient to use the dual transformation, which rewrites the model in terms of the vortex variables instead of the original charge (Cooper pair) variables.¹³⁻¹⁵ In the presence of the quasiparticle dissipation, the resulting effective model reads¹⁶

$$Z = \sum_{\{n^v\}} \int \mathcal{D}[\phi^v] \exp[-(S_0^v + S_D^v)] \quad (5)$$

with

$$S_0^v = \int_0^\beta d\tau \left[-i \sum_i n_i^v \dot{\phi}_i^v + 2\pi^2 K \sum_{i,j} (n_i^v - f_i) G_{ij} (n_j^v - f_j) - \sum_{\langle i,j \rangle} \frac{1}{4\pi^2 K} \cos \phi_{ij}^v \right], \quad (6)$$

$$S_D^v = \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\langle i,j \rangle} \alpha(\tau - \tau') \left\{ 1 - \cos \left[\frac{\theta_{ij}(\tau) - \theta_{ij}(\tau')}{2} \right] \right\}, \quad (7)$$

where i, j now denote the dual lattice sites, i.e., the tilde signs representing the dual lattice sites have been dropped for simplicity. The vortex charge (in units of Φ_0) n_i^v and the vortex phase ϕ_i^v (of the macroscopic vortex wave function) are conjugate to each other, G_{ij} is the lattice Green's function, and $\theta_i \equiv -2\pi \sum_j G_{ij} n_j^v$.

We now investigate the dynamics of a single vortex, focusing on the nature of the dissipation on the vortex. Such a single vortex may actually be introduced by adjusting weak external magnetic field at zero temperature. For a vortex at position $\mathbf{r}(\tau)$, the dissipative action takes the form

$$S_D^v = \int_0^\beta d\tau \int_0^\beta d\tau' \alpha(\tau - \tau') W[\mathbf{r}(\tau) - \mathbf{r}(\tau')], \quad (8)$$

where the nonlocal self-interaction

$$W[\mathbf{r}(\tau) - \mathbf{r}(\tau')] \equiv \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \left[1 - \cos \left(\frac{\theta_{\mathbf{r}\mathbf{r}'}(\tau) - \theta_{\mathbf{r}\mathbf{r}'}(\tau')}{2} \right) \right] \quad (9)$$

can be evaluated to reveal the logarithmic dependence

$$W[\mathbf{r}(\tau) - \mathbf{r}(\tau')] \approx \frac{\pi}{2} \ln \left[\frac{|\mathbf{r}(\tau) - \mathbf{r}(\tau')|}{a} \right], \quad (10)$$

aside from an irrelevant additive constant. The procedures of action minimization and analytic continuation¹² then lead to the semiclassical equation of motion describing the real-time dynamics of a single vortex driven by an applied current I (in units of the Josephson critical current I_J) in the x direction

$$2\pi^2 K \ddot{\mathbf{r}} - 2 \int^t dt' \alpha(t-t') \nabla_{\mathbf{r}} W[\mathbf{r}(t) - \mathbf{r}(t')] = 2\pi K I \hat{\mathbf{u}} \times \hat{\mathbf{z}}. \quad (11)$$

The damping kernel $\alpha(t)$ in the real-time domain may be obtained by the analytic continuation

$$\alpha(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \alpha(i\omega \rightarrow \omega + i0^+). \quad (12)$$

Alternatively, $\alpha(t)$ can also be obtained directly through the use of the real-time formalism,^{9,17} which, for the actions in Eqs. (3,4), gives $\alpha(t)$ related to the quasiparticle tunneling current $I_{qp}(\omega)$ (in units of I_J) via^{12,18}

$$i\alpha(\omega) = K I_{qp}(\omega). \quad (13)$$

Equation (11) is the most general semiclassical equation of motion at length scales larger than the lattice constant. At zero temperature ($T=0$), the quasiparticle tunneling current is simply given by

$$\bar{I}_{\text{qp}}(\omega) = \begin{cases} 0, & |\bar{\omega}| < 2\Delta \\ 2\gamma\omega, & |\bar{\omega}| > 2\Delta, \end{cases} \quad (14)$$

where $\gamma \equiv 1/\omega_p R_N C$. This allows one to write the equation of motion in a more explicit and appealing form. The damping kernel in this case reduces to

$$\alpha(t) = 2\gamma K \frac{d}{dt} [\delta(t) - \Delta \text{sinc}(2\Delta t)], \quad (15)$$

where $\text{sinc}x \equiv (2/\pi x)\sin x$. Here it should be noted that the short-time behavior of Eq. (15) is valid only approximately, because of the high-frequency cutoff in $\alpha(\omega)$. With this simple form of the damping kernel, we finally obtain

$$\begin{aligned} \ddot{\mathbf{r}} + \gamma \dot{\mathbf{r}} - \gamma \frac{\Delta}{\pi^2} \int^t dt' \text{sinc}[2\Delta(t-t')] W''[\mathbf{r}(t) - \mathbf{r}(t')] \dot{\mathbf{r}}(t') \\ = -\frac{1}{\pi} I \hat{\mathbf{y}}, \end{aligned} \quad (16)$$

which, at long time scales, Eq. (16) takes the more explicit form

$$\begin{aligned} \ddot{\mathbf{r}} + \gamma \dot{\mathbf{r}} - \gamma \Delta \int^t dt' \text{sinc}[2\Delta(t-t')] \frac{\dot{\mathbf{r}}(t')}{\pi |\mathbf{r}(t) - \mathbf{r}(t')|^2} \\ = -\frac{1}{\pi} I \hat{\mathbf{y}}. \end{aligned} \quad (17)$$

The semiclassical equation of motion given by Eq. (16) or Eq. (17) possesses two damping terms: One is linear, but the other is nonlinear in the vortex velocity and nonlocal (memory-dependent) in time. When the vortex velocity is large, we have $\dot{\mathbf{r}}(t')/|\mathbf{r}(t) - \mathbf{r}(t')|^2 \sim 1/v(t')$ in Eq. (17), and the nonlinear term becomes sufficiently small compared with the ordinary linear friction term. For small velocities, on the other hand, one must be careful about the short-wavelength cutoff¹⁹ present in the function $W(\mathbf{r})$. (Note that the continuum approximation has been used in the derivation of the equation of motion.) To examine the behavior at low velocities, we have thus numerically integrated the equation of motion, and display the obtained frictional force as a function of the vortex velocity in Fig. 1. It is revealed that the frictional force slightly deviates from the linear behavior at velocities smaller than $v_c \equiv \Delta$. It is of interest to note that v_c may be written in the form $v_c = a\omega_J/\pi$ (in natural unit), which corresponds to vortex hopping by one lattice constant during the characteristic time $1/\omega_J \equiv \hbar/2|e|R_N I_J$ associated with the Josephson oscillation in a resistively shunted Josephson junction.

The above analysis demonstrates that the frictional force on a vortex is *practically* linear in the vortex velocity, in particular in the long-time and long-wavelength scale, where the semiclassical equation of motion is mostly concerned.

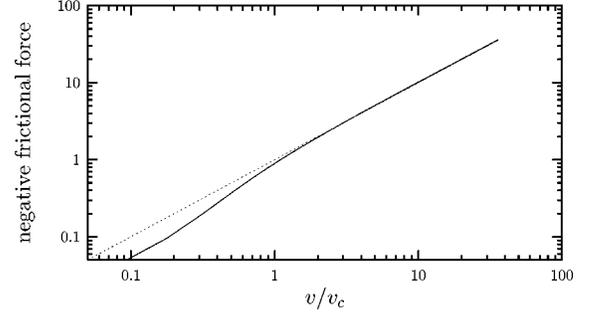


FIG. 1. Behavior of the frictional force (in arbitrary units) as a function of the vortex velocity. The dotted line represents the usual linear frictional force. The logarithmic scale should be noticed. The values of the parameters are $R_N = 10.0$ and $K\Delta = 1/8$.

Neglecting the nonlinear friction and rescaling the time in units of $1/\omega_J$, we have Eq. (17) in the reduced form

$$\pi\beta_c \ddot{\mathbf{r}} + \pi \dot{\mathbf{r}} = -I \hat{\mathbf{y}}, \quad (18)$$

which precisely corresponds to the commonly adopted phenomenological equation of motion describing the resistively and capacitively shunted junction (RCSJ) model with the Stewart-McCumber parameter $\beta_c \equiv \omega_J R_N C$.⁶ This is remarkable in view of the fact that we have considered only quasiparticle tunneling and neither the Ohmic shunt nor any other local damping sources have been included.

In conclusion, we have considered a microscopic model for a two-dimensional array of Josephson junctions, including the quasiparticle degrees of freedom. From the effective action, which has been obtained without any phenomenological presumptions, the semiclassical equation of motion for a single vortex has been deduced. It has been revealed that the quasiparticle tunneling produces friction on the vortex motion. It includes a nonlinear term in addition to the ordinary linear term although the nonlinear friction is in most cases dominated by the latter. At finite temperatures, the quasiparticle tunneling current $I_{\text{qp}}(\omega)$ is smoothed out and approaches the Ohmic behavior [i.e., $I_{\text{qp}}(\omega) \propto \omega$]. In consequence, the nonlinear friction term would become even less pronounced and essentially negligible. Similarly, arrays of Josephson junctions between d -wave superconductors are expected to display essentially linear behavior since d -wave superconductors have nodes in the momentum space at which the energy gap vanishes. This leads to the quasiparticle tunneling current with no sharp threshold in ω , even at $T=0$, separating the high-frequency Ohmic behavior and the low-frequency non-Ohmic behavior.²⁰ It is of interest to compare the nonlinear behavior found in this work with that obtained in Ref. 10. In the latter, numerical integration of the phenomenological RCSJ model has led to effective damping in the vortex motion, which becomes nonlinear as the velocity increases. Here, unlike the existing phenomenological approach, we have started from a microscopic model, and derived explicitly the equation of motion for a vortex in the system. The resulting damping term displays nonlinearity mainly in the low-velocity regime, which is in contrast with that obtained in the phenomenological approach. In addition to the quasiparticle tunneling, spin-wave excitations may

provide another mechanism for the damping of vortex motion.^{21,22} Since the spin-wave excitation requires energy of order of $\hbar\omega_p$ (Ref. 22) for $\hbar\omega_p \geq \Delta$, it is rather irrelevant. Furthermore, in the case of discrete charge states considered here, the region where the spin-wave damping can be ignored gets wider.²¹ Nevertheless for $\hbar\omega_p \ll \Delta$, however, contributions of spin-wave excitations may not be disregarded, and the investigation of the dissipation due to both the quasiparticle and spin-wave excitations will be a challenging topic. Finally, we remark that we have considered only the single vortex motion. In the case of many vortices, the

vortex-vortex interaction becomes crucial, especially for $C_0 \ll C$, which yields the logarithmically long interaction range. The interaction effects, based on the highly suggestive effective action in Eqs. (3) and (4), will be an interesting topic for further study.

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