

Pumps Operating at the Boundary Between the Classical and the Quantum Worlds

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Recently, we proposed a new mechanism to generate a dc current of particles at zero bias based on a noble interplay between coherence and decoherence [S. W. Kim and M-S. Choi, Phys. Rev. Lett. **95**, 226802 (2005)]. We showed that a dc current arises if the transport process in one direction is coherent while the process in the opposite direction is incoherent. Related issues such as the first and the second law of thermodynamics, the law of detailed balances, and the validity of the plane wave description, will be discussed in detail.

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I. INTRODUCTION

Recently, we presented the so-called decoherence-driven quantum transport [1], where a mechanism to generate a dc current of particles at zero bias was proposed based on an interplay between coherence and decoherence. In Ref. [1] we concentrated on presenting the basic working principle and confirming it by showing a simple example. We believe it is worth discussing several issues related to its physical implications and related issues such as the first and the second law of thermodynamics, the law of detailed balances, and the validity of the plane wave description, in a slightly informal way.

II. BASIC PRINCIPLE

First, we briefly summarize the basic principle of decoherence-driven quantum transport. A dc current will arise if the transport process in one direction is coherent while the process in the opposite direction is incoherent. One can easily check the idea by noting that the transmission probability of coherent transport varies with the relative phases of multiple paths, which does not affect incoherent transport. The important question is then how to realize such *spatially* anisotropic, coherent/incoherent transport processes. The scheme

of our implementation is quite general, but for definiteness, here we take a specific example based on the atomic Michelson interferometer.

Let us consider an atomic Michelson interferometer [2–5] and two reservoirs, 1 and 2, of two-level atoms at the ends of the two input/output channels of the interferometer (Fig. 1). The atoms from a reservoir enter the interferometer, experience scattering and/or interference, and are either reflected back to the original reservoir or transmitted to the other reservoir. In addition we have an important component, the microcavity (C) between reservoir 2 and the atomic beam splitter (BS) [2–5]. The cavity is set to be resonant with the level splitting Δ of the two-level atoms so that atoms entering the cavity in the ground state come out of the cavity in an excited state. Therefore, when entering the interferometer, the atoms from reservoir 2 are in an excited state while those from reservoir 1 remain in the ground state. This difference in the energy state between atoms entering the interferometer can cause a significant difference in the coherence of their center-of-mass (CM) motions in the interferometer.

To see this, let $L_\tau = v\tau$, where v is the velocity of the atoms and τ is the lifetime of the excited energy level. Provided that $L_\tau < 2L$, with L being the lengths of the arms (*i.e.*, the paths from the atomic BS to the mirrors) of the interferometer (the lengths of the arms are assumed to be equal), an excited atom in the interferometer will relax back to its ground state by emitting a photon [Fig. 1(b)]. In the ideal case, the photon enables us to locate the atom definitely on one of the two arms

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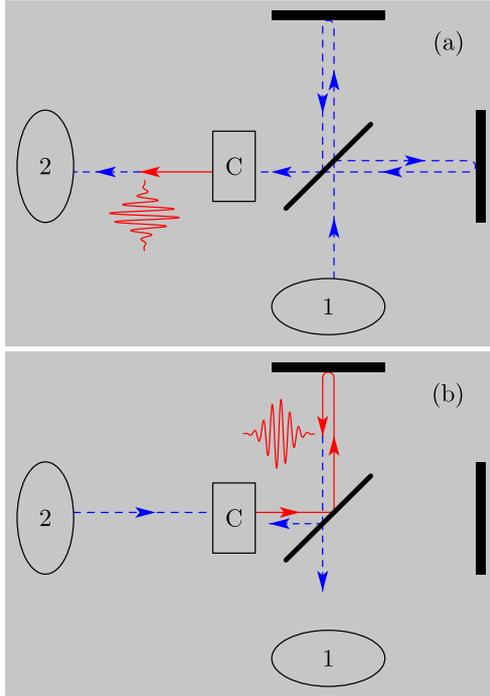


Fig. 1. (color online) Scheme based on the Michelson interferometer. 1 and 2 represent reservoirs. The (blue) dashed and the (red) solid lines represent the trajectories of the ground-state and excited atoms, respectively. The horizontal and the vertical thick lines are mirrors, and the tilted thick lines in the middle an atomic beam splitter. The box with “C” is the microcavity. (a) The coherent process: An atom from reservoir 1 undergoes constructive interference and reaches reservoir 2 with unit probability. The cavity does not affect the transmission of this atom. (b) The incoherent process: An atom from reservoir 2 is excited at the cavity and spontaneously emits a photon within the vertical path. The atom is then transmitted to either reservoir with equal probability 0.5.

of the interferometer. The excited atoms, thus, never experience an interference through the Michelson interferometer. In this sense, the CM motion of the atoms from reservoir 2 is *incoherent*. Furthermore, starting from the just located arm (whichever it is), the atom is transmitted to reservoir 1 with probability 0.5 and reflected back to reservoir 2 with probability 0.5 (we consider a 50:50 BS); see Fig. 1(b).

On the other hand, the atoms from reservoir 1 (ground-state atoms) do not allow such relaxation and will experience coherent interference as long as $L_\phi \gg 2L$, where L_ϕ is the coherence length of the CM motion of *ground-state* atoms; see Fig. 1 (a). Due to constructive interference, an atom from reservoir 1 is perfectly transmitted to reservoir 2; see Fig. 1 (a). Comparing these two transport processes, one can see that 50 % of the incoming atoms contribute to the net dc current. Namely, when the currents from reservoirs 1 and 2 are equal, $I_1 = I_2 = I$, the *net* current from 1 to 2 is given

by

$$I_{12} = I_1 - 0.5I_2 = 0.5I. \quad (1)$$

The maximum current in Eq. (1) can be obtained in the ideal case implied in the above arguments. Several requirements should be satisfied for such an idealistic situation. Otherwise, as we discuss now, the current will be diminished. Firstly, only a fraction of the atoms from reservoir 2 may be excited by the cavity. Then, the current will be reduced to

$$I_{12} = I - 0.5P_{\text{ex}}I - (1 - P_{\text{ex}})I = 0.5P_{\text{ex}}I. \quad (2)$$

Secondly, not all the excited atoms entering the interferometer may relax to the ground state *inside* the interferometer. The probability P_τ for an excited atom to relax to the ground state inside the interferometer is given by

$$P_\tau \approx \int_0^{2L/v} dt e^{-t/\tau}, \quad (3)$$

ignoring the distance between the beam splitter and the cavity. If P_τ is significantly less than 1, then the current in Eq. (1) is reduced to

$$I_{12} = I - 0.5P_\tau I - (1 - P_\tau)I = 0.5P_\tau I. \quad (4)$$

Thirdly, even if the excited atom relaxes inside the interferometer and a photon is emitted, the atom cannot contribute to the net current unless the photon gives information about which path of the interferometer the atom takes. For example, if the wavelength λ of the photon is comparable to or larger than the size of the interferometer L ($\lambda \gtrsim L$), then one cannot get enough information about which path the atom takes, and the CM motion of the atom still remains coherent. Such an effect of insufficient information about the path can be described by assigning the states $|a\rangle$ and $|b\rangle$ with the photon emitted from the vertical and the horizontal arms, respectively. $\langle a|b\rangle = 0$ implies that the which-path information is sufficient, and one can locate perfectly the atom on one of the two arms [6–8]. Finite $\langle a|b\rangle$ will reduce the current in Eq. (1) as

$$I_{12} = 0.5(1 - \text{Re}\langle a|b\rangle)I. \quad (5)$$

We note that the decoherence in our scheme is of a particular type and is clearly distinguished in its role from the *usual* decoherence of the CM motion of the atom when the atom keeps its internal state (either ground or excited state). It is obvious that in our scheme, the *usual* additional decoherence always reduce the net current while it is not necessarily true in quantum pumps. The point here is that the usual decoherence is spatially isotropic while the decoherence of our concern is anisotropic.

The overall effects of these non-ideal situations can then be summarized as

$$I_{12} = 0.5P_{\text{ex}}P_\tau(1 - \text{Re}\langle a|b\rangle)I. \quad (6)$$

Also, to avoid the back reaction of the photon to the atom, the momentum of the CM of the atom p_{CM} must

be sufficiently larger than that of the photon h/λ . A more rigorous mathematical derivation of Eq. (6) can be found in Ref. [1].

III. WAVE PACKET DESCRIPTION

In the above discussions, we described the scattering and the interference of atoms in terms of *plane* waves with a definite momentum within a stationary scattering formalism. Some of readers may doubt the validity of the description because the decoherence process in our scheme should locate the atom within the interferometer, and the subsequent scattering process of the located atom is separated in time from that of the incoming atom. Therefore, one may think that for the incoherent transport process in question, a description in terms of a wave packet within the time-dependent scattering formalism would be required. Indeed, one can think of the incoherent transport process in three steps: For example, in the scheme based on the Michelson interferometer, (i) the incoming wave packet of an excited atom scatters off the BS, (ii) the scattered wave packet undergoes the decoherence process, relaxing to the ground state and emitting a photon, and finally (iii) the packet of the ground-state atom (starting from either arm of the interferometer) scatters off the BS.

It has been well established in scattering theory [9] that since the plane waves form a complete basis as stationary scattering states, the stationary description in terms of the plane waves is equivalent to the time-dependent description for arbitrary scattering states (*e.g.*, Gaussian packets). Furthermore, the decoherence in the CM motion of the atom can be described by the entanglement of the orbital state of the CM motion of the atom and the photonic state [6,7]. Therefore, as long as the scattering matrix is not sensitive to the wave numbers in the range of interest, the description (i)–(iii) is equivalent to that given above, where we took two separate stationary scattering processes of plane waves with the initial conditions of the latter given by a certain *classical* probability (*e.g.*, for a 50:50 BS, 50 % for starting from either arm).

IV. THE FIRST LAW OF THERMODYNAMICS

In our scheme, the energy is, of course, indispensable for exciting the atoms in the microcavity. It should be emphasized, however, that this energy does not contribute directly to generating the dc current of the CM motion of atoms. The energy absorbed by the atom finally returns to the environment via spontaneous emission from which only the which-path information is extracted. In this sense, the energy is not a crucial ingredient for generating a dc current. Even though a tiny

momentum transfer from the emitted photon to the CM of the atom is considered, it can be averaged out because the direction of the spontaneously emitted photon is random. The energy consideration given above, however, raises a subtle problem.

Since the dc current generation of this scheme does not require any energy, it gives rise to *no* density difference arising from the current, which leads us to the conclusion that our pump cannot operate between reservoirs with *finite* sizes. We believe this causes no problem once the reservoir is assumed to be infinitely large, as is usual.

V. THE SECOND LAW OF THERMODYNAMICS

At this point, it will be interesting to address the question: Does this spontaneous dc current violate the second law of thermodynamics? Consider four atoms, two from each reservoir. One will end up with (on average) three atoms in reservoir 1, but one in reservoir 2, which corresponds to a decrease in the entropy by $\log(3/2)$. However, the increase in entropy induced by the decoherence is enough to compensate for this decrease and give a net increase in *total* entropy. To see this, note that complete decoherence makes the off-diagonal components of the density matrix zero, which gives rise to an increase in the entropy by $\log 2$. Therefore, the net increase in total entropy is $(1/2)\log 2 - (1/4)\log(3/2) \approx \log 1.3$ per atom.

In another sense, the entropy increase from decoherence is regarded as that associated with information entropy. This issue has a long history from J. C. Maxwell, the so-called Maxwell's demon (MD) [10]. The MD is a hypothetical being of intelligence, but molecular order of size, imagined to illustrate the limitations of the second law of thermodynamics. Leo Szilard introduced his famous model in which an intelligent being operated a heat engine with a one-molecule working fluid [11]. The Szilard model seems to provide realization of the MD leading to a breakdown of the second law of thermodynamics. It has been proven, however, that in Szilard's engine, the entropy of the environment really increases by erasing the memory on the location of the molecule or by the measurement process [12]. In our pump, the measurement process takes place once the spontaneous decay occurs, which is ascribed to an increase in the entropy by $\log 2$, where "2" obviously originates from the *two* possible classical paths of the atom [13]. It is noted that without the interferometry between the reservoirs, the spontaneous decay itself would not make any entropy available to generate the dc current.

VI. DETAILED BALANCE

Detailed balance tells us that the transmission probability from one side to the other should be equivalent

to that of the opposite direction. To satisfy the law of detailed balance, one needs two assumption: (i) the system should be in equilibrium, and (ii) the system should have a microscopic time-reversal symmetry. For example the so-called Brownian motor [14,15] breaks the equilibrium condition for generating a dc current while the so-called adiabatic quantum pump [16] lacks time-reversal symmetry. We think that our pump satisfies neither of these. The populations of the internal states of the atoms are not described by equilibrium distribution, *i.e.* the Boltzmann factor due to the existence of the Rabi excitation induced by driving in the microcavity. Spontaneous emission obviously breaks time-reversal symmetry. In the viewpoint of detailed balance, it is no wonder that a dc current is generated.

VII. OTHER INTERFEROMETERS

In the above discussion, we have exploited Michelson interferometry to demonstrate a decoherence-driven quantum pump (the ring interferometer was also presented in Ref. [1]). It is worth noting that for the well-known Young and Mach-Zehnder interferometers it is hard to efficiently realize such an idea. For a Young's double-slit interferometer, the intensity of the atomic beam at the center of the screen decreases when which-way information is acquired. However, the total intensity of the atomic beam over the whole screen is still conserved. If all atoms on the screen are collected to the reservoir, a net current cannot be generated. If part of them are chosen, one should lose atoms. For a Mach-Zehnder interferometer an atom finally chooses one of the two exits without backward scattering so that the total transmission probability from one reservoir to another is not changed even when decoherence takes place. If one exit is coupled to the reservoir and the other opens, atoms are also lost.

VIII. SUMMARY

In summary we have proposed a new mechanism to generate a dc current by using a noble interplay between coherence and decoherence. A specific scheme of implementation has been presented based on the Michelson interferometer. The physical implications and the related issues were discussed.

A coherent superposition of states has more information (or equivalently less entropy) than incoherent ones. In some sense, this extra information has been exploited to generate a dc current. Thus, it will be interesting to compare our work with another striking proposal by Scully *et al.* [17], a quantum heat engine operating from a single heat bath prepared in a certain coherent superposition and with a greater efficiency than a classical Carnot engine. The idea presented here may hopefully

shed light on a deeper understanding of the nature of decoherence and on the subtle boundary between classical and quantum physics.

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